The Impact of Oil Shocks in a Small Open Economy
New-Keynesian Dynamic Stochastic General Equilibrium Model for an Oil-Importing Country: The Case of South Africa

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Abstract
This paper studies the effects of foreign (real) oil price shocks on key macroeconomic variables for South Africa: a net-importer of oil. We develop and estimate a small open economy new-Keynesian dynamic stochastic general equilibrium model with a role for oil in consumption and production. The substitutability of oil for capital and consumption goods is low, import price pass-through is incomplete, domestic and foreign prices and wages are sticky, and the uncovered interest rate parity condition holds imperfectly. Foreign real oil price shocks have a strong and persistent effect on domestic production and consumption activities and, hence, are a fundamental driver of output, inflation and interest rates in both the short- and long-run. Oil price shocks also generate a trade-off between output and inflation stabilisation. As a result, episodes of endogenous tightening of monetary policy slow the recovery of South Africa’s real economy. Our findings go further to suggest an important role for oil prices in predicting the South African output during and after the recession that followed the 2008 global financial crisis.

JEL codes: E31, E32, E37, E52, Q41, Q43

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1 Introduction

Following the early works of Rasche and Tatom (1977), Mork and Hall (1980), Hamilton (1983), and Hickman et al. (1987), which investigated the effects of oil shocks on the business cycles in the United States, a large international literature exists that has analyzed the impact of oil price shocks on macroeconomic variables for both developing and developed economies (see for example, Cunado and Perez de Gracia 2003, 2005; Jimenez-Rodriguez and Sanchez 2005; Cologni and Manera 2008, 2009; Baumeister et al. 2010; Sanchez 2011; Gupta and Wohar forthcoming for detailed literature reviews in this regard). Within the set of emerging economies considered, South Africa—an oil importing and inflation targeting small open economy—has featured prominently. A large number of studies have been devoted to analysing the impact of oil shocks on macroeconomic variables of the South African economy (see for example, Dagut 1978; Kantor and Barr 1986; McDonald and van Schoor 2005; Bellamy 2006; Kohler 2006; Nkomo 2006; Swanepoel 2006; Wakeford 2006, 2012; Fofana et al. 2009; Gupta and Hartley 2013; Aye et al. 2014; forthcoming; Balcilar et al. forthcoming; Kin and Courage 2014; Ajmi et al. 2015; de Bruyn et al. 2015; Gupta and Kanda 2015; Tshepo 2015; Chisadza et al. forthcoming; Gupta and Kotze forthcoming). In general, these studies tend to agree to the fact that oil shocks are inflationary for the South African economy. However, the impact of oil shocks on the other variables are exceptionally mixed; ranging from positive to negative, and even neutral (in the statistical sense) depending on the methodology, variables and the sample periods considered.

All these studies rely on macroeconometric models comprising of either (linear or nonlinear) regressions and variations of vector autoregressive (VAR) or vector error-correction (VEC) frameworks. These types of models involve only a few variables and therefore tend to be misspecified (Paetz and Gupta forthcoming), and hence, the results from these studies could be biased and probably differ from the true magnitude of the effects of oil price shocks (Gupta and Sun 2016). In fact, unless a general equilibrium approach is considered, these effects could possibly end up being overestimated (Hou et al. 2016). Further, with these approaches being atheoretical and non-structural, they suffer from the Lucas (1976) critique. Being not microfounded and not grounded in proper theory could also be the reason behind the mixed macroeconomic evidence as reported in the above-discussed South African literature involving oil price shocks. Using a theoretical framework helps identify channels through which oil price affects the economy, quantify its importance, and also provide recommendations for policy-makers, especially central bankers. In addition, recent studies by Paetz and Gupta forthcoming and Gupta and Sun (2016), while analysing the impact of stock and house prices on the South African economy, show that results based on atheoretical
frameworks tend to be way overstated than those obtained under microfounded dynamic stochastic
general equilibrium (DSGE) models.

Against this backdrop, we develop a small open economy New Keynesian DSGE (SOENKDSGE)
model for South Africa with a role for oil in household consumption and firm production activities.
In production, we emphasise capital-oil substitutability (e.g., Kim and Loungani (1992), Rotemberg
than between labour and oil as in Medina and Soto (2005) and Blanchard and Riggi (2013). The
substitutability of oil for physical capital and consumption goods is low. We assume that the law
of one price does not hold for foreign goods and oil so that import price pass-through is incomplete
(Burstein and Gopinath, 2014). The domestic economy follows the standard New Keynesian setup
with nominal price and wage stickiness. Similar to Medina and Soto (2005) and Steinbach et al.
(2009), in a world of complete asset markets we have complete international risk sharing in con-
sumption. Importantly, stochastic risk premiums on both domestic and foreign assets means that
the uncovered interest parity condition holds imperfectly. We estimate the model for the South
African economy over the period 1995:Q2–2015:Q2. The foreign economy macroeconomic data are
aggregated and weighted according to major trading partners. Using this model, we study the role
of foreign (real) oil price shocks on output, inflation, the nominal interest rate and exchange rates.
The relationship between real oil price and South African recessions are shown in Figure 1. As can
be seen, the real oil price tends to be on the rise leading up to each recession, followed by a sharp
drop. The most severe episode occurred over the 2008:Q1–2009:Q3 recession period. Here, a 22%
positive real oil price shock in the first quarter of 2008 likely worsened the downturn.

We also compare alternative models with and without oil to highlight the importance of end-
gogenous oil price and quantity dynamics, as well as the model’s relative forecast performance. To
the best of our knowledge this is the first paper to develop a SOENKDSGE model for South Africa
with an explicit role for oil (energy) usage. In the process, we add to the fast growing international
literature on DSGE models that incorporate oil shocks (Kilian, 2014), and particularly to the small
number of papers that exists on oil shocks in DSGE models for small open oil importing countries
(see for example, Medina and Soto, 2005; An and Kang, 2011; Beidas-Strom and Poghosyan, 2011;
Alba et al., 2013).

The remainder of the paper is organized as follows. Section 2 develops the small open economy
New Keynesian DSGE model wherein oil forms part of the representative household’s consumption
basket and enters as a factor input in firm production. Section 3 discusses the data and calibration of
the model as well as the Bayesian estimation results. Section 4 present results for two alternative
models: a SOENKDSGE model with oil versus one without. In Sub-Section 4.1 we show both the historical and variance decomposition of output, inflation and the nominal interest rate to investigate the importance of oil price shocks. Sub-Section 4.2 compares impulse response functions on key macroeconomic variables. And to provide some additional insight on the merits of the model with oil, Sub-Section 4.3 concludes our comparison analysis with out-of-sample forecasts for output, inflation and the nominal interest rate. Finally, Section 5 concludes.

2 The model

2.1 Domestic households

The domestic economy is populated by a continuum of infinitely lived households, indexed by \( j \in [0, 1] \). Each household \( j \)'s consumption bundle is given by

\[
C_{j,t} = \left[ (1 - \gamma_o) \frac{1}{\eta_o} (Z_{j,t})^{\frac{\eta_o - 1}{\eta_o}} + \frac{1}{\eta_o} (O_{j,t})^{\frac{\eta_o - 1}{\eta_o}} \right]^{\frac{\eta_o}{\eta_o - 1}},
\]

(1)

where the composite consumption index is a constant elasticity of substitution (CES) function consisting of fuel (oil) consumption \( O_{j,t} \) and non-fuel (goods) consumption \( (Z_{j,t}) \). In addition, households consume both domestic and foreign (imported) consumption goods, given by

\[
Z_{j,t} = \left[ (1 - \gamma_c) \frac{1}{\eta_c} (C_{j,t}^h)^{\frac{\eta_c - 1}{\eta_c}} + \frac{1}{\eta_c} (C_{j,t}^f)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}},
\]

(2)
where $C_{j,t}^h$ and $C_{j,t}^f$ represent consumption of domestic and foreign goods. $0 \leq \gamma_c, \gamma_o < 1$ capture the import shares of foreign goods and oil. $\eta_c$ and $\eta_o$ measure the respective intratemporal elasticities of substitution. Each household $j$ chooses her desired combination of oil and core consumption, and domestic and foreign consumption. Minimizing the total cost of each consumption basket subject to Eq. 1 and Eq. 2 gives the demand functions for $Z_{j,t}$, $O_{j,t}^c$, $C_{j,t}^h$, and $C_{j,t}^f$:

$$Z_{j,t} = (1 - \gamma_o)\left(\frac{P_t^z}{P_t}\right)^{-\eta_o} C_{j,t}^h, \quad O_{j,t}^c = \gamma_o \left(\frac{P_t^o}{P_t}\right)^{-\eta_o} C_{j,t}^c$$

$$C_{j,t}^h = (1 - \gamma_c)\left(\frac{P_t^h}{P_t}\right)^{-\eta_c} Z_{j,t}, \quad C_{j,t}^f = \gamma_c \left(\frac{P_t^f}{P_t}\right)^{-\eta_c} Z_{j,t},$$

where $P_t^h$ and $P_t^f$ are the price indices for domestic and foreign goods, and where $P_t^c$ and $P_t^o$ are the price of core consumption goods and the price of oil given by

$$P_t \equiv [(1 - \gamma_o)(P_t^c)^{1-\gamma_o} + \gamma_o(P_t^o)^{1-\gamma_o}]^{1/1-\gamma_o}$$

$$P_t^c \equiv [(1 - \gamma_c)(P_t^h)^{1-\gamma_c} + \gamma_c(P_t^f)^{1-\gamma_c}]^{1/1-\gamma_c}.$$

Household preferences are separable in consumption, labour $N_{j,t}$ and real money balances $M_{j,t}/P_t$, such that each household $j$ maximises their discounted lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta_p^t \left[\frac{(C_{j,t} - \phi C_{t-1})^{1-\sigma_c}}{1 - \sigma_c} - \frac{(N_{j,t})^{1+\sigma_n}}{1 + \sigma_n} + \frac{\alpha}{\sigma_m} \left(\frac{M_{j,t}}{P_t}\right)^{\sigma_m}\right],$$

where $\beta_p^t$ is the subjective discount factor. The coefficient of relative risk aversion $\sigma_c$ measures the curvature of the household’s utility function with respect to its argument $C_{j,t} - \phi C_{t-1}$, where $C_{j,t}$ is real consumption at time $t$ and external habit formation is parameterized by $\phi$. $\sigma_n$ is the elasticity of labor supply ($N_{j,t}$) measured as hours worked. Households derive direct value from the liquidity services of real money holdings ($M_{j,t}/P_t$), where $\sigma_m$ is the interest elasticity of money demand.

Households have access to three types of assets: money $M_{j,t}$, domestic bonds $B_{j,t}$, and foreign bonds $B_{j,t}^f$. Domestic bonds pay a gross nominal rate of return $I_t^b$ in domestic currency, whereas foreign bonds pay an exchange rate adjusted, $\varepsilon_t$, gross nominal rate of return $I_t^{b*f}$. While capital mobility is flexible (i.e., no portfolio adjustment costs) domestic households face a risk premium $\mu_t^{b*f}$ when borrowing abroad. Similarly, the stochastic disturbance term $\mu_t^h$ represents the domestic risk.

