Time-Varying Persistence of Inflation: Evidence from a Wavelet-based Approach

Heni Boubaker [a] Giorgio Canarella [b] Rangan Gupta [c]
Stephen M. Miller [b,*]

June 2016

Abstract

We propose a new long-memory model with a time-varying fractional integration parameter, evolving non-linearly according to a Logistic Smooth Transition Autoregressive (LSTAR) specification. To estimate the time-varying fractional integration parameter, we implement a method based on the wavelet approach, using the instantaneous least squares estimator (ILSE). The empirical results show the relevance of the modeling approach and provide evidence of regime change in inflation persistence that contributes to a better understanding of the inflationary process in the US. Most importantly, these empirical findings remind us that a "one-size-fits-all" monetary policy is unlikely to work in all circumstances.

Keywords: Time-varying long-memory, LSTAR model, MODWT algorithm, ILSE estimator

JEL classification: **C13, C22, C32, C54, E31**

* Corresponding author.

[a] IPAG LAB, IPAG Business School, 184 Boulevard Saint-Germain, 75006 Paris, France.

[b] University of Nevada, Las Vegas, 4505 S. Maryland Parkway, Las Vegas, Nevada 89154-6005, USA.

[c] University of Pretoria, Pretoria, 0002, South Africa.

1 Introduction

The dynamics of inflation, as defined by its moments, volatility, and persistence affect the ability of central banks to control it. This paper focuses on inflation persistence in the US using monthly data from 1871:02 to 2016:01. The data span the modern history of the international monetary systems, including the classical gold standard era (1870-1914), the interwar period (1915-1944), the Bretton Woods system (1945-1971), and the post Bretton Woods system (1971-present), and thus provide a unique opportunity to appraise how inflation persistence may vary across different monetary regimes and institutions.

As frequently noted in the literature, inflation persistence plays an important role in the conduct of monetary policy and in the development of macroeconomic theories. At the monetary policy level, inflation persistence determines how the monetary authorities respond to shocks over time (e.g., Gerlach and Tillmann, 2012; van der Cruijsen, *et al.*, 2010). Inflation persistence measures the speed at which the inflation rate returns to its equilibrium level after an inflationary shock. If inflation returns to its equilibrium level more quickly (i.e., inflation exhibits less persistence) after a shock, then the more effectively the monetary authorities can reduce inflation fluctuations, all else equal (Fuhrer, 1995).[1] High inflation persistence causes shocks to exert long-lasting effects and may require a strong policy response to bring inflation under control. In the worst case, inflation may follow a random walk (i.e., an I(1) process) making it impossible for central banks to bring it under control. In the best case, inflation may

[1] Recent contributions on inflation persistence in the US include Kumar and Okimoto (2007), Pivetta and Reis (2007) and Mehra and Reilly (2009). Beechey and Osterholm (2009), Batini (2006) and Meller and Nautz (2012) consider inflation persistence in the Euro area, while Gadea and Mayoral (2006) examine inflation persistence in 21 OECD countries.

follow an I(0) process, implying that it reverts to its equilibrium level rapidly after a shock occurs. In this case, the response to the inflationary shock may not require an active monetary policy.[2] As a consequence, the optimal timing and size of monetary policy crucially depend on the knowledge of how shocks affect the dynamics of inflation. Importantly, inflation persistence plays an important role in the current debate on inflation targeting (IT). Under inflation targeting, when a central bank successfully anchors inflationary expectations, it reduces or eliminates inflation persistence, since well-anchored inflationary expectations depend less on past inflation.[3] At the theoretical level, inflation persistence plays an important role, mainly because it associates with the theory of inflationary expectations and nominal anchors. New-Keynesian dynamic stochastic general equilibrium (DSGE) macroeconomic models that incorporate lags of inflation in the new Keynesian Phillips curve (NKPC)[4] identify inflationary expectations as the main determinant of inflation persistence, suggesting that inflation persistence may decline through enhanced anchoring of inflation expectations (Mishkin, 2007; Nautz and Strohsal, 2015).

A large number of studies model inflation as an autoregressive process and

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[2] Whether inflation follows a stationary or nonstationary process possesses theoretical implications, since a number of macroeconomic models (Dornbusch, 1976; Taylor, 1979, 1980; Calvo, 1983; and Ball, 1993) assume stationary inflation. Additionally, models such as Fuhrer and Moore (1995) and Blanchard and Gali (2007) suggest that inflation persistence captures structural characteristics of the economy that do not likely respond to policy actions. Thus, a policy of IT should exert no effect on inflation persistence. Others such as Batini (2006), Beechey and Osterholm (2007), Benati (2008), and Mehra and Reilly (2009) present evidence that inflation persistence varies across monetary regimes.

[3] Significant evidence documents the fall of inflation persistence in the last twenty years in developed IT countries (Walsh, 2008; Kuttner and Posen, 2001; Benati, 2008). Benati (2008) estimates the parameters of a sticky-price DSGE model for the U.K. and Canada before and after the introduction of IT and finds that inflation persistence falls significantly during inflation targeting periods. In other words, implementation of an IT policy lowers persistence. Evidence from Canada, Sweden, and the U.K. also supports the view that an IT policy anchors expectations (Gurkaynak, *et al.*, 2006, 2007). In developing countries, scant evidence exists that inflation persistence declines after the adoption of IT (Kuttner and Posen, 2001; Gurkaynak, *et al.*, 2006, 2007).

[4] Fuhrer and Moore (1995), Gali and Gertler (1999) and Christiano, *et al.* (2005) develop theoretical models that justify the inclusion of lags of inflation in the new Keynesian Phillips curves.

measure persistence using different metrics, such as the integer order of integration, the half-life of responses to shocks, the largest autoregressive root, and the sum of the autoregressive coefficients.[5] One portion of the inflation persistence literature considers whether shocks to the inflation process decay rapidly, as with non-integrated processes, or whether they decay much more slowly, as with fractionally integrated processes.[6] In the latter case, inflation rates display evidence of long-range dependence, or long-memory (i.e., follow a fractional integration process, denoted by I(d)).[7]

Most previous studies of inflation persistence assume that the fractional integration parameter d does not change over time, which implies that the long-range dependence structure of the underlying phenomenon persists over time. This assumption seems too restrictive due, for example, to the potential presence of structural breaks in the fractional integration parameter (Granger and Hyung, 2004; Morana and Beltratti, 2008; and Baillie and Morana, 2009). In the same context, some authors find that the

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[5] Examples include Nelson and Plosser (1982), Fuhrer and Moore (1995), Cogley and Sargent (2001), Stock (2001), Cecchetti and Debelle (2006), Pivetta and Reis (2007), and Zhang *et al.* (2008) for the U.S.; O'Reilly and Whelan (2005) and Beechey and Osterholm (2009) for Europe and Levin and Piger (2004) and Levin *et al.* (2004) for a group of OECD countries. Barsky (1987), Ball and Cecchetti (1990), and Brunner and Hess (1993) suggest that the U.S. inflation contains a unit root. The unit-root property appears to occur in a wide array of countries examined in O'Reilly and Whelan (2005) and Cecchetti *et al.* (2007).

