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Effective Optical Depth as a Means for Simplifying Radiation-Conduction Analysis

Edward E. Anderson and Zirong Hu
Department of Mechanical Engineering
Texas Tech University
Lubbock, Texas 79409
ed.anderson@ttu.edu

Abstract

Analysis of any heat transfer problem that includes thermal radiation with absorption and emission is always complicated by the "action-at-a-distance" nature of thermal radiation. This implies that the entire temperature field has to be determined simultaneously rather than focusing on a single location and the immediate neighborhood about that location as in a conduction-only problem. This paper introduces the concept of the effective optical depth (EOD) which limits the range about a location over which a solution is conducted rather than solving the entire temperature field at once. In this study, we investigate the error introduced in the solution of a planar, gray, radiation-conduction heat transfer problem with black and grey boundaries for a range of EOD/optical thickness ratio from 0.01 to 10 and conduction-radiation parameter from 0.01 to 10. In general, the accuracy of the predicted temperature profiles and total heat flux was within a few percentage points and was observed to be as high as 10% for small EOD/optical thickness ratios. Computational times using the finite element method were estimated to be 12.5% or less using the EOD as compared to solving the entire temperature field for each element. This savings in computational time may justify the small errors introduced by using the EOD approximation.

Nomenclature

A – Area, m^2
EOD – Effective Optical Depth measured in optical distance
G – Incident radiant flux – kJ/s-m^2
I_b – Blackbody function, kJ/s-m^2
k – Thermal conductivity, kJ/K-s-m
N – Conduction-radiation parameter, $k\kappa/4\sigma T^4$
q_r – Radiative heat flux, kJ/s-m^2
r – Radius, m
r_0 – Origin, m
T – Temperature, K
T_w – Bounding wall temperature, K
V – Volume, m^3
κ – Absorption coefficient, m^{-1}
ε – Wall emissivity
θ – angle, radians
τ – Optical coordinate or distance, κx
τ_L – optical thickness

Introduction

The "at-a-distance" nature of volumetric radiation heat transfer in absorbing, emitting, and scattering media makes the analysis of the energy transfer in these situations difficult. Heat transfer analysis becomes even more difficult when radiative transfer is coupled with conduction and convection. The primary difficulty with analyzing radiative transfer coupled with conduction and convection is the difference in their governing equations. Conduction and convection are described by differential equations because a point in the medium is only influenced by its immediate neighborhood while volumetric radiative transfer is governed by integral equations since the entire medium impacts the energy balance at a point in the medium. Thus, one must solve integro-differential equations when analyzing heat transfer situations that include volumetric radiative transfer. An approximate numerical method for solving combined conduction-radiation without scattering in a one-dimensional planar geometry under steady-state conditions with temperature-independent properties is presented in this paper. The limitations of this approximation are also determined for varying boundary conditions and radiation-conduction parameter *N*.

Extensive reviews of heat transfer in semitransparent media have been presented by Viskanta and Anderson [1] and Howell [2]. A variety of numerical methods, including finite-difference (FD), P-N, Zonal, discrete-ordinate, Monte Carlo, and successive approximations, have been applied to the planar, conduction-radiation in an absorbing medium problem [3] with varying degrees of success. All have been demonstrated to be successful when the problem is conduction dominant, large *N*, and tend to become problematic and inaccurate when the field is radiation dominant, small *N*. All of these methods are difficult to apply to more realistic multi-dimension, temperature-variable properties, unsteady, and etc. situations. These complexities also increase the computer resources (and consequently cost) that are needed to effect a solution. Highly refined finite-element methods are now extensively available for heat transfer analysis [4]. Razzaque [5], Utreja, and Chung [6], Chung and Kim [7] have developed and applied the finite element (FE) method to more complicated combined mode heat transfer situations.

Various approximations, such as the Rosseland diffusion [8], Taylor series [9] and exponential integral approximations, have been introduced to simplify the analysis of radiative heat transfer. These approximations have limited ranges of applicability, typically $N \approx 1$ and greater, and do introduce errors in the analysis. These limitations are offset by their simplification and reduction in analytical resources required to obtain solutions.

This study proposes and investigates the effective optical depth (EOD) approximation as a technique for reducing the computational resources required for combined conduction-radiative heat transfer analysis. The basic premise of this approximation is that one only need consider the finite elements or finite-difference nodes that are within the EOD of an element or node rather than include the entire field when conducting FE or FD analysis. This approximation is motivated by the exponential decay of the radiative intensity and heat flux with distance from the source. Hence, the emitting media sufficiently far from the absorbing media makes but a small contribution to the energy balance of the absorbing node or element.