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1The households decision problem can be characterised by three stages. (1) each household $j$ minimizes the total cost of its consumption basket, $P_t^c Z_{j,t} + P_t^o O_{j,t}^c$, subject to Eq. 1 where $P_t^c$ and $P_t^o$ are the price of core consumption goods (i.e., the core consumption deflator) and the price of oil. (2) similarly, we minimize the core consumption basket, $P_t^h C_{j,t}^h + P_t^f C_{j,t}^f$, subject to Eq. 2 and (3) we maximise utility subject to the budget constraint.

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5
premium (spread) over the monetary policy rate for domestic asset holdings. The representative household’s budget constraint is as follows:

\[
P^o_{t}O_{j,t}^c + P^h_{t}C^h_{j,t} + P^f_{t}C^f_{j,t} + \frac{B_{j,t}}{P_{t}^b I_{t}^b} \frac{\epsilon_t B^*_{j,t} I_{t}^*}{\mu^*_t} + M_{j,t} = \\
B_{j,t-1} + \epsilon_t B^*_{j,t-1} + M_{j,t-1} + W_{t}N_{j,t} + \Pi_{j,t} + T_{j,t},
\]

where \( W_t \) is the nominal wage set by labour unions, \( \Pi_{j,t} \) are dividends received from domestic firms, and \( T_{j,t} \) represents lump-sum net transfers from government. Given the pricing functions \( Eq. \, 5 \) and \( Eeq. \, 6 \) we can re-write the budget constraint as

\[
P^c_{t}Z_{j,t} \equiv P^h_{t}C^h_{j,t} + P^f_{t}C^f_{j,t}, \\
P^o_{t}O_{j,t}^c \equiv P^o_{t}O_{j,t}^c + P^c_{t}Z_{j,t}.
\]

Households optimize their consumption-savings decision by maximizing \( Eeq. \, 7 \) subject to \( Eeq. \, 9 \). The aggregated first order conditions for domestic and foreign bonds give the standard Euler equations

\[
1 = \beta E_t \left[ \frac{\Lambda_{t+1} P_{t}}{\Lambda_t \mu^*_t} \right], \\
1 = \beta E_t \left[ \frac{\Lambda_{t+1} P_{t} \epsilon_{t+1}}{\Lambda_t \mu^*_t} \right],
\]

where \( \Lambda_t \) is the marginal utility of consumption and the LaGrangian multiplier of the budget constraint.

Similar to Medina and Soto (2005) and Steinbach et al. (2009), complete international asset markets implies complete international consumption risk sharing. Eqs. \( 12 \) and \( 13 \) together give the standard uncovered interest parity (UIP) condition:

\[
(1 + i^b_{t}) = (1 + i^*_{t}) \frac{\epsilon_{t+1}}{\epsilon_t} \Phi_t,
\]

\( ^2 \)This feature allows for a well-defined steady-state for the small open economy (see, e.g., Medina and Soto (2005, p.5), Schmitt-Grohé and Uribe (2003), and Steinbach et al. (2009)).

\( ^3 \)With the assumption of complete markets (i.e., a complete set of contingent claims), the decision problem is identical for all households.
where $\Phi_t = (\mu_t^{bs}/\mu_t^b)$ is the prevailing stochastic risk premium. A positive shock to $\Phi_t$, equivalent to a negative demand shock, raises the return on domestic bonds relative to foreign bonds and reduces current consumption (Smets and Wouters, 2007; Steinbach et al. 2009).

2.2 Labour supply decisions and the wage-setting equation

The wage-setting equilibrium stems from the work of Gali et al. (2007). Monopolistically competitive unions set the optimal wage at the prevailing labour demand equilibrium. There is a continuum of unions, and each union represents workers of a certain type $\tau$. The labour demand schedule that each household type $\tau$ faces is determined by

$$N^\tau_t = \left( \frac{W^\tau_t}{W_t} \right)^{-\varepsilon_w} N_t,$$

where $\varepsilon_w$ is the elasticity of substitution across different types of households.

Following Calvo (1983), in each time period only a random fraction $1 - \theta_w$ of unions have the opportunity to reset their wages ($\tilde{W}_t$), whereas those unions that cannot reset their wages simply index to the lagged wage rate, as in Christiano et al. (2005) and Smets and Wouters (2007).

Therefore, the aggregate wage index is given by:

$$W_1^{1-\varepsilon_w} = \theta_w \left( \frac{P_{t-1}^\tau}{P_{t-2}} \right)^{\gamma_w} W_{t-1}^{1-\varepsilon_w} + (1 - \theta_w) (\tilde{W}_t)^{1-\varepsilon_w}$$

where $\gamma_w$ is the degree of wage indexation. The re-optimizing union’s problem is to therefore choose $\tilde{W}_t$ to maximize the consumption-weighted wage income:

$$\max_{\tilde{W}_t} E_t \sum_{i=0}^{\infty} (\theta_w^\beta)^i \left[ \Gamma_{t+i} \tilde{W}_t N^\tau_{t+i} - \frac{(N^\tau_{t+i})^{1+\sigma_n}}{1 + \sigma_n} \right]$$

subject to the labour demand schedule Eq. 15.

Assuming a constant wage elasticity of substitution, the first-order condition for the optimal reset wage $\tilde{W}_t$ is:

$$E_t \sum_{i=0}^{\infty} (\theta_w^\beta)^i \left[ \frac{\Gamma_{t+i} \tilde{W}_t}{P_{t+i}} \left( \frac{1}{MRS_{t+i}} \right) \right] = E_t \sum_{i=0}^{\infty} (\theta_w^\beta)^i \left[ \mu^w \left( \frac{\tilde{W}_t}{W_{t+i}} \right)^{-\varepsilon_w} \sigma_n \right]$$

where $MRS_{t+i} = -\Lambda N_{t,i}/\Lambda_t = \tilde{C}_{t+i} N^\tau_{t+i}$ is the marginal rate of substitution between consumption and leisure for households, and $\mu^w = \varepsilon_w - 1$ is the steady-state wage markup.

\[^4\text{i.e., when wages cannot be reset: } W_t^* = \Gamma_t W_{t-1}, \text{ where } \Gamma_t = \Pi^w_t = (P_{t-1}/P_{t-2})^\gamma_w.\]

\[^5\tilde{C}_{t+i} = (C_{t+i} - \phi C_{t-1+i})^\gamma.\]
Log-linearizing and solving for $\tilde{w}_t$ gives the optimal reset wage equation:

$$\tilde{w}_t = \frac{(1 - \theta_w \beta)}{(1 + \varepsilon^w \sigma_n)} E_T \sum_{i=0}^{\infty} (\theta_w \beta)^i \left( \chi_m r s_{t+i} + \varepsilon^w \sigma_n w_{t+i} + p_{t+i} - \gamma w \pi_{t+i-1} \right)$$

(19)

where $\chi \equiv \frac{W}{MRS_{w}}$. Combining (19) with the log-linearized wage index equation (16) gives the aggregate sticky real wage ($\hat{w}_t = w_t - p_t$) equation, which we can re-write in nominal wage inflation form as:

$$\hat{\pi}_w t - \gamma \hat{w}_t \hat{\pi}_t - 1 = \beta E_T \hat{\pi}_w t + 1 - \theta_w \beta \gamma \hat{\pi}_t + \Phi^* \left( \chi_{s} \hat{m} r s_{t} + \chi_{b} \hat{m} r s_{t} - \tilde{w}_t \right),$$

(20)

where $\Phi^* = \frac{(1 - \theta_w)(1 - \theta_w \beta)}{\theta_w (1 + \varepsilon^w \sigma_n)}$ and $\hat{m} r s_t = \frac{\sigma_{e}}{1 - \phi} (c_t - \phi c_{t-1}) + \sigma_n n_t$.

### 2.3 Investment and capital goods

The capital goods producing firm chooses a path for investment that maximises the present value of its profits:

$$\max_{K_{t+1}, V_t} E_T \sum_{i=0}^{\infty} \Lambda_{t,t+i} \left[ R_k t_i + P h t_i K_{t+i} - P v t_i V_{t+i} - P h t_i - V_{t+1} - V t_i K_{t+i} \right],$$

(21)

where $R_k t_i$ is the gross (real) return on rented capital holdings, and $\Lambda_{t,t+i}$ denotes the stochastic discount factor for real profits, $i$-periods ahead given by

$$\Lambda_{t,t+i} = \beta^i \left( C_{t+i} / C_t \right)^{-\sigma_e}.$$

(22)

$\phi(\cdot)$ captures the adjustment cost of capital installation.$^6$

The first order conditions for the capital goods producer problem are:

$$Q_t = E_T \left\{ \Lambda_{t,t+1} \left[ R_k t_{t+1} + Q_{t+1} (1 - \delta) + \phi v_{t+1} V_{t+1} - \phi v_{t+1} \right] \right\}$$

(23)

$$Q_t = \frac{P v t_i}{P h t_i} + \phi v_i K_t,$$

(24)

where $\phi v_i = (\kappa_v / \delta)(V_t / K_t - \delta)^i / (1 / K_t)$. As in Adolfson et al. (2005, 2007), we assume that the prices of domestically produced consumption goods and investment goods coincide ($P v t_i = P h t_i$).$^7$

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$^6$ $\phi' > 0, \phi'' < 0, \phi'(\delta) = 0, \phi(\delta) = 0$. Specifically, $\phi(V_t / K_t) K_t = (\kappa_v / 2\delta)(V_t / K_t - \delta)^2 K_t$.

$^7$ In Adolfson et al. (2005, 2007) and Medina and Soto (2007, 2014) there are monopolistically competitive firms in the import and export markets for both investment and consumption goods. When nominal rigidities (as in Smets and Wouters 2002) are zero, exchange rate pass-through to import and export prices is complete.
capital accumulation equation is given by

\[ K_{t+1} = (1 - \delta)K_t + V_t , \]  

(25)

where \( \delta \) measures the depreciation rate of capital, and \( K_t \) represents the physical capital stock at the beginning of period \( t \).

### 2.4 Domestic production

The domestic goods-producing sector is made up of a continuum of infinitely lived firms indexed by \( j \in [0,1] \). Each of these domestic firms combines labour, capital and oil to produce intermediate goods for final good production. Notably, we emphasise the substitutability between capital and oil in the production process (e.g., Kim and Loungani (1992), Rotemberg and Woodford (1996), Backus and Crucini (2000) and Frondel and Schmidt (2002, 2004)) rather than between labour and oil as in Medina and Soto (2005) and Blanchard and Riggi (2013). Capital and oil therefore enter as a CES function within a Cobb-Douglas production function:

\[ Y_{j,t} = A_t(N_{j,t})^\alpha[\vartheta(K_{j,t})^{1-\nu} + (1 - \vartheta)(O_{j,t}^h)^{1-\nu}/(1-\nu)]^{1-\alpha}/(1-\nu), \]

where \( 1/\nu \) captures the elasticity of substitution between physical capital and oil.

The decision problem can be characterised by two stages. First, firm \( j \) minimizes the total cost of production subject to the production constraint. Second, each firm \( j \) maximizes its profit function subject to both foreign and domestic demand. Following Calvo (1983), all firms face a probability \( \theta_H \) of not being able to optimally adjust prices. In this market, final goods producers are monopolistically competitive.