[6] Baillie (1996) provides an extensive review of the concepts of fractional integration in economic time series. Long-memory processes are defined in both time and frequency domains. In the time domain, a process Y_t exhibits long-memory, if its autocorrelation function ρ (k), k = 1, 2, . . . , decreases at a hyperbolic rate rather than the exponential decay in a covariance-stationary ARMA process. In the frequency domain, the spectrum for a long-memory process diverges to infinity at the zero frequency. In practical applications, long-memory emerges when the series possesses a pole on a part of the spectrum close to the zero frequency (Granger and Joyeux, 1980).

[7] The empirical literature employs both autoregressive and fractionally integrated measures of inflation persistence. Examples of the autoregressive approach to estimate inflation persistence include Pivetta and Reis (2007), Gamber, Liebner, and Smith (2012), Stock (2001), O'Reilly and Whelan (2005), Levin and Piger (2004), and Gerlach and Tillman (2012). Examples of fractional integration analysis include Hassler and Wolters (1995), Siklos (1999), Barkoulas, *et al.*, (1999), Bos, *et al.*, (1999, 2002), Kuttner and Posen (2001), Baillie, *et al.*, (2002), Arize *et al.*, (2005), Levin, *et al.*, (2004), Petursson (2005), Gadea and Mayoral (2005), Kumar and Okimoto (2007), Beechey and Osterholm (2009), Meller and Nautz (2012), Caporin and Gupta (forthcoming), Canarella and Miller (2015), and Plakandaras *et al.* (2015).

fractional integration parameter varies over time $d(t)$ and that persistence to shocks also varies over time (Jensen, 1999a,b; Whitcher and Jensen, 2000; Beran, 2009; and Roueff and von Sachs, 2011) and across expansions and recessions (Caporin and Gupta, forthcoming). Thus, the degree of persistence to shocks varies over time, implying that inflation may change from a stationary process to a non stationary process, or vice versa, within the same sample period.

These findings relate to transitional and stochastic events such as financial crises, market collapses, important news announcements, and speculative bubbles that produce different responses to positive and negative shocks. To accommodate such behavior, several authors (Chandler and Polonik, 2006; Beran, 2009; Palma and Olea, 2010; and Roueff and von Sachs, 2011) find that the stochastic process exhibits non-stationarity. Assuming, however, that the time variability of the model exhibits enough smoothness, we can approximate it locally by stationary processes (i.e., locally-stationary models). Nevertheless, these studies do not specify the evolutionary process of $d(t)$. The existence of time-varying behavior in $d(t)$ could reflect a variety of issues, including the heterogeneity of agents in time horizons and strategies (Lux and Marchesi, 2000; Kirman and Tyssiere, 2002; Iori, 2002; and Alfarano and Lux, 2002) the presence of multiple attractors or “intermittent” volatility clustering (Gaunersdorfer, 2001), and changes in financial market structure such as the creation of new financial products.

A few recent studies develop models where the time-varying fractional integration parameter $d(t)$ follows several regime switching models. Among these models, Beine and Laurent (2001) find that a Markov-switching process drives $d(t)$. Dufrénot *et al.* (2005 a, b; 2008) discover that $d(t)$ follows a Self-Exciting Threshold

Autoregressive (SETAR) process. Recently, Boutahar *et al.* (2008) and Aloy *et al.* (2013) determine that $d(t)$ evolves according to a Smooth Transition Regression (STR) process (Teräsvirta 1994, 1998).

In this paper, we propose a long-memory model of inflation persistence where the fractional integration parameter varies over time. This approach has, among other things, the additional benefit of bypassing the rather complex problem of structural breaks. In particular, we assume, following Boutahar *et al.* (2008) and Aloy *et al.* (2013), that the fractional integration parameter varies according to two regimes with smooth transition from one regime to the other. Thus, the model interestingly allows for the presence of both long-range dependence (long-memory) in inflation and asymmetry in the degree of inflation persistence. To estimate the time-varying fractional integration parameter, Boutahar *et al.* (2008) consider the arranged regression, which orders the observations of endogenous and exogenous variables in ascending order of magnitude of the observations of another variable.

We adopt an alternative method, based on a two-step approach. The first step estimates a time-varying fractional integration parameter, using the wavelet method. Wavelet analysis decomposes a time series into several scales and, thus, preserves the informational content present in the series. This is known in the technical literature as multiresolution analysis (Mallat, 1989). Compared to standard Fourier analysis, wavelets provide a localized analysis in the time domain as well as in the frequency domain. Most interestingly, wavelets represent functions that exhibit discontinuities and sharp peaks and decompose and reconstruct finite nonstationary signals (Boubaker and Boutahar, 2011).[8] The advantage of estimating the fractional integration parameter in a time-varying framework (i.e., $d = d(t)$) is that it allows the implementation of tests to check for the presence of nonlinearity and the determination of the appropriate transition function and transition variable.

The second step reproduces the dynamics of $d(t)$ using an LSTAR model (Teräsvirta and Anderson, 1992; Granger and Teräsvirta, 1993; Teräsvirta, 1994; Teräsvirta, 1998). When considering aggregate economic series such as the rate of inflation, we more likely capture the time path of any structural change by a model whose dynamics undergoes a gradual rather than an instantaneous adjustment between regimes. The strength of the wavelet approach lies in its capacity to localize simultaneously a process in time and scale. At high scales, the wavelet exhibits a small centralized time support, enabling it to focus on short-lived time phenomena. By moving from low to high scales, the wavelet zooms in on a process's behavior, identifying singularities, jumps, and cups. In this paper, we consider the Maximum Overlap Discrete Wavelet Transform (MODWT) advanced by Percival and Walden (2000) that provides an approximate log-linear relationship between the time-varying variance of the MODWT coefficients and the time-varying parameters $d(t)$. Then, we apply the instantaneous least squares estimator (ILSE) to obtain local estimates for time-varying fractional integration parameters.