Analysis

The steady-state heat balance for a differential element of a gray absorbing, emitting, and conducting medium gives:

$$\nabla\left(k\nabla T\right)-\nabla q_r = 0 \qquad (1)$$

where the radiative heat flux divergence at location r is given by:

$$\nabla q_r = \kappa\left[4\pi I_b(T)-G(r)\right]. \qquad (2)$$

Equation 2 accounts for both the local emission of thermal radiation through the black-body function $I_b(T)$ and absorption of the radiation incident, $G(r)$ on the medium element. The incident radiation consists of that which is leaving the bounding surfaces and arrives at the medium element, and that which is emitted by all the other medium elements and arrives at the medium element,

$$\begin{aligned} G(r) &= \int_A I^*(r_0)\frac{\cos\theta}{\left|r-r_0\right|^2}e^{-\kappa(r-r_0)}dA \\ &+\int_V \kappa I_b(T(r'))\frac{1}{\left|r-r'\right|^2}e^{-\kappa(r-r')}dV \end{aligned} \qquad (3)$$

Observe how the radiant intensity leaving a boundary surface element, $I^*(r_0)$, is attenuated by the optical distance, $\kappa\left(r-r_0\right)$, between it and the medium element under consideration. The same attenuation occurs over the optical distance, $\kappa\left(r-r'\right)$, between a volume emitting element and the element under consideration.

These equations need to be solved subject to the boundary conditions:

$$T(r_0) = T_w \qquad (4)$$

And

$$I^*\left(r_0\right) = \varepsilon I_b\left(T\left(r_0\right)\right)+\frac{(1-\varepsilon)}{2\pi}G^*\left(r_0\right) \qquad (5)$$

Equation 4 is the conduction boundary condition that sets the medium temperature to that of the bounding surfaces where it contacts the boundaries. Equation 5 is for a gray diffusely emitting and reflecting boundary which relates the radiant intensity leaving the bounding surface to that emitted by the surface and the reflection of the radiant intensity incident upon the bounding surface. The exponential divided by distance squared kernel of the two integrals of Equation 3 suggests that these integrals only need be performed over bounding surface and emitting volume elements that are within a limited number of optical depths, κr, of the medium element being considered.

The governing equations given above were further reduced to fit a one-dimensional planar geometry with gray, diffuse, isothermal walls maintained at temperatures T_1 and T_2 [10]. Chung's [6, 7] development and implementation of the general Galerkin finite element methodology was then applied to develop the finite element model for the heat transfer between the bounding walls. This model incorporated the optical thickness, effective optical depth, conduction-radiation interaction parameter, wall temperatures, and wall emittance as parameters.

The natural nonlinearities of radiative transfer were accommodated in this model with successive iterations and over-relaxation. The integrals were resolved using Gaussian quadratures. Solutions were accepted when the individual element difference between iterations were $1(10^{-5})$ or less and the integrated difference was $1(10^{-3})$ or less. The code was verified by comparing the predicted temperature profiles and heat fluxes with the EOD equal to the layer optical thickness to those results of other researchers [1, 3, 11]. These verifications were conducted for the conditions where the temperature of one wall is one-half that of the other wall, $\theta_2 = 0.5\theta_1$, the layer optical thickness, τ_L, was 1, the conduction-radiation parameter, N, varied from 0.00001 to 10, and the wall emittance, ε, was either 0.5 or 1.0. The maximum temperature error for N = 10, τ_L = 1, and ε = 1 was 0.15%. At higher layer optical thickness, τ_L = 10, the temperature error rose to 3.5%. With grey walls, ε = 0.5, N = 0.03, and τ_L = 10, the maximum temperature error was 1.5%. Total heat flux comparisons were made for black walls, $0.01 \le N \le 10$, and $0.01 \le \tau_L \le 10$. The heat flux predicted by the code was exact to within 3 significant digits over this entire range. These errors are not inconsistent with the errors of other numerical methods [1].

Results

Once the code was verified, parametric studies were conducted to examine the temperature distribution and total heat flux errors produced when the EOD optical depth was one-half of the layer optical depth or less, $\tau_{EOD} \le 0.5\tau_L$. Results predicted by the code when $\tau_{EOD} = \tau_L$ were considered to be exact.

Figure 1 illustrates the error in the temperature distribution for N = 10, τ_L =10, ε =1, and the right-hand boundary temperature is one-half that of the left-hand boundary. This is a conduction dominant situation and the temperature profile is almost linear. Inspection of Figure 1 demonstrates that the maximum error is quite reasonable and acceptable even when only one-tenth of the radiation field is included in the calculations.