### 2.4.1 Demand for inputs and marginal cost

Each intermediate goods-producing firm \( j \) therefore chooses its factor inputs—labour \( N_{j,t} \), capital \( K_{j,t} \), and oil \( O_{j,t}^h \)—to minimize the total cost of production, taking prices as given:

\[ \min_{\{N_{j,t},K_{j,t},O_{j,t}^h\}} TC_{j,t} + \lambda_t(Y_{j,t}^h - A_t(N_{j,t})^\alpha[\vartheta(K_{j,t})^{1-\nu} + (1 - \vartheta)(O_{j,t}^h)^{1-\nu}/(1-\nu)]^{1-\alpha}/(1-\nu)), \]

(26)
where $TC_{j,t} = \frac{W_t}{P_t} N_{j,t} + R_t^b K_{j,t} + \frac{P_t}{P_t^h} O_{j,t}^h$ is the total real cost of production.\(^8\) The first order efficiency conditions for labour, capital, and oil are:

$$\frac{W_t}{P_t} = \lambda_t \frac{\partial Y_{j,t}^h}{\partial N_{j,t}} = \alpha \lambda_t \frac{Y_{j,t}^h}{N_{j,t}}$$

$$R_t^b = \lambda_t \frac{\partial Y_{j,t}^h}{\partial K_{j,t}} = (1 - \alpha) \partial (1 - \alpha) \lambda_t \left[ \frac{Y_{j,t}^h}{(K_{j,t})^\nu \vartheta (K_{j,t}) \nu + (1 - \vartheta) (O_{j,t}^h) \nu} \right]$$

$$\frac{P_t^o}{P_t} = \lambda_t \frac{\partial Y_{j,t}^h}{\partial O_{j,t}^h} = (1 - \alpha) (1 - \vartheta) \lambda_t \left[ \frac{Y_{j,t}^h}{(O_{j,t}^h)^\nu \vartheta (K_{j,t}) \nu + (1 - \vartheta) (O_{j,t}^h) \nu} \right],$$

where $\lambda_t$ is the real marginal cost of domestic production and the Lagrangian multiplier of the production function.

### 2.4.2 Price setting

Each firm $j$ is monopolistically competitive in its intermediate good $Y_{j,t}^h$. The firm is able to brand and sell its good at a markup $P_t^h$ over marginal cost, taking into account their individual demand curves from domestic and foreign consumers. Here, we assume that both foreign and domestic consumers have identical elasticities with respect to domestic goods. Following Calvo (1983), we assume that only a random fraction $(1 - \theta_H)$ of firms can adjust their retail price in each period. Therefore, each firm $j$ faces the following decision problem:

$$\max E_t \sum_{i=0}^{\infty} \theta_{H,t+i} \left[ \left( \frac{P_{t+i}}{P_{t+i}^h} \right)^{\lambda_{t+i}} - \lambda_{t+i} \right] Y_{j,t+i}^h$$

subject to the consumer demand schedule for goods

$$Y_{j,t+i}^h = \left( \frac{P_{t+i}^h}{P_{t+i}^h} \right)^{-\varepsilon^p} Y_{t+i}^h,$$

where $\Lambda_{t+i} = \beta^i(\Lambda_{t+i}/\Lambda_t)$ is the consumption-based relevant discount factor, and $\varepsilon^p$ is the steady state price-elasticity of demand for intermediate good $Y_t^h$.\(^9\) $P_t^h$ denotes the optimal price set by

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\(^8\)Notice that while households consider total (headline) price adjustments ($\Delta P_t$), firms operate in the domestic goods sector only. That is, they consider the price of domestic goods ($P_t^h$) only, and therefore only indirectly internalize oil and foreign price shocks. Therefore, when derivation is complete, the system of equilibrium conditions (specifically the real price of domestic goods in production) must account for the relative price differences conditioned in each sector. In other words, the real price in the domestic economy is in terms of $P_t$, and therefore the domestic sector must account for the relative price difference $P_t^h / P_t$.

\(^9\)Note that, the above assumption concerning domestic and foreign elasticities implies that $\varepsilon^p$ incorporates both domestic and foreign elasticities of demand with $Y_t^h = C_t^h + C_t^f$, where $C_t^f$ is foreign consumption of domestic goods. However, as we will see below, the foreign demand for domestic goods in the foreign economy has its own demand curve. As such, it serves as an exogenous foreign demand shock with uncertainty (i.e., foreign demand shocks are not substitutable and have complete pass-through to the real economy).
firms who are able to adjust the price in period $t$, and $\lambda_t$ is the real marginal cost of production.

The aggregate price level is determined by

$$
(P^h_t)^{1-\varepsilon^h_t} = \theta_H(\frac{P^h_{t-1}}{P^h_{t-2}})^\gamma^p (P^h_t)^{1-\varepsilon^h_t} + (1-\theta_H)(\tilde{P}^h_t)^{1-\varepsilon^h_t},
$$

(32)

where $\gamma^p$ determines the degree of price indexation for non-optimizing retailers. Solving and linearizing the optimization problem and combining it with Eq. 32 gives the forward-looking New-Keynesian Phillips curve, as in the literature.

### 2.5 Domestic importing retailers and incomplete pass-through

Extensive empirical evidence indicates the tendency for a high degree of pass-through to import prices, whereas the pass-through to domestic prices is more dampened (see [Burstein and Gopinath, 2014](#)). For local importing retailers we therefore introduce incomplete pass-through of exchange-rate movements in the short-run (Calvo-type price setting). Specifically, they are import price takers (given the exchange rate) but face a downward sloping domestic demand curve. The law of one price (l.o.p) gap, Eq. 33 therefore measures deviations from the l.o.p. ([Monacelli, 2003](#)):

$$
\Psi^f_t \equiv \frac{\varepsilon_t P_{f,t}^f}{P^f_t}.
$$

(33)

The domestic demand schedule for foreign good $j$ is given by:

$$
C^f_{j,t} = \left(\frac{P^f_{j,t}}{P^f_t}\right)^{-\varepsilon^f_t} C^f_t,
$$

(34)

where, similar to domestic firms, import firms operate in a Calvo-type stick price environment. Specifically, prices are adjusted with probability $1-\theta_f$ in each period. The aggregate import price index is therefore determined by

$$
(P^f_t)^{1-\varepsilon^f_t} = \theta_F(\frac{P_{t-1}}{P_{t-2}})^\gamma^f (P^f_t)^{1-\varepsilon^f_t} + (1-\theta_F)(\tilde{P}^f_t)^{1-\varepsilon^f_t},
$$

(35)

$P^f_t$ is the price of foreign goods in the domestic currency (or the domestic currency price of imports). If $\Psi^f_t \equiv 1$ then the l.o.p holds with the foreign price of foreign produced goods (imports) traded with the domestic country $P^{f,s}_t$, adjusted for the spot nominal exchange rate $\varepsilon_t$ (i.e., the price of one unit of foreign currency in terms of the domestic currency). For simplicity, we set $\gamma^f$ to zero, which implies that import prices are not indexed to headline inflation. As such, the Calvo price-setting parameter $\theta_F$ governs the degree of import price pass-through of foreign goods.
2.6 Terms of trade and the real exchange rate

The foreign demand for domestic goods is captured by the following demand schedule:

\[ C_{t}^{hs} = \gamma_{c}^{*} \left( \frac{P_{t}^{hs}}{P_{t}^{f}} \right)^{-\xi_{f}^{*}} C_{t}^{*}. \]  (36)

where \( \xi_{f}^{*} \) is the foreign price elasticity of demand for domestic goods. A higher elasticity implies larger changes in foreign demand for domestic goods given the foreign price of domestic goods relative to foreign goods.

The terms of trade of an economy (excl. oil imports) is defined as the price of imports relative to the price of domestically produced goods:

\[ S_{t} = \frac{P_{t}^{f}}{P_{t}^{h}} \]  (37)

This implicitly assumes that that domestic firms cannot price discriminate across markets and that the l.o.p holds for domestic export prices, \( \varepsilon_{t}P_{t}^{hs} = P_{t}^{h} \). Medina and Soto (2005) also note that this assumes that the foreign consumption bundle excludes oil and that the share of domestic goods \( \gamma^{*} \) in \( C_{t}^{*} \) is negligible.

The definition of the real exchange rate can be written as:

\[ RER_{t} = \frac{\varepsilon_{t}P_{t}^{f}}{P_{t}^{*}} \]  (38)

which is the price of foreign headline CPI denominated in domestic currency relative to domestic headline CPI. Given that the l.o.p for oil does not hold (\( \Psi_{o}^{t} \equiv \varepsilon_{t}P_{t}^{os}/P_{t}^{o} \), where \( \Psi_{o}^{t} \) is an AR(1) process), we have the following expression for the domestic real price of oil:

\[ \frac{P_{t}^{o}}{P_{t}} = RER_{t} \frac{P_{t}^{os}}{P_{t}^{f}} \frac{1}{\Psi_{t}^{o}}, \]  (39)

where \( P_{t}^{os} \) is the foreign currency price of oil in the rest of the world. \( P_{t}^{os}/P_{t}^{f} \) and \( \Psi_{t}^{o} \) both follow exogenous stochastic AR(1) processes.

2.7 International risk sharing and the UIP

We can combine the definition of the real exchange rate in Eq. 38 with the UIP condition of Eq. 14 to describe the equation of motion for the relative purchasing power parity condition (i.e., the real
exchange rate):

\[
E_t[RER_{t+1}] = RER_t \frac{(1 + i^b_t)}{(1 + i^{bt}_t)} E_t \left[ \frac{\Pi^{f*}_t}{\Pi_{t+1}} \right] \frac{1}{\Phi_t}.
\]

(40)

Note that this condition holds only under complete asset markets (i.e., international risk sharing in consumption (see also, Steinbach et al., 2009, p. 219)). The evolution of the real exchange rate—a measure of trade competitiveness—is rising in the domestic real interest rate \((I^b_t/\Pi_{t+1})\) and falling in the foreign real interest rate \((I^{f*}_t/\Pi^{f*}_{t+1})\). A positive shock to the prevailing stochastic risk premium \((\Phi_t)\) reduces the real exchange rate of the domestic economy. The domestic short-term nominal interest rate is determined by the following Taylor-type monetary policy reaction function:

\[
I^b_t = (I^b_{t-1})^{\rho_i} \left( \frac{\Pi_t}{\Pi_{target}} \right)^{\kappa_\pi(1-\rho_i)} \left( \frac{Y_t}{Y_{t-1}} \right)^{\kappa_y(1-\rho_i)} \epsilon^i_t,
\]

(41)

where \(\rho_i\) captures the degree of interest rate smoothing, \(\kappa_\pi\) is the weight on inflation, and \(\kappa_y\) is the weight on output growth. \(\epsilon^i_t\) is the i.i.d monetary policy shock.