We find evidence of two distinct regimes in the persistence of inflation depending on the values of the transition variable (i.e., the lagged fractional integration parameter $d(t\text{-}1)$). In the lower regime, inflation is anti-persistent, while in the higher regime,

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[8] Percival and Walden (2000) provide an exhaustive overview of the wavelet methodology in time series analysis.

inflation is persistent. These empirical findings provide important implications for the conduct of monetary policy. Knowledge that inflation persistence changes over time implies that the response to inflationary shock also changes over time. This may provide the monetary authorities with alternative instruments to intervene in the economy. Most importantly, these empirical findings remind us that a "one-size-fits-all" monetary policy will probably not work in all circumstances.

The rest of the paper is organized as follows. Section 2 presents the time-varying long-memory model. Section 3 proposes a wavelet-based estimator used to estimate the time-varying long-memory parameter. Section 4 gives an empirical application to the consumer price (CPI) inflation rate time series and section 5 concludes the paper.

2 The time-varying long-memory model

In this section, we first outline the classical constant-parameter ARFIMA model, ARFIMA(p, d, q) , and then describe the generalized time-varying ARFIMA model, denoted as TV-ARFIMA $(p(t), d(t), q(t))$

2.1 The constant-parameter ARFIMA model

Let $X_t, t = 1,\ldots,T$ denote a time series process. Following Granger and Joyeux (1980), the conventional ARFIMA(p,d,q) model is given by:

$$\Phi(B)(1-B)^d X_t = \Theta(B)\varepsilon_t, \tag{1}$$

where B is the back-shift operator such that $B^i X_t = X_{t-i}$, $\Phi(B)$ and $\Theta(B)$ are polynomials in B involving autoregressive and moving average processes of orders p and q, respectively, with their roots strictly outside the unit circle and no common factors, d is the fractional integration parameter, and ε_t is a white-noise process with zero mean and variance σ^2. The fractional differencing lag operator $(1-B)^d$ is defined by the binomial expansion:

$$(1-B)^d = \sum_{i=0}^{\infty} \frac{\Gamma(i-d)}{\Gamma(i+1)-\Gamma(d)} \tag{2}$$

where $\Gamma(.)$ denotes the gamma function. The parameters found in $\Phi(B)$ and $\Theta(B)$ constitute the short memory parameters and affect only the short-run dynamics of the process while the fractional integration parameter d detects the long-memory behavior of the process. Fractionally integrated processes possess different characteristics depending on the value of d. Various cases exist. If $-0.5 < d < 0$, the process is stationary, but anti-persistent, indicating that the process reverses itself more frequently than a random process. If $0 < d < 0.5$, the process is stationary and persistent, possesses long-memory, and causes shocks to the system to disappear hyperbolically. If $0.5 \leq d < 1$, the process is non-stationary, but mean reverting with finite impulse response weights. Finally, when $d = 0$, the process reduces to the standard ARMA process, while when $d = 1$, the process contains a unit root and reduces to the conventional ARIMA process, with infinite persistence to a shock.

2.2 The time-varying ARFIMA model

The time-varying ARFIMA model (TV-ARFIMA) assumes that the fractional integration parameter d varies over time (i.e., $d(t)$). Let $X_t, t = 1,\ldots,T-1$ denote a stochastic process. The TV-ARFIMA $(p(t), d(t), q(t))$ is then defined by:

$$\Phi_t(B)X_t = \Theta_t(B)(1-B)^{-d(t)}\varepsilon_t, \tag{3}$$

where $d(t) < 0.5$ is the time-varying fractional integration parameter, $\Phi(B)$ and $\Theta(B)$ are stable polynomials (i.e., their roots are strictly outside the unit circle), and ε_t is a white noise process with zero mean and variance σ^2.

We can define the time-varying long-memory model in both the frequency and time domains. In the frequency domain X_t is a locally stationary long-memory process, if there exists a time-varying spectral density function $SDF(t,\lambda)$ such that (Boubaker, 2014)

$$SDF(t,\lambda) \sim \lambda^{-2d(t)} \text{as } \lambda \to 0^+. \quad (4)$$

Thus, if $d(t) > 0$, $SDF(t,\lambda)$ is smooth for frequencies close to zero, but is unbounded when $\lambda = 0$. If $d(t) < 0$, then $SDF(t,\lambda) = 0$ and X_t is a locally stationary series that is anti-persistent. X_t is smoother and exhibits less variability in its amplitude during time periods where $d(t) > 0$, while it experience large fluctuations when $d(t) < 0$. In the time domain, X_t is a locally-stationary long- memory process, if there exists a local autocovariance function $\text{cov}_X(t, g-h)$ such that (Boubaker, 2014)

$$\text{cov}_X(t, g-h) \sim |g - h|^{2d(t)-1} \text{as } |g-h| \to \infty. \quad (5)$$

The slow hyperbolic decay of $\text{cov}_X(t, g-h)$ is the feature most often observed in the discussion of the dynamics of long-memory processes. In our empirical analysis, for simplicity, we assume that *d(t)* appears on a finer grid (i.e., we rescale *d(t)* on the closed interval [0, 1] so that we can denote it by *d(t/T)*). In equation (3), we also model the short-memory parameters as time-varying functions of t (i.e., $\Phi(B)$ and $\Theta(B)$). In this paper, however, to simplify the estimation process, we assume that the short-memory parameters are constant, and set $\Phi_t(B) = \Phi(B)$ and $\Theta_t(B) = \Theta(B)$ for all t, resulting in the following TV-ARFIMA model:

$$\Phi(B)(1-B)^{d(t)} X_t = \Theta(B)\varepsilon_t . \quad (6)$$

The long-memory model in equation (6) is a locally-stationary process in the sense of Dahlhaus (1996) (see Dahlhaus, 1996 and Whitcher and Jensen, 2000 for further details). Although this model indicates that the fractional integration parameter varies over time, it does not provide information about the evolution of $d(t)$. Following Boutahar *et al.* (2008) and Aloy *et al.* (2011), we assume that $d(t)$ evolves according to the two-regime Smooth Transition Autoregressive (STAR) model proposed by Teräsvirta (1994, 1998):

$$d(t) = d_1[1-F(s(t);\gamma,c)] + d_2 F(s(t);\gamma,c) , \quad (7)$$

where d_1 and d_2 are the values of the fractional integration parameter in the first and second regimes, respectively. $F(s(t);\gamma,c)$ is the transition function that is continuous, bounded between 0 and 1, with $s(t)$ denoting the transition variable, $s(t) = d(t-i)$, $\forall t > i$, $t = 1, \ldots, T$. The slope parameter γ measures the speed of the transition between the two regimes (associated with the extreme values 0 and 1 of the transition function), which can be either positive or negative depending upon whether the transition function is increasing or not. The parameter c represents the threshold for the transition variable, $s(t)$, which defines the two underlying regimes: the first (lower) regime $s(t) \leq c$ and the second (higher) regime $s(t) > c$.