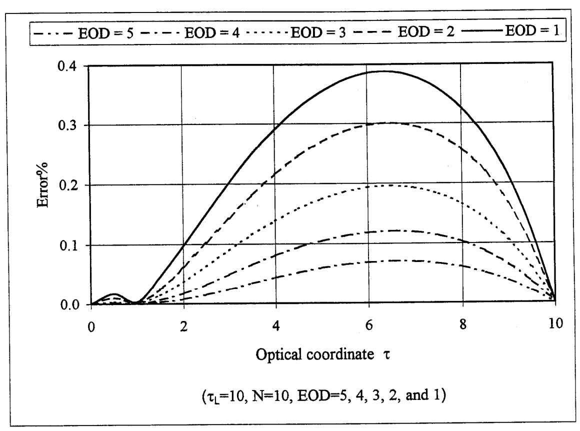

Figure 1 Temperature Prediction Error for Various EOD (N = 10, τ_L =10, ε =1)

The error in the temperature distribution, as shown in Figure 1, goes to zero as more and more of the layer, EOD approaching τ_L, is included in the calculation of the radiative hat flux. But, since this is a conduction-dominant situation and the radiative heat transfer is small, this error is acceptable even for EOD =1.

A more radiation dominant situation, N = 0.10, is shown in Figure 2. The error for small EOD is now large and may be unacceptable and most of the radiation field must be included in the model. At least 30% or more of the radiation field must be included in the calculations if temperature errors of 1.5% or less are required.

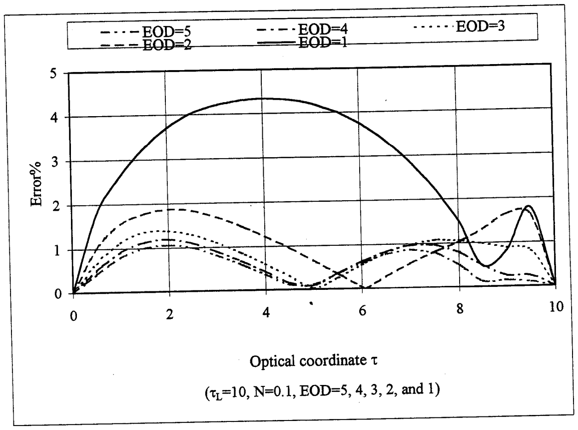

Figure 2 Temperature Prediction Error for Various EOD (N = 0.1, τ_L =10, ε =1)

Figure 2 demonstrates that the greatest temperature error occurs in the hottest portions of the layer near the left-hand hot boundary. This is the region where the radiative heat flux and the blackbody function are the greatest. Only the radiative heat flux is approximated. Thus, errors can be expected to be the largest where the radiative heat flux is the greatest.

Maximum temperature prediction errors for small layer optical thicknesses are presented in Figure 3. These results demonstrate that temperature prediction errors tend to decrease with increasing layer optical thickness. The error also tends to be the largest at conduction-radiation parameter values at or near 0.1.

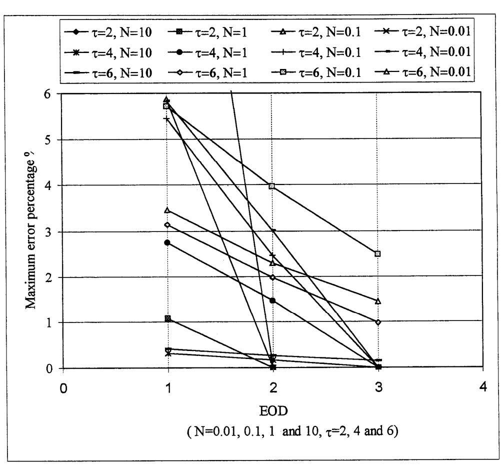

Figure 3 Maximum Temperature Prediction Errors

The total heat fluxes predicted by the finite-element model are presented in Table 1. Conditions under which the error exceeded 2% or more are indicated by the shaded cells in the table. This table indicates that total heat flux predictions are quite accurate as is typical with most numerical solutions of the conduction-radiation heat transfer problem. Heat fluxes are accurately predicted when the EOD is 30% or more of the layer optical thickness.

Layer Optical Thickness = 10				
EOD	10	5	2	1
N				
10	2.1147	2.1133	2.1046	2.0901
1	0.3146	0.3122	0.2988	0.2825
0.1	0.1326	0.1279	0.1023	0.06342
0.01	0.1099	0.105	0.07796	0.03272

Layer Optical Thickness = 6				
10			3.5066	3.499
1			0.5006	0.4909
0.1		0.2053	0.1819	0.1575
0.01		0.1671	0.1399	0.1013

Layer Optical Thickness = 2				
10			10.403	10.398
1			1.4008	1.3921
0.1			0.4845	0.4697
0.01			0.3691	0.3437

Table 1 Total Heat Fluxes, kJ/s-m^2

Conclusions

It has been demonstrated that the EOD is an accurate means of predicting coupled conduction – radiation heat transfer over a wide range of the conduction-radiation parameter and layer optical thicknesses of 2 or more. Acceptable error in the predictions typically occurs when the EOD is 30% or more of the layer optical thickness. Thus, only about 30% of the radiation field impacts the energy balance of any finite element in the medium for the conditions investigated. Temperature predictions are more sensitive to this approximation than total heat flux predictions. Depending upon the error requirements of a given application, this approximation should be acceptable and does reduce the computing resources required. Similar findings are expected in problems involving more complicated geometries and situations.

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