2.8 Aggregate equilibrium and the foreign sector

In a symmetric equilibrium, all households and firms make identical decisions, so that \(C^c_{jt} = C_t\), \(O^c_{jt} = O^c_t\), \(B^c_{jt} = B_t\), \(B^s_{jt} = B_s^t\), \(Y^h_{jt} = Y^h_t\), \(N_{jt} = N_t\), \(K_{jt} = K_t\), \(O^h_{jt} = O^h_t\), \(P^h_{jt} = P^h_t\), \(P^f_{jt} = P^f_t\) for \(j \in [0,1]\) and \(t = 0,1,2\ldots\). Equilibrium in the domestic goods producing sector therefore requires that

\[
P^h_t Y^h_t = P^h_t C^h_t + \epsilon_t P^{hs}_t C^{hs}_t
\]

\[
\therefore Y^h_t = C^h_t + C^{hs}_t.
\]

(42)

The total value of exports and imports are given by

\[
\frac{P^X_t}{P_t} X_t = \frac{\epsilon_t P^{hs}_t}{P_t} C^{hs}_t = \frac{P^h_t C^{hs}_t}{P_t}.
\]

(43)

\[
\frac{P^M_t}{P_t} M_t = \frac{\epsilon_t P^{fs}_t}{P_t} C^f_t + \epsilon_t P^{os}_t O_t = RER_t C^f_t + \Psi_t P^o_t O_t,
\]

(44)

where \(O_t = O^c_t + O^h_t\) is total oil imports used in consumption and production. The aggregate resource constraint then follows as \(Y_t = C_t + V_t + X_t - M_t\).
We deviate from the Medina and Soto (2005) model, where foreign economy dynamics are captured and characterised by exogenous processes, and rather follow Steinbach et al. (2009, pp.216-7) and assume a large open economy for the foreign market. This allows us to specify the foreign rate $I^*_t$, foreign inflation $\Pi^*_{t+1} = \Pi^*_{t+1}$, and foreign consumption $Y^*_t = C^*_t$ according to the standard 3-equation New-Keynesian model, namely: an IS curve, a Phillips curve, and a Taylor-type policy rate rule. Foreign oil price shocks are assumed to not have a direct affect on the foreign economy.\footnote{See Appendix A for the full linearized system of equilibrium conditions.}

### 2.9 Exogenous shocks

We include 10 exogenous shocks in the model. The two oil shocks are the foreign real price of oil: $\hat{p}^{ro\alpha}_t = \rho_{o\alpha}\hat{p}^{ro\alpha}_{t-1} + \epsilon^{ro\alpha}_t$ and the domestic deviations from l.o.p shock: $\hat{p}^{\epsilon}_t = \rho_{\epsilon}\hat{p}^{\epsilon}_{t-1} + \epsilon^{\epsilon}_t$.\footnote{Distinguishing these two shocks is necessary for the model estimation, and, we believe, helps match reality. Firstly, it alleviates the effect of large and volatile oil price fluctuations (in foreign currency terms) on the adjustment of the real effective exchange rate and the domestic real price of oil (see Eq. A.21). In this sense, the two shocks are able to better capture the pass-through of foreign oil price shocks (captured by $\hat{p}^{ro\alpha}_t$). Secondly, oil enters as an input in the production process directly, and following the aforementioned approach allows large foreign oil price shocks to be dampened by domestic deviations from the l.o.p. (see also, Medina and Soto, 2005).} For the domestic economy, the monetary policy shock ($\epsilon^{i}_t$), as given in Eq. 11 is i.i.d, whereas the domestic technology shock ($\hat{a}_t$) and domestic price markup shock ($\hat{\xi}^{p}_t$) follow AR(1) processes. The foreign economy follows analogously with an i.i.d monetary policy shock ($\epsilon^{i\ast}_t$) and AR(1) processes for the foreign technology shock ($\hat{a}_t^\ast$) and the foreign price markup shock ($\hat{\xi}^{p\ast}_t$). In addition, the risk premium shock for domestic borrowing abroad, equivalent to a negative demand shock, follows: $\hat{\mu}^{b\ast}_t = \rho_{b}\hat{\mu}^{b\ast}_{t-1} + \epsilon^{b\ast}_t$.\footnote{Note: for both the oil and risk premium cases, adding the extra shocks improves the fit (measured from log-marginal data density) and attenuates possible misspecification of the oil price and foreign risk premium shocks—that is, it counteracts overestimation of the contribution of these shocks in the model.} Following the recommendation of Steinbach et al. (2009), and in similar motivation as with the two shocks associated with oil price, we add a risk premium shock on domestic assets relative to the policy rate:\footnote{See the appendix for data and sources.} $\hat{\mu}^{b}_t = \rho_{b}\hat{\mu}^{b}_{t-1} + \epsilon^{b}_t$.

### 3 Model estimation

#### 3.1 Data and calibration

We estimate the model over the sample period 1995:Q2–2015:Q2, with the start and end dates being driven by data availability at the time of writing this paper. The dataset contains 7 observable variables.\footnote{For the domestic economy, South Africa, we have gross domestic product (GDP) per capita, the total consumer price index, and the 3-month treasury rate. The foreign economy macroeconomic data are calculated using a trade-weighted average for the USA, UK, Euro area and}
Japan. Combined, we have the foreign gross domestic product (GDP) per capita, the foreign total consumer price index, and the foreign 3-month Treasury Bill (Government Bond) rate. Finally, we include the foreign relative (real) price of oil: international price of WTI oil deflated by the foreign consumer price index. All data are log-differenced except interest rates—which are in quarterly terms.

Table 1 presents the calibrated parameters. Table 2 shows the corresponding implied steady-state values from the model setup. For households, the share of imported goods in the non-fuel (core) consumption basket is set to 0.27, whereas the import share of oil in consumption is 0.07. Both values correspond to the aggregate South African trade statistics and the implied steady-state values from the model (in Table 2). Following the small open economy models of Faia and Monacelli (2008) and Steinbach et al. (2009), we calibrate the external habit formation parameter $\phi$ to be 0.7 and the elasticity of labour supply parameter $\sigma_n$ to be 3. Similarly, the discount factor $\beta$ equals 0.99. For firms, the share of labour in production is 0.7, whereas the relative share of capital to oil in production is 0.9 (Alba et al., 2013). To ensure a steady-state return on capital of 4%, the rate of physical capital depreciation is set to 0.03. Following Bernanke et al. (1999), we fix the elasticity of the price of capital with respect to the investment-capital ratio ($v_k$) to 0.25. We assume wage contracts are reset, on average, every 4 quarters ($\theta_w = 0.75$) with a moderate degree of price indexation ($\gamma_w = 0.5$). A wage elasticity of substitution of 5 implies a steady-state markup of 25% ($\epsilon_w \cdot \epsilon_{w-1}$). The remaining domestic economy steady-state parameters are calibrated directly from the aggregate data and implied model values.

### 3.2 Prior and posterior parameters

Tables 3 and 4 present the prior and posterior statistics for the estimated parameters. We follow Medina and Soto (2005) and estimate the AR(1) process for the foreign oil price shock separately. Doing so avoids misspecification of the domestic real price of oil (Eq. A.21), and allows us to more accurately estimate stochastic deviations from the law of one price. Similar to Medina and Soto (2005) the estimation of the persistence parameter and exogenous shock for the foreign real price of oil gives $\rho_o = 0.9$ and $\epsilon_o = 0.135$. For the remaining nine stochastic shocks, we set the prior means of the autoregressive coefficients to 0.75, each with a standard deviation of 0.1. The variances of the shocks follow inverse gamma distributions with a prior mean of 0.01.

---

14 The USA, UK, Euro area and Japan make up 67% of total trade over the sample period. From 1994 to 2002 the average was 77.65%; from 2003 to 2009 the average was 70.53%; from 2010 to 2012 the average was 54.83%. The recent drop is due to China’s current 20% share of trade with South Africa. (SARB Quarterly Bulletin, December 2008 and June 2014).

15 Estimation results in Table 6 show an improvement of 13.5 log-marginal points compared to the model in which we simultaneously estimate the foreign real oil price shock (mod.1 versus mod.2).
### Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_c$</td>
<td>Import share of foreign goods in non-fuel goods consumption</td>
<td>0.27</td>
</tr>
<tr>
<td>$\gamma_o$</td>
<td>Import share of oil in consumption</td>
<td>0.07</td>
</tr>
<tr>
<td>$\beta_h$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Habit formation</td>
<td>0.70</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Elasticity of labour supply</td>
<td>3.00</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of labour in firm production</td>
<td>0.70</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>Relative share of capital to oil in production</td>
<td>0.90</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Rate of depreciation</td>
<td>0.03</td>
</tr>
<tr>
<td>$\kappa_v$</td>
<td>Physical capital adjustment costs</td>
<td>0.25</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Sticky wage adjustment</td>
<td>0.75</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>Wage indexation</td>
<td>0.50</td>
</tr>
<tr>
<td>$\epsilon_w$</td>
<td>Wage elasticity of substitution</td>
<td>5.00</td>
</tr>
<tr>
<td>$\phi^*$</td>
<td>Habit formation</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma_n^*$</td>
<td>Elasticity of labour supply</td>
<td>3.00</td>
</tr>
<tr>
<td>$\gamma_p^*$</td>
<td>Price indexation</td>
<td>0.00</td>
</tr>
<tr>
<td>$V/Y$</td>
<td>Investment-output</td>
<td>0.20</td>
</tr>
<tr>
<td>$X/Y$</td>
<td>Export-output</td>
<td>0.28</td>
</tr>
<tr>
<td>$M/Y$</td>
<td>Import-output</td>
<td>0.27</td>
</tr>
<tr>
<td>$O/M$</td>
<td>Import share of fuel to total merchandise imports</td>
<td>0.16</td>
</tr>
<tr>
<td>$O\ell/O$</td>
<td>Household’s consumption share of fuel imports</td>
<td>0.75</td>
</tr>
</tbody>
</table>

### Table 2: Implied steady-state values from the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\beta - (1 - \delta)$</td>
<td>Return on capital</td>
<td>0.040</td>
</tr>
<tr>
<td>$\frac{V}{\delta}$</td>
<td>Capital-output ratio</td>
<td>6.670</td>
</tr>
<tr>
<td>$(1 - \frac{V}{\delta} - \frac{\phi}{\delta} + \frac{\gamma_w}{\delta})$</td>
<td>Total consumption-output ratio</td>
<td>0.795</td>
</tr>
<tr>
<td>$(1 - \frac{\phi}{\delta})$</td>
<td>Domestic consumption-production ratio</td>
<td>0.720</td>
</tr>
<tr>
<td>$(1 - \frac{\gamma_w}{\delta})$</td>
<td>Consumption of foreign goods to total imports</td>
<td>0.840</td>
</tr>
<tr>
<td>$(1 - \frac{\epsilon_w}{\delta})$</td>
<td>Firm’s usage share of fuel imports</td>
<td>0.250</td>
</tr>
</tbody>
</table>
In line with the literature, we assume the substitutability of oil in household consumption and firm production is low (see, e.g., Backus and Crucini 2000). The inverse elasticity of substitution between capital and oil ($\nu$) is set to 2 with a standard deviation of 0.25, close to the values of 1.43 and 1.54 given in Kim and Loungani (1992) and Alba et al. (2013). Following Medina and Soto (2005), we set the elasticity of substitution between oil and consumption ($\eta_o$) to 0.2 with a standard deviation of 0.05. A foreign demand elasticity ($\xi_f$) of 4 falls within the range of estimates (from 1.36 to 4.59) in Adolfson et al. (2005). Given this wide range we choose a standard deviation of 0.5 for $\xi_f$. The prior distributions for domestic prices and the monetary policy rule conform closely to other estimates for the South Africa economy and open economy models in general (see, e.g., Adolfson et al. 2007; Steinbach et al. 2009).

The pass-through of import prices into domestic retail prices tends to be low (Monacelli, 2005; Burstein and Gopinath, 2014). Import prices therefore exhibit higher price stickiness, to which we set $\theta_f$ a prior mean of 0.8 and standard deviation of 0.05. As a result, the price adjustment mechanism required to bring real relative prices into equilibrium falls more heavily on the nominal exchange rate. That is, consistent with small open economies, low pass-through is associated with higher exchange rate variability.