The existing empirical literature uses two types of transition functions – the logistics and exponential functions. The logistics function is given by:

$$F(s(t);\gamma,c) = (1+\exp(-\gamma(s(t)-c)))^{-1} , \ \gamma > 0 , \quad (8)$$

where combining equations (7) and (8) yields the logistic smooth transition autoregressive (LSTAR) model. The logistic function monotonically increases in $s(t)$

with $F(s(t);\gamma,c) \to 0$ as $(s(t)-c) \to -\infty$ and $F(s(t);\gamma,c) \to 1$ as $(s(t)-c) \to +\infty$. When $\gamma \to \infty$, $F(s(t);\gamma,c)$ becomes a step function and the transition between the regimes is abrupt. In that case, the model approaches a Self-Exciting Threshold autoregressive (SETAR) model (Tong, 1990). The exponential function is given by:

$$F(s(t);\gamma,c) = 1 - \exp(-\gamma(s(t)-c)^2),\ \gamma > 0, \tag{9}$$

where combining equations (7) and (9) produces the exponential STAR (ESTAR) model.

In this paper, we adopt a two-step approach to estimating the dynamics of the US inflation persistence during the time period 1871:02-2016:01. The approach merges two strands of the empirical literature. First, we estimate the TV-ARFIMA model of inflation persistence using the wavelet approach:

$$\Phi(B)(1-B)^{d(t)} X_t = \Theta(B)\varepsilon_t \tag{10}$$

Second, we estimate the dynamics of the time-varying inflation persistence using an LSTAR model:

$$d_t = d_1[1 - F(s(t);\gamma,c)] + d_2 F(s(t);\gamma,c) \tag{11}$$

where

$$F(s(t);\gamma,c) = (1 + \exp(-\gamma(s(t)-c))^{-1}, \gamma > 0 \tag{12}$$

First, we estimate the entire fractional integration parameter sequence $d(t)$ using a wavelet approach (equation 9). Then, second, we estimate the LSTAR model applied to the derived series $d(t)$ (equations 10 and 11), using a nonlinear approach.

3 Wavelet-based estimation of the time-varying fractional integration parameter

The existing literature develops numerous estimation methods for the fractional integration parameter d in fractionally integrated process *I(d)* for stationary series.

Among these methods, we find the parametric methods, which use (approximate or exact) likelihood methods in the time or frequency domains, the semi-parametric estimators, which rely on spectral density, and the nonparametric methods. In this paper, we adopt an alternative estimation method based on the wavelet approach. First, we review some basics of wavelets. Then, second, we present the instantaneous least squares estimator that we use to estimate the fractional integration parameter $d(t)$.

3.1 Wavelet methodology

Wavelets are mathematical tools that are widely applied for analyzing time series.[9] The starting point in such analysis decomposes a time series on a scale-by-scale basis. Wavelets are orthonormal bases (Daubechies, 1992) obtained from a dyadic grid by dilating and translating a pair of specially constructed functions φ and ψ, which are called the *father* and *mother wavelets*, respectively, such that

$$\int_{-\infty}^{\infty} \varphi(t)\mathrm{d}t = 1 \tag{13}$$

and

$$\int_{-\infty}^{\infty} \psi(t)\mathrm{d}t = 0 \tag{14}$$

Thus, the father wavelet (the scaling function) integrates to 1 and reconstructs the smooth and the low-frequency parts of the series, while the mother wavelet (the wavelet function) integrates to 0 and describes the details and high-frequency components of the series. The father and mother wavelets are more formally the functions

$$\varphi_{j,k}(t) = 2^{j/2}\varphi(2^j t - k), \text{ and} \quad (15)$$

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), \quad (15)$$

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9 For a review on wavelet analysis from a time-series perspective, see Ramsey (1999), Schleicher (2002), Crowley (2007), Percival and Walden (2000), Gençay, Selçuk, and Whitcher (2002) among others.

where $j = 1,\ldots,J$ indexes the scale (or multiresolution levels) and $k = 1,\ldots 2^j$ indexes the translation (i.e., ranges from 1 to the number of coefficients in the specified level). The parameter j dilates the wavelet function and adjusts the support of $\psi_{j,k}(t)$ to capture locally the characteristics of high or low frequencies, while parameter k relocates the wavelets in the temporal scale (Boubaker, 2014). Thus, applying a *J*-level multiresolution decomposition, wavelet analysis provides a complete reconstruction of the series partitioned into a set of *J*-frequency components with each component corresponding to a particular range of frequencies.

One special property of the wavelet expansion is the localization property that the coefficient of $\psi_{j,k}(t)$ reveals the information content of the function at the approximate location $k2^{-j}$ and frequency 2^j.Thus, using wavelets, we can uniquely expand any function in $L^2(\mathbb{R})$ over the wavelet basis, as a linear combination at arbitrary level $J_0 \in N$ across different scales (Boubaber, 2014). The wavelet representation of a discrete time series $x(t)$ in $L^2(\mathbb{R})$ is then given by

$$x(t) = \sum_k c_{J_0,k}\varphi_{J_0,k}(t) + \sum_{j>J_0}\sum_k d_{j,k}\psi_{j.k}(t), \tag{17}$$

where the coefficients $c_{J_0,k}$, known as the *smooth coefficients*, are coarse scale coefficients and represent the underlying smooth behavior of the time series at the coarse scale 2*J*, while the coefficients $d_{j,k}$, known as *detailed coefficients*, are the fine scale coefficients. They represent the wavelet transform coefficients, which measure the contribution of the corresponding wavelet to the function $x(t)$. We approximate these coefficients by the following relations:

$$c_{J_0,k} = \int_{-\infty}^{\infty} x(t)\varphi_{J_0}(t)\mathrm{d}t \quad (18)$$

$$_0\, d_{J_0,k} = \int_{-\infty}^{\infty} x(t)\psi_{j,k}(t)\mathrm{d}t \quad (19)$$

The expression in equation (17) represents the decomposition of $x(t)$ into orthogonal components at different resolutions and constitutes the so-called wavelet multiresolution analysis (MRA) (Mallat, 1989; Boubaker and Boutahar, 2011).