For the foreign economy, we restrict the standard 3-equation New-Keynesian model with zero habit formation and no price indexation. Structural persistence in consumption is therefore governed by the foreign risk aversion coefficient $\sigma^*_c$, to which we set the prior mean to 1 with a standard deviation of 0.2. Similarly, Calvo foreign prices control the degree of price stickiness. The prior mean for $\theta^*$ is set to 0.75 with a standard deviation of 0.1.

The posterior parameter estimates in Tables 3 and 4 are based on standard Bayesian techniques (e.g., Adolfson et al. 2005, 2007). Most of the prior distributions are shown to be robust to the data. Notably, domestic households exhibit a relatively higher degree of risk aversion (2.59) and therefore respond more smoothly to interest rates. A difference of 0.4 between the posterior means of $\eta$ and $\eta_o$ implies that households raise their consumption of domestic goods in response to real exchange rate increases (i.e., an improved competitiveness) and reduce their consumption of domestic goods when relative domestic prices increase (see Eq. A.1). The reverse holds for the consumption of foreign goods in Eq. A.2. The data also predicts a foreign demand elasticity close to 0.4, which is lower than the estimates identified in Adolfson et al. (2007) for the euro area as well as Medina and Soto (2005) for the Chilean economy. In both Adolfson et al. (2007 p.488) and Medina and Soto (2005 p.9-10), however, the foreign economy is identified exogenously by autoregressive processes. In our model, the foreign sector contains endogenous frictions which likely reduce the need for a
Table 3: Structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type</td>
<td>Mean</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>Gamma</td>
<td>0.60</td>
</tr>
<tr>
<td>$\eta_o$</td>
<td>Gamma</td>
<td>0.20</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Gamma</td>
<td>1.00</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>Gamma</td>
<td>1.00</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Gamma</td>
<td>2.00</td>
</tr>
<tr>
<td>$\xi^*$</td>
<td>Gamma</td>
<td>4.00</td>
</tr>
<tr>
<td>Prices</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>Beta</td>
<td>0.60</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>Beta</td>
<td>0.80</td>
</tr>
<tr>
<td>$\theta_f$</td>
<td>Beta</td>
<td>0.75</td>
</tr>
<tr>
<td>Monetary policy rule</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_\pi$</td>
<td>Gamma</td>
<td>1.50</td>
</tr>
<tr>
<td>$\kappa_y$</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>$\kappa^{*}_\pi$</td>
<td>Gamma</td>
<td>1.50</td>
</tr>
<tr>
<td>$\kappa^{*}_y$</td>
<td>Beta</td>
<td>0.50</td>
</tr>
</tbody>
</table>

In order to highlight the role of oil in a small open oil-importing economy, we estimate the model described in Section 2 excluding oil as a factor of production and as a commodity for consumption. The baseline model without oil (no oil hereafter) is obtained by setting the shares of oil in consumption and production to zero ($\gamma_o = 0; \vartheta = 1$). Table 6 compares the posterior parameter estimates for the no oil model to that of the model with oil (oil hereafter). Section 4.3 discusses these results in more detail.

4 Results

4.1 Historical and variance decomposition

Table 5 reports the variance decompositions for the three main variables of interest: output, total (headline) inflation, and the nominal interest rate. The results are shown for 1-quarter, 1-year, 2-
Table 4: Exogenous processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AR coefficients</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{o}^*$ Foreign oil price shock</td>
<td>-</td>
<td>0.90 -</td>
</tr>
<tr>
<td>$\rho_{\psi}$ Oil l.o.p shock</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{a}$ Technology</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{p}$ Price markup</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{i}$ Monetary policy</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{a}^*$ Foreign technology</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{p}^*$ Foreign price markup</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{i}^*$ Foreign monetary policy</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{a}^*$ Foreign technology</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{p}^*$ Foreign price markup</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{i}^*$ Foreign monetary policy</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td>$\rho_{b}^*$ Domestic risk premium</td>
<td>Beta</td>
<td>0.75 0.1</td>
</tr>
<tr>
<td><strong>Standard deviations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{o}^*$ Foreign oil price shock</td>
<td>Inv.Gamma 0.01 inf</td>
<td>0.135 -</td>
</tr>
<tr>
<td>$\epsilon_{\psi}$ Oil l.o.p shock</td>
<td>Inv.Gamma 0.01 inf</td>
<td>0.017 0.010 0.016 0.024</td>
</tr>
<tr>
<td>$\epsilon_{a}$ Technology</td>
<td>Inv.Gamma 0.01 inf</td>
<td>0.007 0.002 0.006 0.014</td>
</tr>
<tr>
<td>$\epsilon_{p}$ Price markup</td>
<td>Inv.Gamma 0.01 inf</td>
<td>0.004 0.003 0.004 0.005</td>
</tr>
<tr>
<td>$\epsilon_{i}$ Monetary policy</td>
<td>Inv.Gamma 0.01 inf</td>
<td>0.002 0.002 0.002 0.003</td>
</tr>
<tr>
<td>$\epsilon_{a}^*$ Foreign technology</td>
<td>Inv.Gamma 0.01 inf</td>
<td>0.006 0.003 0.006 0.009</td>
</tr>
<tr>
<td>$\epsilon_{p}^*$ Foreign price markup</td>
<td>Inv.Gamma 0.01 inf</td>
<td>0.001 0.001 0.001 0.002</td>
</tr>
<tr>
<td>$\epsilon_{i}^*$ Foreign monetary policy</td>
<td>Inv.Gamma 0.01 inf</td>
<td>0.002 0.001 0.002 0.002</td>
</tr>
<tr>
<td>$\epsilon_{b}$ Domestic risk premium</td>
<td>Inv.Gamma 0.01 inf</td>
<td>0.003 0.002 0.003 0.003</td>
</tr>
</tbody>
</table>

year, and 5-year horizons. Columns 2 to 5 report the results for the model with oil (oil hereafter). Columns 6 to 9 report the results for the baseline model without oil (no oil hereafter). Figure 2 shows the results for the historical decompositions of the same three variables over the sample period 1995:Q2–2015:Q2. The purpose of this section is to provide a formal assessment of the contribution of each structural shock to fluctuations in the endogenous variables, firstly, at different horizons, and secondly, at each observation of the actual data. We find that oil price shocks have a significant effect on all three macro variables.

The effect of a foreign real oil price shock ($\epsilon_{o}^*$) on output is strong and persistent across all horizons (between 15.3 and 22.3%). Given the model setup, a decline in the oil price has an important direct real effect on domestic production and consumption activities. The largest contributor to output variation in the short-run, however, is the domestic risk premium shock ($\epsilon_{b}$): 37.4% after 1-quarter and 24.2% after 1-year. The impact of this domestic demand shock declines quickly, contributing 17.3% and 11.2% in the medium and long-run respectively. In addition, the domestic monetary policy shock shows a similar pattern, but at approximately half the magnitude. In contrast, the domestic technology shock contributes 17.4% in the first quarter to almost half the variance of output after 5-years. This long-run versus short-run effect of domestic demand and supply shocks described here follows intuitively from traditional supply and demand dynamics.

---

161-4 quarters captures the short-run, 8 quarters the medium-run, and 20 quarters the long-run.
Notably, other than the oil price, foreign economy shocks have a negligible impact on the domestic real economy. Compared to the no oil model, we find that the technology and risk premium shock are reduced by approximately 20 and 10 percentage points across all horizons, whereas the monetary policy shock increases in the short run by about 5 percentage points.

The contribution of both oil price shocks (the foreign real price and the deviation from l.o.p) have a significant impact on total headline inflation. It is important to note, however, that these two shocks tend work against each other (-0.55 correlation). From Eq. A.21:

\[
\hat{p}_t^o - \hat{p}_t^o\ast = \hat{r}\hat{e}_t + \hat{\psi}_t,
\]

we can see that when deviations from the l.o.p for oil are zero (\(\hat{\psi}_t = 0\)) the difference between the domestic and foreign real price of oil are fully absorbed by the real exchange rate (\(\hat{r}\hat{e}_t\)). Given the contribution of l.o.p deviations for oil, the net effect of the foreign oil price shock is therefore more closer in magnitude to that of the domestic productivity shock and the domestic risk premium shock. That said, it is clear that the impact of the technology shock is significantly reduced compared to the no oil model. Interestingly, monetary policy surprises and price markup shocks are the least important domestic shocks in both models, and the impact of foreign economy shocks are again negligible.

The nominal interest rate is mainly driven by the monetary policy and the domestic risk premium shocks. The net effect of foreign real oil price shocks is smaller but noticeable over all horizons. In summary, it is clear that oil price shocks are a key driver of output, inflation and interest rates in a small open economy. This is further substantiated by the minimal explanatory power of real and nominal foreign economy shocks in describing the domestic business cycle.

The historical decompositions of the same three variables over the sample period 1995:Q2–2015:Q2 are shown in Figure 2. Intuitively, positive oil price shocks should feed through into higher headline inflation and lower output, and vice versa. This effect can be clearly seen over the whole sample period: most notably around the periods of large declines in the foreign real price of oil in 1997/8, 2001, 2008 and 2014. Also, foreign oil price shocks tend to offset technology and risk premium shocks. The recent Great Recession is a case in point, whereby the decline in oil prices dampened the negative impact of technology and risk premium shocks on output and inflation. Turning to the short-term nominal interest rate in the bottom-right of Figure 2, we find a similar relationship between oil price shocks and domestic shocks. Here, oil prices influence interest rates through inflation and real wealth affects on household consumption. Overall, episodes of positive (negative) oil price shocks tend to put upward (downward) pressure on the nominal interest rate, which suggests that oil’s effect on inflation is greater than on household consumption.

17Historical decompositions of the variables and the impulse responses shown in Figures 2 to 5(b) reiterate this point.
Table 5: Forecast error variance decomposition of output, total inflation and nominal interest rate

<table>
<thead>
<tr>
<th>Variance decomposition of output</th>
<th>Oil model: Time Horizons</th>
<th>No oil model: Time Horizons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shocks</td>
<td>1-quarter</td>
<td>1-year</td>
</tr>
<tr>
<td>( \epsilon_o )</td>
<td>15.32</td>
<td>19.83</td>
</tr>
<tr>
<td>( \epsilon_a )</td>
<td>0.77</td>
<td>0.67</td>
</tr>
<tr>
<td>( \epsilon_p )</td>
<td>0.14</td>
<td>0.11</td>
</tr>
<tr>
<td>( \epsilon_i )</td>
<td>0.22</td>
<td>0.11</td>
</tr>
<tr>
<td>( \epsilon_b )</td>
<td>17.35</td>
<td>29.83</td>
</tr>
<tr>
<td>( \epsilon_\psi )</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance decomposition of total inflation</th>
<th>Oil model: Time Horizons</th>
<th>No oil model: Time Horizons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shocks</td>
<td>1-quarter</td>
<td>1-year</td>
</tr>
<tr>
<td>( \epsilon_o )</td>
<td>47.51</td>
<td>43.44</td>
</tr>
<tr>
<td>( \epsilon_a )</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>( \epsilon_p )</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( \epsilon_i )</td>
<td>5.00</td>
<td>9.94</td>
</tr>
<tr>
<td>( \epsilon_b )</td>
<td>0.49</td>
<td>0.76</td>
</tr>
<tr>
<td>( \epsilon_\psi )</td>
<td>1.31</td>
<td>1.66</td>
</tr>
<tr>
<td>( \epsilon_b )</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>( \epsilon_\psi )</td>
<td>3.94</td>
<td>5.87</td>
</tr>
<tr>
<td>( \epsilon_b )</td>
<td>41.66</td>
<td>38.24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance decomposition of nominal interest rate</th>
<th>Oil model: Time Horizons</th>
<th>No oil model: Time Horizons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shocks</td>
<td>1-quarter</td>
<td>1-year</td>
</tr>
<tr>
<td>( \epsilon_o )</td>
<td>36.47</td>
<td>32.94</td>
</tr>
<tr>
<td>( \epsilon_a )</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>( \epsilon_p )</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon_i )</td>
<td>1.36</td>
<td>4.80</td>
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<tr>
<td>( \epsilon_b )</td>
<td>0.11</td>
<td>0.36</td>
</tr>
<tr>
<td>( \epsilon_\psi )</td>
<td>14.09</td>
<td>10.71</td>
</tr>
<tr>
<td>( \epsilon_b )</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( \epsilon_\psi )</td>
<td>14.22</td>
<td>23.18</td>
</tr>
<tr>
<td>( \epsilon_b )</td>
<td>35.73</td>
<td>27.98</td>
</tr>
</tbody>
</table>
Figure 2: Historical decomposition: Output (top-left); Total inflation (bottom-left); Nominal interest rate (bottom-right)
Figure 3: Impulse responses (solid line: oil model; dashed line: no oil model). Figure 2(a): top panel. Figure 2(b): bottom panel.
4.2 Impulse response function analysis: oil versus no oil

Figures 3(a) and 3(b) show the impulse response function results to a domestic technology shock (column 1), a domestic monetary policy shock (column 2), a domestic price markup shock (column 3), and a foreign real oil price shock (column 4). To highlight the role of oil, we compare the responses of the oil model to that of the no oil model. The no oil model is obtained by setting the shares of oil in consumption and production to zero (γo = 0; θ = 1). In the Appendix of the paper, we present the impulse responses for the various shocks for the model with and without oil along with the confidence bands, which in turn, highlights the significance, in general, of the effects of the various shocks.

A positive 13.5% shock to the foreign real price of oil raises oil inflation in the domestic economy by 12.95% (row 4, column 4 in Figure 3(a)). Total inflation rises 1% which implies a pass-through of 7.72%, wherein the second-round effect on domestic inflation accounts for one-fifth of total inflation. The higher real price of oil therefore induces households to reduce their consumption of both oil (by 2.5 percentage points) and domestic goods. Similarly, the demand for oil in domestic production declines 6.7 percentage points (pp) in response to the oil price shock. In aggregate, domestic oil usage falls 3.6 pp (row 4 in Figure 3(b)). Given that foreign goods and physical capital are imperfect substitutes for oil, domestic output declines 2.24% to it’s peak in the fourth quarter. In response to declining output and rising inflation, the monetary authorities raise the short-term nominal interest rate 20 basis points. Oil price shocks therefore generate a trade-off between output and inflation stabilization when the substitutability of oil in consumption and production is low (Montoro 2012; Natal 2012). In our estimated model of the South African economy, the endogenous tightening of monetary policy slows the recovery of the real economy. Compared to the estimated model of Medina and Soto (2005) for the Chilean economy, the responses of inflation, output and the real exchange rate are closely comparable. Although the responses of the policy rate are qualitatively similar, including a risk premium on domestic assets (μb) in our model reduces the emphasis on endogenous tightening of monetary policy (see Table 5).

Under the monetary policy shock (column 2), the difference between the oil model and no oil model is small. A 15 basis points rise in the policy rate reduces output and total inflation by 0.4 and 0.2 pp, compared to 0.35 and 0.12 pp in the no oil model.

Conversely, the model with oil significantly reduces the effect of a domestic price markup shock (column 3), confirming the variance decomposition results in Table 5. Comparing columns 3 and 4 in Figures 3(a) and 3(b) show that both the domestic price markup shock and the oil price shock

18Just like South Africa, Chile too is a small open economy with an inflation targeting monetary policy and a floating exchange rate regime. Both country’s are also net importers of oil.
have qualitatively analogous impacts on nominal and real variables in the domestic economy. As a result, including oil in the model framework highlights the relative importance of oil price shocks in distorting relative prices. It also clearly explains the dampened effect of price markup shocks in the oil model.

A positive technology shock (column 1) raises output by 0.9 pp and reduces total inflation by 0.6% in the no oil model. In response, the policy rate is cut 18 basis points. Compared to the oil model, we see that oil has a strong and persistent effect on output from 2 quarters (peaking at 1.1 pp by the fourth quarter). Initially, domestic real price of oil ($\hat{pr}_t^o$) rises in response to higher household consumption and firm production, offsetting total inflation. As a result, the magnitude of the monetary policy response is approximately halved.

For the open economy variables in Figure 3(b), the competitiveness of the home country improves through a rise (depreciation) in the real effective exchange rate ($REER$). For example, a positive technology shock that reduces unit costs of production leads to domestic goods being relatively cheaper than foreign goods. As a result, the real exchange rate depreciates. In the model economy, international risk sharing in consumption implies that rising domestic consumption relative to foreign consumption must be accompanied by a rising real exchange rate (see Eq. A.5). As such, the results confirm strong co-movement between the real exchange rate and domestic output (e.g., Steinbach et al. 2009). We can also think of nominal effective exchange rate changes ($\Delta NEER$) as the price adjustment mechanism that maintains equilibrium between foreign and domestic goods markets; in relative purchasing power parity (PPP) terms, a change in the real effective exchange rate must equate with changes in $NEER$ plus the foreign-domestic inflation differential (Eq. A.22): $\Delta REER = \Delta NEER + (\pi^* - \pi)$. Row 2 shows the well-known phenomenon of nominal exchange rate overshooting, in that initial changes in $NEER$ tend to be greater than foreign-domestic inflation differentials before returning to relative PPP with $\Delta REER \approx 0$. Specifically, arbitrage in international asset markets requires that the uncovered interest parity (UIP) condition holds (Eq. 14), after which the goods market takes time to clear. Corresponding to an extensive literature, in all four shocks we see strong initial co-movement between real and nominal exchange rates (e.g., Finn 1999; Burstein and Gopinath 2014). The foreign oil price shock mimics that of a domestic price markup shock: a rise in headline inflation relative to the foreign economy leads to an appreciation in the real exchange rate. Similarly, a rise in the nominal interest rate relative to the foreign interest rate induces an initial appreciation in $\Delta NEER$ due to capital inflows. Subsequently, this leads to an expected depreciation in the nominal exchange rate, which satisfies

\[ \hat{\pi}_t^o = \hat{pr}_t^o - \hat{pr}_{t-1}^o + \hat{\pi}_t. \]
the UIP condition.

Given the impulse response results, it is clear that the model with oil is robust to the base-line small open economy model, and that oil in production and consumption are both important determinants of nominal and real variables in the South African economy.

Table 6: Alternative model parameter estimates

<table>
<thead>
<tr>
<th>Posterior Distribution Means</th>
<th>Baseline</th>
<th>mod.1</th>
<th>mod.2</th>
<th>Posterior Distribution Means</th>
<th>Baseline</th>
<th>mod.1</th>
<th>mod.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal density</td>
<td>2007.8</td>
<td>2011.1</td>
<td>1997.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structural Parameters</td>
<td></td>
<td></td>
<td></td>
<td>Shock parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>0.211</td>
<td>0.591</td>
<td>0.658</td>
<td>$\rho_{o*}$</td>
<td>-</td>
<td>-</td>
<td>0.967</td>
</tr>
<tr>
<td>$\eta_o$</td>
<td>-</td>
<td>0.195</td>
<td>0.195</td>
<td>$\rho_o$</td>
<td>-</td>
<td>0.247</td>
<td>0.306</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>3.430</td>
<td>2.591</td>
<td>2.995</td>
<td>$\rho_{c*}$</td>
<td>-</td>
<td>0.695</td>
<td>0.751</td>
</tr>
<tr>
<td>$\sigma_{c*}$</td>
<td>0.945</td>
<td>1.052</td>
<td>0.959</td>
<td>$\rho_{c}$</td>
<td>-</td>
<td>0.755</td>
<td>0.844</td>
</tr>
<tr>
<td>$\nu$</td>
<td>-</td>
<td>2.061</td>
<td>2.013</td>
<td>$\rho_1$</td>
<td>-</td>
<td>0.692</td>
<td>0.757</td>
</tr>
<tr>
<td>$\xi_{f*}$</td>
<td>0.335</td>
<td>0.389</td>
<td>0.387</td>
<td>$\rho_{o*}$</td>
<td>0.928</td>
<td>0.957</td>
<td>0.944</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>0.505</td>
<td>0.621</td>
<td>0.582</td>
<td>$\rho_{t*}$</td>
<td>0.819</td>
<td>0.918</td>
<td>0.836</td>
</tr>
<tr>
<td>$\gamma_{fr}$</td>
<td>0.547</td>
<td>0.593</td>
<td>0.576</td>
<td>$\rho_{r*}$</td>
<td>0.822</td>
<td>0.841</td>
<td>0.836</td>
</tr>
<tr>
<td>$\theta_f$</td>
<td>0.752</td>
<td>0.790</td>
<td>0.763</td>
<td>$\rho_0$</td>
<td>0.867</td>
<td>0.893</td>
<td>0.879</td>
</tr>
<tr>
<td>$\theta^*$</td>
<td>0.633</td>
<td>0.641</td>
<td>0.626</td>
<td>$\epsilon_a*$</td>
<td>-</td>
<td>-</td>
<td>0.136</td>
</tr>
<tr>
<td>$\epsilon_o$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$\epsilon_{c*}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$\epsilon_{c}$</td>
<td>0.011</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>$\epsilon_p^*$</td>
<td>1.468</td>
<td>1.536</td>
<td>1.503</td>
<td>$\epsilon_{c*}$</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>$\epsilon_b$</td>
<td>0.602</td>
<td>0.789</td>
<td>0.784</td>
<td>$\epsilon_{c}$</td>
<td>0.003</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>$\epsilon_{p*}$</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>$\epsilon_{c}$</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$\epsilon_{b*}$</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>$\epsilon_{c}$</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Note: We exclude parameter descriptions, prior means and standard deviations (see Tables 3 and 4), and statistic confidence intervals in the table due to the limited space. Model 2 (mod.2) are the estimates for the model with both the foreign real oil price shock and the deviations from l.o.p for oil shock.

4.3 Comparing the fits of the DSGE model with and without oil

In this subsection, we compare both the in-sample and out-of-sample performances of the DSGE model with and without oil. To assess in-sample fit, as well as robustness, Table 6 compares the posterior parameter estimates for the no oil model to that of the oil model. When measured in terms of log marginal data densities, including oil shows an overall improvement in fit by 3.3 log points. Notably, the change in parameter values for $\eta_c$ and $\sigma_c$ seem to be key for the structural improvement of the oil model: while the majority of structural parameters are robust to the alternative model estimations, excluding oil from the model reduces $\eta_c$, the intratemporal elasticity of substitution between domestic goods and foreign goods, from 0.59 to 0.21, and raises the risk aversion coefficient $\sigma_c$ from 2.59 to 3.43. As a result, the effect of relative price movements on
the consumption of foreign and domestic goods is dampened, as well as the response of aggregate consumption to the real interest rate. Both the intra-temporal and inter-temporal consumption decisions of households are therefore more muted in the no oil model. As noted in Section 3.2, we also report results for the model that estimates both the foreign oil price shock and the stochastic process for deviations from l.o.p (mod.1 versus mod.2 in Table 6). Estimation results show an improvement of 13.5 log points for the oil model. This suggests that following the approach of Medina and Soto (2005) does well to avoid misspecification of the net effect of foreign oil price shocks on the domestic economy, and in doing so improves the in-sample model fit.