In practice, we invariably deal with sequences of values indexed by integers rather than functions defined over the entire real axis. In this case, we use short sequences of values rather than actual wavelets, referred to as wavelet filters. The number of values in the sequence is called the width of the wavelet filter. Thus, the wavelet analysis, viewed from a filtering perspective, is well adapted for time-series applications. For the discrete wavelet transform, we use the MRA scheme to calculate the wavelet coefficients. The recursive MRA scheme, which is implemented by a two-channel filter (low-pass and high-pass filters) representation of the wavelet transform, is divided into decomposition and reconstruction schemes, according to the forward and inverse wavelet transform.

Daubechies (1992) constructs a class of wavelet functions, where the smallest support for a given number of vanishing moments distinguishes between two choices: the extremal phase filters $D(L)$ and the least asymmetric filters $LA(L)$. Two main wavelet algorithms are discussed in the literature: the Discrete Wavelet Transform (DWT) and the Maximal Overlap Discrete Wavelet Transform (MODWT) (Percival and Walden, 2000). The MODWT algorithm modifies the DWT method. The MODWT algorithm carries out the same filtering steps as the standard DWT; however, in the MODWT the time series $x(t)$ is not subsampled (not decimated). The MODWT is generally preferred to the DWT, since the MODWT can handle any sample of size T, while the DWT restricts the sample size to a multiple of 2^{J_0} for a partial DWT or to be exactly a power of 2 for the full transform. Consequently, with the MODWT, the number of scaling and wavelet coefficients at each level of the transform is the same as the number of sample observations. See Percival and Walden (2000) for a complete analytical scrutiny of the two transforms and a list of properties that distinguish the MODWT from the DWT. Mathematically, decomposing a time series $x(t)$, using the MODWT, to J-levels involves the application of J pairs of filters. The filtering operation at the j^{th} level consists of applying a rescaled father wavelet to yield a set of detailed coefficients and a rescaled mother wavelet to yield a set of scaling coefficients.

3.2 Instantaneous least squares estimator

The basic idea behind estimating the fractional integration parameter d via a wavelet transform of the time series involves the wavelet variance. Wavelet variance analysis consists in partitioning the variance of a time series into pieces that associate with different time scales. This approach substitutes the notion of variability over certain scales for the global measure of variability estimated by the sample variance, which tells us what scales importantly contribute to the overall variability of a series.

In particular, consider the time series, $X_1,\ldots,X_T$, which is a realization of a stationary process with variance σ_X^2. If the scaling coefficients for level j associate with averages of length 2^j, then the level j wavelet coefficients, which are differences of averages half this length, associate with changes at scale $\tau_j \equiv 2^{j-1}\Delta t$, where Δt is the sampling interval of X_t. Thus, the wavelet variance $V_X^2(\tau_j)$ for scale $\tau_j \equiv 2^{j-1}$ is defined as follows:

$$\sigma_X^2 \equiv \sum_{j=1}^{J} V_X^2(\tau_j). \tag{20}$$

Note that in the present analysis, we consider $\Delta t = 1$. For estimating the fractional integration parameter via the wavelet approach, many methods exist in the literature. We can, in general, classify them into three computationally efficient schemes. First, we can use a wavelet-based approximation to the maximum likelihood estimator (MLE) of *d* under the assumption of multivariate Gaussianity (McCoy and Walden, 1996; Jensen, 1999a, 2000). Second, we can use the fact that the relationship between the variance of the wavelet coefficients across scales is dictated by *d*. In this framework, we construct a least squares estimator (LSE) of *d* (Abry and Veitch, 1998; Jensen, 1999b). Third, we can use only certain coefficients that are co-located in time, which we call the instantaneous least squares estimator (ILSE) (Percival and Walden, 2000; Boubaker, 2014).

This third estimator, however, depends on the entire time series. The instantaneous least squares estimator uses a single wavelet coefficient from each scale of resolution. That is, we only use $\tilde{d}_{j,t_j}$ to estimate $V_X^2(\tau_j)$, where t_j is a time index of the j^{th} level MODWT coefficient associated with time t in X_t (t=1,...,T). We can only meaningfully determine the time index, t_j, if we use a linear phase wavelet filter. Formally, let the vector of dimension containing the wavelet coefficients obtained by the MODWT transform. The instantaneous least squares estimator (ILSE) is given by:

$$\hat{d}_{ILSE,t} = \frac{\Delta_J \sum \ln(\tau_j) Y_t(\tau_j) - \sum \ln(\tau_j) \sum Y_t(\tau_j)}{2\left(\Delta_J \sum \ln^2(\tau_j) - (\sum \ln(\tau_j))^2\right)} + \frac{1}{2}, \tag{21}$$

where $\Delta_J = J - J_0 + 1$. In equation (11), all the sums run over $j = J_0, \ldots, J$, and $Y_t(\tau_j) \equiv \ln\left(d_{j,t_j}^2\right) - \psi(1/2) - \ln(2)$, where ψ is the digamma function (i.e., the logarithmic derivative of the gamma function). See also the MODWT-based weighted least squares estimator developed by Percival and Walden (2000).

4 Empirical results

This section contains an empirical application of the wavelet methodology (MODWT), the instantaneous least squares estimator (ILSE), and the smooth transition autoregressive (LSTAR) model to modeling the dynamics of the US rate of inflation. Our data consists of monthly observations on the seasonally adjusted Consumer Price Index (CPI) and are obtained from the data segment on the R. J. Shiller website[10] and cover the period of 1871:01 until 2016:01, corresponding to T = 1741 observations. We compute the rate of inflation as the monthly logarithmic difference of the CPI, expressed as a percentage. Figure 1 shows the inflation rate series.

Over the entire sample, inflation averages approximately 2 percent. But, computed over the different monetary regimes, average inflation shows some interesting characteristics. For example, during the period of the classical gold standard, the average inflation is -0.47 percent, although the period was characterized by two decades of secular deflation, followed by two decades of secular inflation (Bordo and Redish, 2001), with a maximum of 81 percent, and a minimum of -81 percent. This is, however, the only period when inflation, on average, is negative. In the interwar period, inflation averages 1.88 percent, while during the Bretton Wood era inflation averages about 3 percent. The post-Bretton Woods period, on the other hand, witnesses an

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[10] *http://www.econ.yale.edu/˜shiller/data.htm.*

average inflation of approximately 4 percent, with a maximum and minimum of 21 sand -23 percent, respectively. Both the skewness (-0.1381) and kurtosis (9.5216) statistics indicate that the inflation series exhibit a fatter-tail distribution than a normal distribution. The Jarque-Bera teat statistic (3089.1) provides further evidence of the departure from normality, which indicates that we can reject the null hypothesis of normality at any conventional significance level. This result is not surprising and it is frequently found in the empirical literature on inflation persistence. It implies several extreme values relative to the standard normal distribution.