Next we turn our attention to a more robust comparison between the models by looking at one- to eight-quarter-ahead out of sample forecasts for output, inflation and interest rate. For our purpose, we use an out-of-sample period of 2008:Q2 to 2015:Q2, over which the DSGE model is estimated recursively to produce the forecasts at various horizons. The choice of the out-of-sample period corresponds to the start in the recent decline of oil prices and also when the South African economy was deep in its latest recession. Table 7 presents the ratio of root mean square errors (RMSEs) from the model with oil relative to the same without oil. Understandably, if the ratio is less than one, the model with oil outperforms the model without it. As can be seen from the Table, the model with oil consistently outperforms the model without it for output at all horizons. The same holds for horizons two and eight for inflation, whereas, for the interest rate, the model with oil does not outperform the DSGE model without it at any horizons. Based on McCracken’s 2007 MSE-F statistic suitable for nested models, we find that the forecasts for output from the oil model are significantly better than those from the no oil version at the one percent level of significance for horizons 1 to 6, and at ten percent level for horizon 7. For inflation, the MSE-F statistic is significant at the five percent level for horizon of two-quarter-ahead.

\footnote{When we used an out-of-sample horizon that started in 2010:Q2, i.e., after the South African economy got out of its recession, results are qualitatively similar for output. But there are gains for inflation from the DSGE model with oil relative to the one without it at horizons one- to four-quarters-ahead, and at horizons of one-quarter-ahead, and then three- to six-quarters-ahead for the interest rate. Complete details of these results are available upon request from the authors.}
Table 7: Relative Root Mean Square Errors (RMSEs): 2008:Q2–2015:Q2

<table>
<thead>
<tr>
<th>Horizon</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.94***</td>
<td>0.90***</td>
<td>0.88***</td>
<td>0.89***</td>
<td>0.94***</td>
<td>0.95***</td>
<td>0.98*</td>
<td>0.99</td>
<td>0.93</td>
</tr>
<tr>
<td>Total (Headline) Inflation</td>
<td>1.04</td>
<td>0.98**</td>
<td>1.02</td>
<td>1.01</td>
<td>1.02</td>
<td>1.01</td>
<td>1.01</td>
<td>0.99</td>
<td>1.01</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>1.10</td>
<td>1.17</td>
<td>1.11</td>
<td>1.05</td>
<td>1.04</td>
<td>1.04</td>
<td>1.05</td>
<td>1.05</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Notes: A RMSE ratio < 1 means that the oil model outperforms the no oil model. ****, ***, and * indicate significance at the 1%, 5% and 10% levels.

5 Concluding remarks

Just like most economies around the world, there is also a huge literature on the role of oil prices in affecting the macroeconomy (and financial market) of South Africa—an oil importing and inflation targeting country. While, these studies generally conclude that the impact of positive oil shocks is inflationary for the South African economy, the evidence is mixed for output, interest rates and exchange rates. This we believe is possibly due to the fact that the South African literature on the effects of oil price is based on atheoretical models and hence, is not robust to choice of variables, models and sample sizes. Given this, in this study, we aim to develop a SOENKDSGE model to provide definitive answers to the impacts of oil shocks on the macroeconomic variables of South Africa as obtained from the theoretical framework.

Upon estimating the SOENKDSGE model using quarterly data over the period of 1995:Q2 to 2015:Q2, we can draw the following conclusions. Foreign real oil price shocks have a strong and persistent effect on domestic production and consumption activities and, hence, are a fundamental driver of output, inflation and interest rates in both the short- and long-run. Oil price shocks also generate a trade-off between output and inflation stabilisation. As a result, episodes of endogenous tightening of monetary policy slow the recovery of South Africa’s real economy. Accounting for oil (energy) demand in firm production and household consumption is therefore crucial for policymakers in oil-importing small open economies. In fact, the historical decomposition results show a clear pattern for oil price shocks on output and inflation, most notably around the periods of large declines in the foreign real price of oil in 1997/8, 2001, 2008 and 2014: lower (higher) oil prices feed through into lower (higher) headline inflation and improved (deteriorated) output conditions. For example, declining oil prices in the recent 2008/9 recession benefited the economy by offsetting adverse demand (risk premium) and supply (technology) shocks. As a result, oil

\[\text{Negative oil price shocks, on the other hand, will tend to lead to an accommodative monetary policy response, a widening output gap and likely excessive asset price imbalances.}\]
prices influence interest rates through inflation, real wealth effects on household consumption and production capacity. Overall, episodes of positive (negative) oil price shocks tend to put upward (downward) pressure on the nominal interest rate. We also find that the SOENKDSGE model with oil significantly improves the out-of-sample forecast for output over the period 2008:Q2–2015:Q2, i.e., during and after the recession that followed the 2008 global financial crisis.

As Kilian (2009) points out, not all oil price fluctuations have the same macroeconomic impacts. For instance, if oil demand and oil prices rise because of strong foreign aggregate demand, worldwide activity expands rather than contracts—as in the case of price increases resulting from foreign oil supply disruptions. Given this, the international dimension of oil trade matters, and the structure of the oil market in DSGE models must be rich enough to identify different kinds of oil demand and supply shocks. While some attempt have been made to enrich the oil-based DSGE models for the US economy (see for example, Peersman and Stevens 2010, Bodenstein and Guerrieri 2011), it would be interesting to incorporate such a structure of the oil market in a small open economy model for South Africa as part of future research.
References


Appendix

A The linearized system

A.1 Aggregate demand

\[ \dot{c}_t^h = \gamma_c(\eta_c - \eta_o)(\dot{r}_t - \dot{\psi}_t^f) - (\gamma_c(\eta_c - \eta_o) + \eta_o)\dot{\rho}_r^h + \dot{c}_t \quad (A.1) \]

\[ \dot{c}_t^f = (1 - \gamma_c)(\eta_c - \eta_o)\dot{\rho}_r^h + (\gamma_c - \eta_c)(\dot{r}_t - \dot{\psi}_t^f) + \dot{c}_t \quad (A.2) \]

\[ \dot{o}_t = -\eta_o\dot{\rho}_o + \dot{c}_t \quad (A.3) \]

\[ \dot{c}_t = \frac{1}{1 - \phi}(\dot{e}_t - \phi^*\dot{e}_{t-1}) - \frac{\sigma_c}{1 - \phi}(\dot{e}_t - \phi\dot{e}_{t-1}) \quad (A.4) \]

\[ \dot{r}_t = 1 - R_k = \frac{1}{\beta} - \frac{1 - \delta}{\beta} \]

Eq[A.1] domestic consumption of home goods; Eq[A.2] domestic consumption of foreign goods; Eq[A.3] domestic consumption of oil; Eq[A.4] Euler eqn; Eq[A.5] is the international risk sharing condition\(^{22}\)

A.1.1 Investment schedule

\[ \dot{v}_t - \dot{k}_t = \beta E_t(\dot{v}_t + 1 - \dot{k}_t + 1) + \beta R_k^k \frac{E_t(r_k^k + 1)}{\kappa_v} + \sigma_c \frac{E_t(c_t - c_{t-1})}{\kappa_v} \]

where \( R_k = 1/\beta - (1 - \delta) \).

A.2 Aggregate supply & inflation

A.2.1 (real) wage setting equation:

\[ \dot{w}_t = \Phi E_t(\dot{w}_t + 1) + \Phi \dot{w}_{t-1} + \Phi \Phi^*(\dot{m}_s - \dot{w}_t) \]

\[ + \Phi E_t(\dot{\pi}_t + 1 - \Phi \theta_w \beta \gamma_w \dot{\pi}_t + \Phi \gamma_w \dot{\pi}_{t-1} \]

The real wage setting equation can be re-written in nominal wage inflation form as:

\[ \dot{\pi}^w_t - \gamma_w \dot{\pi}_{t-1} = \beta E_t \dot{\pi}^w_{t+1} - \theta_w \beta \gamma_w \dot{\pi}_t + \Phi^*(\dot{m}_s - \dot{w}_t), \quad (A.7) \]

where \( \Phi^* = \frac{(1 - \theta_w)(1 - \theta_w \beta)}{\theta_w(1 + \tau^n)} \), \( \Phi = \frac{1}{1 + \phi} \), and \( \dot{m}_s t = \frac{\sigma_c}{1 - \phi}(c_t - \phi c_{t-1}) + \sigma_n c_t \).

\(^{22}\)The UIP condition holds from the Euler equations of the domestic and foreign sectors: \( \dot{e}_t = \dot{e}_t + E_t(\Delta \dot{\pi}_{t+1} + \dot{\mu}_t) \), which implies that the real exchange rate equates the marginal utilities of consumption between the domestic and foreign households.

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A.2.2 Domestic production and inflation (for consumption goods)

\[ \hat{\pi}^h_t = \frac{\gamma_p}{1 + \gamma_p \beta h} \hat{\pi}^h_{t-1} + \frac{\beta h}{1 + \gamma_p \beta h} E_t \hat{\pi}^h_{t+1} + \kappa_h (\hat{mc}^h_t + \xi^h_t) , \]  
(A.8)

where \( \hat{mc}^h_t = \hat{\lambda}_t \) is the real marginal cost of production, and \( \kappa_h = \frac{(1 - \theta h)(1 - \theta f h)}{\theta h} \).

\[ \hat{\lambda}_t = (\hat{w}_t - \hat{p}^h_t) - (\hat{y}^h_t - \hat{n}_t) \]  
(A.9)

\[ \hat{\lambda}_t = \hat{r}^k_t - (\hat{y}^h_t - \nu \hat{k}_t - \hat{y}^h x_t) \]  
(A.10)

\[ \hat{\lambda}_t = (\hat{p}^o_t - \hat{p}^h_t) - (\hat{y}^h_t - \nu \hat{o}_t - \hat{y}^h x_t) , \]  
(A.11)

where \( \hat{y}^h x_t = \hat{\psi}^f_t \) measures the l.o.p gap:

\[ \hat{\psi}^f_t = \hat{\epsilon}_t + \hat{p}^f_t - \hat{p}_t \]  
(A.14)

A.2.3 Imported inflation (for foreign consumption goods)

\[ \hat{\pi}^f_t = \beta f E_t [\hat{\pi}^f_{t+1}] + \kappa f \hat{\psi}^f_t , \]  
(A.13)

where \( \kappa f = \frac{(1 - \theta f)(1 - \theta f f)}{\theta f} \), and \( \hat{\psi}^f_t \) measures the l.o.p gap:

\[ \hat{\psi}^f_t = \hat{\epsilon}_t + \hat{p}^f - \hat{p}^f_t \]  
(A.14)