As shown in Figure 1, the inflation rate appears to exhibit stationary behavior, in the sense of converging towards a long-run equilibrium. To formally assess the stationarity of the inflation rate, we apply three types of unit root tests: the Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1981), the Phillips-Perron (PP) test (Phillips and Perron, 1988) and the KPSS test (Kwiatkowski *et al.*, 1992). The ADF and PP test statistics (with and without a constant) overwhelmingly reject the null hypothesis of unit root at the 1 percent level. The KPSS test (with and without a constant) confirms these results by failing to reject the null hypothesis of stationarity at the-1 percent level. Detailed results are available on request. One should exercise some caution in accepting these findings at face value, since it is well known that these tests possess low power against a fractionally integrated I(d) process with $d < 1$.

4.1 Constant-parameter estimates of inflation persistence

We next consider, as a benchmark, the estimation of the fractional integration parameter over the entire sample using constant-parameter conventional estimators. This allows us to verify the presence of long-range dependence and persistence in the inflation data. To this end, we apply several estimators in both the frequency and wavelet domains. In the frequency domain, we apply the Geweke and Porter-Hudak (GPH) estimator, introduced by Geweke and Porter-Hudak (1983), and the Exact Local Whittle estimator (Shimotsu and Phillips, 2005). Geweke and Porter-Hudak (1983) prove consistency and asymptotic normality of the log-periodogram estimator for the range -0.5 < d < 0, while Robinson (1995) extends consistency and asymptotic normality for -0.5 < d < 0.5. Velasco (1999) and Kim and Phillips (1999) recently extend the properties of the log-periodogram. The Exact Local Whittle estimator is an exact form of the local Whittle estimator that does not rely on tapering or differencing prefilters. The estimator is consistent (Shimotsu and Phillips, 2004) and to possess the same $N(0,1/4)$ limit distribution for all values of *d,* if the optimization covers an interval of width less than 9/2 and the initial value of the process is known.

In the wavelet domain, we consider three estimators. First, the d_{LSE_AV} estimate is a semi-parametric wavelet-based estimator for the Hurst parameter as proposed by Abry and Veitch (1998). Under the general conditions and Gaussianity assumptions, this estimator is unbiased and efficient. Second, the d_{LSE_J} estimate is developed by Jensen (1999b) based on the fact that a log-linear relationship exists between the variance of the wavelet coefficients from the long-memory process and its scale equal to the long-memory parameter. This log-linear relationship yields a consistent ordinary least squares estimator. Finally, the d_{ILSE} estimate formulates the instantaneous least squares estimator that does not depend on the size of the sample, and checks for departures from statistical consistency within a proposed block size. Indeed, we use only a single wavelet coefficient from each scale. Table 1 reports the estimation results of fractional integration parameter under the assumption of temporal constancy.

The estimates of inflation persistence in Table 1 are robust to different estimation methods and overwhelming support the long-memory, $\hat{d} > 0$, mean-reversion, $\hat{d} < 1$, and stationarity, $\hat{d} < 0.5$, properties of US inflation. In turn, these results imply that inflationary shocks dissipate at a hyperbolic, not geometric, rate.

4.2 Time-varying parameter estimates of inflation persistence

One obvious limitation of the findings reported in Table 1 is that they come from estimators that assume constancy of the fractional integration parameter. If inflation persistence varies over time, then the estimates assuming a constant parameter are subject to misspecification error. It is hard to argue that the fractional integration parameter does not vary over time, especially when the data span more than a century. In what follows, as previously noted, we estimate the fractional integration parameter in a time-varying framework, using the instantaneous least squares estimator (ILSE).

We must address two preliminary issues, however. The first issue concerns the constancy of the fractional integration parameter (i.e., the test of $d(t) = d$). For that, we apply a Lagrange Multiplier (LM) test. Teräsvirta (1994, 1998) previously developed this test to consider the linearity of the autoregressive model. Aloy *et al.* (2011) extended this test to consider the constancy of the fractional integration parameter.

We can express the null hypothesis of the test as $H_0 : \gamma = 0$ or, equivalently, as $H'_0 : d_1 = d_2$, the equality of the parameters in the two regimes, against $H_1 : \gamma > 0$. Under the null hypothesis, however, the nuisance parameters γ and c prevent identification. Consequently, we cannot implement the standard LM test. As suggested by Luukkonen *et al.* (1988) and implemented by Aloy *et al.* (2011) to solve this problem, we replace the transition function $F(s(t); \gamma, c)$ by its third-order Taylor approximation around $\gamma = 0$. In the reparameterized model, the identification problem no longer exists and we can test the constancy of the fractional integration parameter using the LM-type test.

More precisely, the test refers to the following auxiliary regression:

$$\begin{aligned}\hat{\varepsilon}(t) = \beta_0 + \beta_{\partial} d(t) + \beta_2 d(t)s(t) + \beta_3 d(t)s(t)^2 \\ + \beta_4 d(t)s(t)^3 + \eta(t).\end{aligned} \quad (22)$$

where $\hat{\varepsilon}(t)$ are the residuals obtained from the first-order linear autoregressive model estimated by OLS for $d(t)$ as follows:

$$d(t) = \alpha_0 + \alpha_1 d(t-1) + \varepsilon(t). \quad (23)$$

The null hypothesis of constancy of the fractional integration parameter ($d(t) = d$) then becomes $H''_0 : \beta_2 = \beta_3 = \beta_4 = 0$. As in standard cases, the LM statistic is asymptotically distributed under the null hypothesis H''_0 as χ^2 with one degree of freedom. We implement the test using the Fisher version.

Terasvirta (1994, 1998) suggests testing the null hypothesis for several candidate transition variables. If we reject the null hypothesis for more than one transition variable, Terasvirta (1994, 1998) suggests choosing the variable with the strongest rejection of linearity, the smallest p-value. Nevertheless, we stress that the statistical inference based on the asymptotic approximation under the null hypothesis depends on the residuals of the auxiliary regression (23). Consequently, to accommodate this issue, we rely on an alternative method based on bootstrapping to correct the distortions of the significance level of the test. (For more details regarding the econometric problems posed by the use of constructed variables in a regression, see Pagan, 1984). Table 2 reports the test results.

We strongly reject the null hypothesis of constancy of $d(t)$ at any conventional significance level with $d(t-1)$ (i.e., $d(t-1)$ exhibits the smallest p-value against the STAR specification). In actuality, we conducted the test with $d(t-i)$, $i=1,...,8$. We obtained the lowest p-value with $d(t-1)$. Table 2 does not report the results for the other lags, which are available on request.

The second issue concerns the selection of the appropriate form of the transition function. To discriminate the LSTAR model from the ESTAR model, we use a sequence of nested hypotheses that test for the order of the polynomial in the auxiliary regression as follows:

$$H_4 : \beta_4 = 0, \tag{24}$$

$$H_3 : \beta_3 = 0 | \beta_4 = 0, \text{ and} \tag{25}$$

$$H_2 : \beta_2 = 0 | \beta_3 = \beta_4 = 0. \tag{26}$$

The rejection of H_4 implies that the LSTAR model is the appropriate model. Conversely, if H_3 exhibits the smallest p-value, then the ESTAR model is the appropriate model. If we cannot reject both H_4 and H_3, but reject H_2, then the LSTAR model is the appropriate model. Table 3 displays the results of the tests, where the F_2, F_3, and F_4 statistics are the Fisher tests for the null hypotheses H_2, H_3, and H_4, respectively. We reject H_4 while we do not reject H_3 and H_2. We, therefore, conclude that $d(t)$ varies according to the LSTAR model.

We conclude that the dynamics of the fractional integration parameter is a long memory LSTAR process. We apply the LSTAR model to model the dynamics of the fractional integration parameter, using the nonlinear least squares (NLS) method. We apply the instantaneous least squares estimator to estimate the fractional integration parameter $d(t)$ at each point in time. Figure 2 plots the sequence of estimates of the fractional integration parameter. We clearly see that $\hat{d}(t)$ changes substantially over time, alternating between phases of anti-persistence, with negative values of d, and persistence, with positive values of d.[11] In particular, this parameter decreases in value, where intervals of increased variability at a variety of large scales. Moreover, the large values of $\hat{d}(t)$ correspond to scheduled economic information announcements. We see negative values of $\hat{d}(t)$ corresponding to new announcements and unexpected market crashes or political upheavals (Whitcher and Jensen, 2000). The existence of anti-persistence signals that the economy remains far-from equilibrium and may experience significant macroeconomic turbulence. According to Figure 2, one period of anti-persistence and four periods of persistence exist. The period of anti-persistence takes place mainly during the classical gold standard.[12] This is followed by four waves of persistence. The first wave comprises the interwar period, ending approximately with the Bretton Woods Agreement in 1944 and the Treasury Accord of 1951.[13] The Treasury Accord appears to define an unexpected event, which most likely explains why inflation temporarily turns anti-persistent. The second wave covers the next three decades, including the Bretton Woods era and ending approximately with the start of the Great

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[11] We do not report the confidence intervals of $d(t)$ in Figure 2. Suffice to say, however, that the standard errors indicate that we reject the null hypothesis of $d(t) = 0$ for 1,663 significant *d's* out of 1740 estimates. A conspicuous detail of the estimation of $d(t)$, on the other hand, is the extremely small standard errors of the parameter estimates (yielding enormous t-ratios) for negative fractional integration estimates.

[12] During the period of the classical gold standard (1871-1914), the mean of the estimates of inflation persistence is -0.1272, with a maximum of 0.5017 and a minimum of -0.5268. Thus, during the classical gold standard era inflation is fundamentally an anti-persistent process.

[13] In the interwar period (1915-1944) the mean of the estimates of inflation persistence is 0.0623, with a maximum of 0.7421 and a minimum of -0.5236. The interwar period is the first time when inflation exhibits a few episodes of non-stationarity.

Moderation.[14] The third wave takes place during the Great Moderation, ending approximated in 2006.[15] Finally, the fourth wave still remains in progress and broadly includes the recent financial crisis, the Great Recession, and its aftermath. [16] We observe that in general anti-persistence correlates with high variability of inflation, while persistence correlates with low variability. Next, we model the time-varying fractional integration parameter by an LSTAR model. Table 4 reports the results.

The empirical results show significant evidence of two regimes for the fractional integration parameter, depending on the size of long-range dependence. The threshold value defines two regimes and divides inflation between persistence and anti-persistence. The estimated threshold parameter $\hat{c}$ is significantly different from zero, implying that increases (decreases) in the fractional integration parameter above (below) 0.2735 produces asymmetric effects on inflation persistence. In the first, lower regime, which occurs when $\hat{d}(t-1) < \hat{c}$, the estimated fractional integration parameter $\hat{d}_1$ is significantly negative (i.e., inflation in the lower regime is anti-persistent). Conversely, in the second, higher regime, which occurs when $\hat{d}(t-1) > \hat{c}$, the estimated fractional integration parameter $\hat{d}_2$ is significantly positive (i.e., inflation in the higher regime is persistent). We observe that in the second, higher regime, the significant positive fractional integration parameter indicates a stationary, long-memory process and implying that the effects of long-range dependence tend to persist, while in the first

[14] During this period (1972-1983) the mean of the estimates of inflation persistence is 0.2467, with a maximum of 0.8195 and a minimum of -0.2462. The period is the only time when inflation exhibits a unit-root episode.

[15] During this period (1984-2006) the mean of the estimates of inflation persistence is 0.1185, with a maximum of 0.7975 and a minimum of -0.3865.

[16] The mean of the estimates of inflation persistence in this period (2007-2016) is 0.2518, with a maximum of 0.5882 and a minimum of -0.0454.

regime $-0.5 < \hat{d}_1 < 0$, indicating a stationary but not long-memory process and implies that the effects of long-range dependence tend to anti-persist. The slope parameter estimate, $\hat{\gamma}$, indicates a smooth transition from one regime to the other. This contrasts to the simple threshold models, which assume a sharp switch or jump.

5. Conclusion

This paper contributes to the debate on the persistence of inflation in the US. We propose a new framework of analysis, which applies the wavelet methodology (MODWT), executed via the instantaneous least squares estimator (ILSE), in conjunction with the smooth transition autoregressive (LSTAR) model to modeling the time-varying long-memory dynamics of the US rate of inflation.

The empirical results show significant evidence of long-range dependence of the US inflationary process and time-variability of inflation persistence. Inflation alternates between phases of persistence and phases of anti-persistence, switching from periods where positive (negative) changes follow other positive (negative) changes, implying persistence, to periods where positive (negative) changes follow negative (positive) changes, implying anti-persistence). We identify one broad period of inflation anti-persistence, which coincides with the classical gold standard era, and four broad periods of inflation persistence, which coincide with the more recent economic history of the US. Intertwined with the period of anti-persistence, however, are episodes of persistence; similarly, intertwined with the four periods of persistence are a few episodes of anti-persistence. This alternations reveal an important point: the inadequacy of the constant-parameter approach to characterize the local dynamics of inflation. The empirical application of the LSTAR model to the series of fractional integration estimates reveals significant evidence of two regimes of inflation persistence. Interestingly, the threshold value of the LSTAR model helps to explain the switching mechanism between persistence and anti-persistence. The first, lower regime defines inflation as an anti-persistence process. Conversely, the second, higher regime defines inflation as a persistence process.