A.2.4 Inflation aggregation equations

From the inflation aggregation equations we have:

\[ \hat{\pi}^z_t = (1 - \gamma_c) \hat{\pi}^h_t + \gamma_c \hat{\pi}^f_t \]  
(A.15)

\[ \hat{\pi}_t = (1 - \gamma_o) \hat{\pi}^z_t + \gamma_o \hat{\pi}^o_t \]  
(A.16)

\[ \therefore \hat{\pi}_t = (1 - \gamma_o)(1 - \gamma_c) \hat{\pi}^h_t + (1 - \gamma_o) \gamma_c \hat{\pi}^f_t + \gamma_o \hat{\pi}^o_t . \]  
(A.16)

Eq. A.15 and Eq. A.16 can be re-written as (see Medina and Soto 2005)

\[ \hat{\pi}^z_t = \hat{\pi}_t - \frac{\gamma_o}{1 - \gamma_o} (\hat{p}^o_t - \hat{p}^o_{t-1}) \]  
(A.17)

\[ 0 = \gamma_o \hat{p}^o_t + (1 - \gamma_o)(1 - \gamma_c) \hat{p}^h_t + (1 - \gamma_o) \gamma_c (\hat{p}^f_t) . \]  
(A.18)

\[ \hat{\epsilon}_t + \hat{p}^f_t - \hat{p}_t \]  
and \( \hat{p}^f_t = \hat{p}^f_t - \hat{p}_t . \)
A.2.5 Evolution of relative prices

\[
\hat{p}_t^h = \hat{p}_t^h - \hat{p}_{t-1}^h + \hat{\pi}_t,
\]
(A.19)

\[
\hat{p}_t^f = \hat{p}_t^f - \hat{p}_{t-1}^f + \hat{\pi}_t,
\]
(A.20)

\[
\hat{p}_t^o = \hat{r}_e + \hat{\pi}_t - \hat{\pi}^o_t + \hat{\psi}_t^o,
\]
(A.21)

\[
r_e = r_e - \Delta \hat{\xi}_t + \hat{\pi}_t^f - \hat{\pi}_t,
\]
(A.22)

\[
\hat{\pi}_t^o = \hat{p}_t^o - \hat{p}_{t-1}^o + \hat{\pi}_t,
\]
(A.23)

\[
\hat{s}_t = \hat{p}_t^f - \hat{p}_t^h,
\]
(A.24)

\[
\hat{w}_t = \hat{w}_{t-1} + \hat{\pi}_t - \hat{\pi}_t,
\]
(A.25)

where \(\hat{p}_t^o\) (the relative (real) foreign price of oil, \(\hat{p}_t^o - \hat{p}_t^f\)) and \(\hat{\psi}_t^o\) (deviations from l.o.p on relative (real) domestic price of oil, \(\hat{p}_t^o\)), are AR(1) processes.\(^{24}\) Eq. (A.22) is the equation of motion for the relative purchasing power parity condition.\(^{25}\) Here, we can think of nominal exchange rate changes (\(\Delta \hat{\xi}_t\)) as the price adjustment mechanism that maintains equilibrium between foreign and domestic goods markets. We can derive an equation for oil inflation in nominal dollar (i.e., foreign currency) terms:

\[
\hat{\pi}_t^o = \hat{p}_t^o - \hat{p}_{t-1}^o + \hat{\pi}_t^f,
\]
(A.26)

where \(\hat{p}_t^o\) is a stochastic process capturing shocks to the price of oil relative to the foreign price level. Eq. (A.26) therefore capture both changes in real oil price movements and the endogenous evolution of price, productivity and risk premium shocks from the foreign economy.

A.2.6 Evolution of capital

\[
\hat{k}_{t+1} = (1 - \delta)\hat{k}_t + \delta \hat{v}_t
\]
(A.27)

A.2.7 Policy rule

\[
\hat{l}_t^h = \rho_l \hat{l}_{t-1} + (1 - \rho_l)\hat{c}_t + (1 - \rho_i)\kappa_y (\hat{y}_t - \hat{y}_{t-1}) + \epsilon_t^i.
\]
(A.28)

\(^{24}\)Specification of stochastic processes in Eq. (A.21) is important. It depends on how we treat the price of oil in estimation: if it enters as \(\hat{p}_t^o\) then we can separate the shocks; if we introduce as \(\hat{p}_t^o\), then we must combine them. As we are interested in foreign real oil price shocks we opt for the former.

\(^{25}\)Derived from \(r_e + \hat{\pi}_t = \hat{r}_t + (\hat{l}_t^o - \hat{\pi}_t) - (\hat{l}_t^o + \hat{\pi}_t^f + \mu_t^o)\), where \((\hat{l}_t^o - \hat{\pi}_t)\) and \((\hat{l}_t^o + \hat{\pi}_t^f)\) are the domestic and foreign real interest rates on bonds, i.e., the Fisher equations.
A.3 Foreign economy

We assume a large open economy for the foreign market. This allows us to specify the foreign rate, $\hat{i}_t^b$, foreign inflation $\hat{\pi}_{t+1}^* = \hat{i}_{t+1}^b$, and foreign consumption $\hat{y}_t^* = \hat{c}_t^*$ according to the standard 3-equation New-Keynesian model, namely: an IS curve, a Phillips curve, and a Taylor-type policy rate rule.

\[
\hat{y}_t = \frac{1}{(1 + \phi^*)} \hat{y}_{t+1}^* + \frac{\phi^*}{(1 + \phi^*)} \hat{y}_{t-1}^* - \frac{(1 - \phi^*)}{\sigma_c^*(1 + \phi^*)} (\hat{i}_t^b - E_t[\hat{\pi}_{t+1}^*] + \hat{\mu}_t^b) \\
\hat{\pi}_t = \frac{\gamma^*}{(1 + \gamma^* \beta)} \hat{\pi}_{t-1}^* + \frac{\beta}{(1 + \gamma^* \beta)} E_t[\hat{\pi}_{t+1}^*] + \kappa_* (n^c_t^* + \xi^p_t^*),
\]

where $\hat{mc}_t^*$ is the real marginal cost of production, and

\[
\hat{mc}_t^* = \left( \frac{\sigma_c^*}{1 - \phi^*} + \sigma_n^* \right) \hat{y}_t^* - \left( \frac{\sigma_c^* \phi^*}{1 - \phi^*} \right) \hat{y}_{t-1}^* - (1 + \sigma_n^*) \hat{a}_t^*,
\]

\[
\hat{i}_t^b = \rho_i^* \hat{i}_{t-1}^b + (1 - \rho_i^*) \kappa_* \hat{\pi}_t^* + (1 - \rho_i^*) \kappa_* (\hat{y}_t^* - \hat{y}_{t-1}^*) + \epsilon_t^i,
\]

A.4 Aggregate equilibrium

\[
\hat{y}_t = \frac{C^h}{Y^c} \hat{c}_t^h + \frac{C^h}{Y^c} \hat{c}_t^* = \frac{C^h}{Y^c} \hat{c}_t^h + \frac{1 - C^h}{Y^h} \left( \hat{c}_t^* - \hat{\xi}^f (\hat{m}_t^h - r \hat{r}_t) \right),
\]

where $\hat{\xi}^f$ is the foreign price elasticity of demand for domestic goods (i.e., the change in foreign demand for domestic goods given the foreign price of domestic goods relative to the foreign price of foreign goods).

\[
\hat{y}_t = \frac{C^c}{Y} \hat{c}_t + \frac{V}{Y} \hat{b}_t + \frac{X}{Y} \hat{x}_t - \frac{M}{Y} \hat{m}_t \\
\hat{x}_t = \hat{c}_t^h = \hat{c}_t^* - \hat{\xi}^f (\hat{m}_t^h - r \hat{r}_t) \\
\hat{m}_t = \frac{C^f}{M} \hat{c}_t^f + \frac{O}{M} \hat{a}_t \\
\hat{a}_t = \frac{O^c}{O} \hat{d}_t + \frac{O^h}{O} \hat{a}_t^h,
\]

where $O/M = (M - C^f)/M$. 

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A.5 Exogenous shocks

We include 10 shocks in the model. The two oil shocks are the foreign real price of oil and the domestic deviations from l.o.p shock:

\[ \hat{p}_t^{\text{pr}} = \rho_o \hat{p}_t^{\text{pr}} + \epsilon_t^{\text{pr}}; \hat{\psi}_t^{\text{pr}} = \rho_p \hat{\psi}_t + \epsilon_t^{\psi} \]

For the domestic economy, the monetary policy shock \((\epsilon_t^i)\), as given in Eq. A.28, is i.i.d, whereas the domestic technology shock and domestic price markup shock follow AR(1) processes:

\[ \hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_t^a; \hat{\xi}_t^p = \rho_p \hat{\xi}_{t-1} + \epsilon_t^p. \]

The foreign economy follows analogously with an i.i.d monetary policy shock \((\epsilon_t^i^*)\), and the following technology and price markup processes:

\[ \hat{a}_t^* = \rho_a \hat{a}_{t-1}^* + \epsilon_t^{a^*}; \hat{\xi}_t^p^* = \rho_p \hat{\xi}_{t-1}^p + \epsilon_t^{p^*}. \]

In addition, the risk premium shocks on domestic assets relative to the policy rate and for domestic borrowing abroad (equivalent to a negative demand shocks) are described as follows:

\[ \hat{\mu}_t^b = \rho_b \hat{\mu}_{t-1}^b + \epsilon_t^b \]

B Data and sources

Data sources retrieved from the Federal Reserve Bank of St. Louis (FRED), the South African Reserve Bank (SARB), US. Energy Information Administration, and World Development Indicators:

1. Consumer Price Index of All Items in United Kingdom [GBRCPIALL], Euro area [EZ17M086-NEST], Japan [JPNCPIAL1] and in South Africa [ZAFCPIALL] retrieved from FRED (Copyright, 2016, OECD and Eurostat)

2. Gross Domestic Product by Expenditure in Constant Prices for United States [GDPC1], United Kingdom [GBQ661S], Euro area [EZQ661S], Japan [JPQ661S] and South Africa [ZAQ661S], retrieved from FRED (Copyright, 2016, OECD )


4. 3-Month Treasury Constant Maturity Rate [GS3M], retrieved from FRED, Federal Reserve Bank of St. Louis (Board of Governors of the Federal Reserve System US)

5. Working Age Population: Aged 15-64: All Persons for the United States [CNP16OV], United Kingdom [GBQ647N], Euro area [EZQ647N] and Japan [JPQ647N] retrieved from FRED

\(^{26}\)Distinguishing these two shocks is necessary for the model estimation, and, we believe, helps match reality. Firstly, it alleviates the effect of large and volatile oil price fluctuations (in foreign currency terms) on the adjustment of the real effective exchange rate and the domestic real price of oil (see Eq. A.21). In this sense, the two shocks are able to better capture the pass-through of foreign oil price shocks (captured by \(\hat{p}_t^{\text{pr}}\)). Secondly, oil enters as an input in the production process directly, and following the aforementioned approach allows large foreign oil price shocks to be dampened by domestic deviations from the l.o.p. (see also, Medina and Soto 2005).
6. SARB, Balance of payments statistics [KBP5000L - KBP5010L]

7. SARB, Final consumption expenditure by households: Total (PCE) [KBP6007L]

8. SARB, Gross fixed capital formation (Investment) [KBP6009L]

9. US. Energy Information Administration, Crude Oil Prices: West Texas Intermediate (WTI) - Cushing, Oklahoma [DCOILWTICO]

10. World Development Indicators, Fuel imports (% of merchandise imports), South Africa [TM.VAL.FUEL.ZS.UN]
Figure C.1: Impulse responses with 90% highest posterior density interval (Bayesian confidence bands)