These empirical findings lead to important implications for the conduct of monetary policy. Knowledge that inflation persistence changes over time implies that the response to inflationary shock also changes over time. This may provide the monetary authorities with alternative instruments to control the economy. Most importantly, these empirical findings remind us that a "one-size-fits-all" monetary policy does not likely work in all circumstances.

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Table 1: Constant-parameter estimates of inflation persistence

$\hat{d}_{GPH}$	$\hat{d}_{GPH}$	$\hat{d}_{LSE\_AV}$	$\hat{d}_{LSE\_J}$	$\hat{d}_{ILSE}$
0.2589***	0.2589***	0.1978***	0.2324***	0.2486***

Note: *** indicate 1 percent significance level. $\hat{d}_{GPH}$ is the GPH estimator of the Geweke and Porter-Hudak (1983), $\hat{d}_{ELW}$ is the Exact Local Whittle estimator of Shimotsu and Phillips (2005), $\hat{d}_{LSE\_AV}$ is the estimator of Abry and Veitch (1998), $\hat{d}_{LSE\_J}$ is the estimator of Jensen (1999b), $\hat{d}_{ILSE}$ is the mean of the instantaneous least squares estimator with boundary correction, $\hat{d}_{ILSE} = \frac{1}{T}\sum_{t=1}^{T}\hat{d}_{ISLE}(t)$, and T is the number of observations.

Table 2: Test of constancy of the fractional integration parameter

F_1	p-value
31.73e+3	8.35e-76***

Note: F_1 is the statistic of the Fisher test where the null hypothesis is given by H''_0. *** indicates rejection of the null hypothesis at the 1 percent level.

Table 3: Test of the nested hypothesis

F_2	F_3	F_4
3.29e-5	1.1e-25	11.74e+31
(10.71e+3)	(11.73e+3)	(1.065e-50)***

Note: *** indicates rejection of the null hypothesis at the 1 percent level. p-values appear in parentheses.

Table 4: Estimates of the LSTAR model of inflation persistence

$\hat{d}_1$	$\hat{d}_2$	$\hat{\gamma}$	$\hat{c}$
-0.0479***	0.3726***	5.6438***	0.2735***

Note: ***indicate significances at the 1-percent significance. The standard deviations used to calculate the t-statistics are based on bootstrapping to correct the distortions of the significance levels.

Figure 1: Monthly Inflation Rate

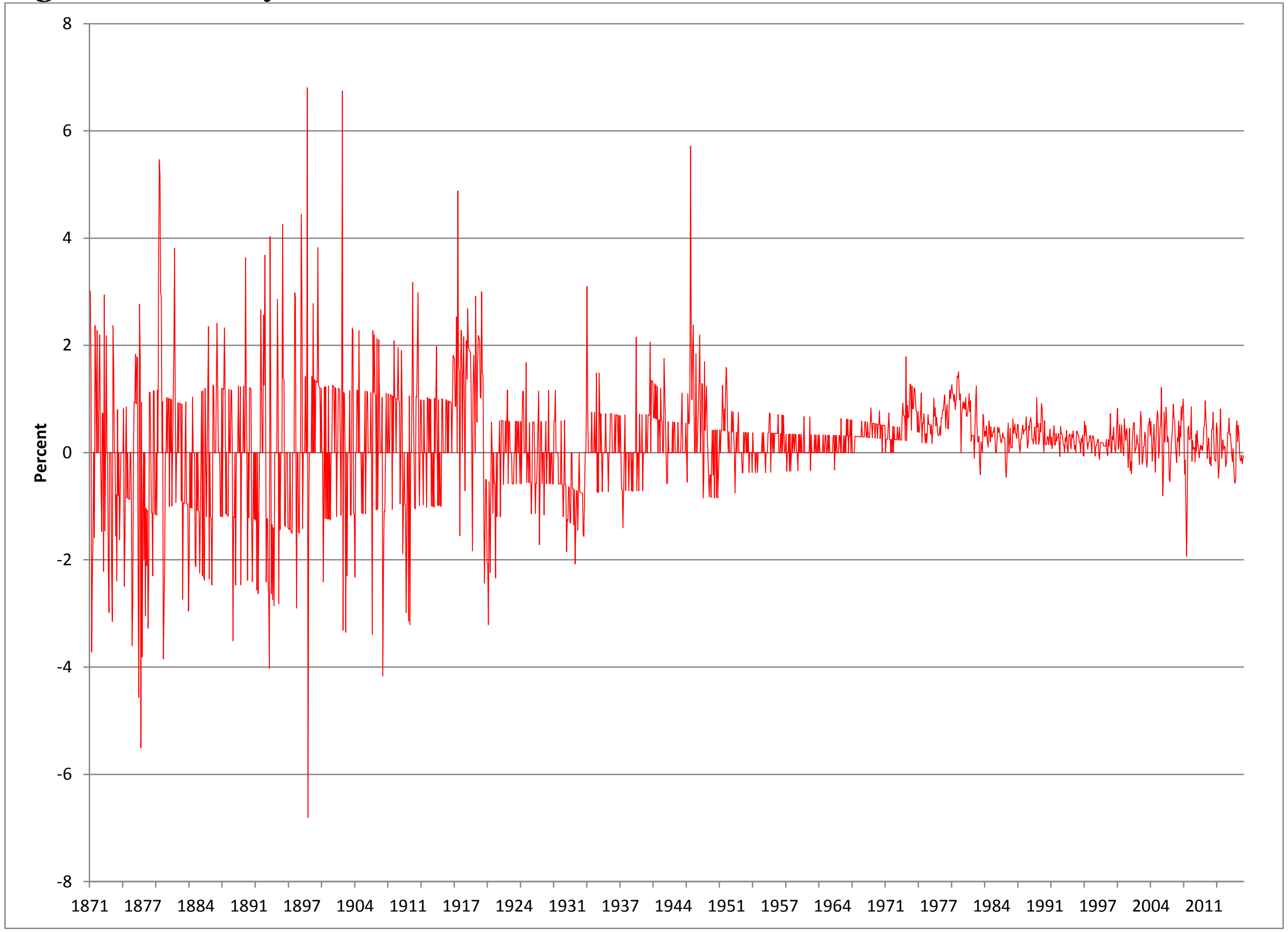

Figure 2: Time-Varying Fractional Integration Parameter