A BVAR MODEL FOR THE SOUTH AFRICAN ECONOMY

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Abstract
The paper develops a Bayesian vector autoregressive (BVAR) model of the South African economy for the period of 1970:1-2000:4 and forecasts GDP, consumption, investment, short-term and long term interest rates, and the CPI. We find that a tight prior produces relatively more accurate forecasts than a loose one. The out-of-sample-forecast accuracy resulting from the BVAR model is compared with the same generated from the univariate and unrestricted VAR models. The BVAR model is found to produce the most accurate out of sample forecasts. The same is also capable of correctly predicting the direction of change in the chosen macroeconomic indicators.

1. INTRODUCTION

This paper develops a Bayesian vector autoregressive model (BVAR) for the South African economy to forecast real Gross Domestic Product (GDP), consumption, investment, short-term (91 days Treasury bill rate) and long term interest rates (10 years or longer government bond rate), and the Consumer Price Index (CPI), based on quarterly data for the period of 1970:1 to 2000:4. The accuracy of the out-of-sample forecasts from the BVAR model, over the period of 2001:1 to 2005:4, is compared with that of the same generated by the (benchmark) univariate models for each variable, and an unrestricted VAR model.

Generally, in the literature,[1] the multivariate BVAR models have been found to produce the most accurate short- and long-term out-of-sample forecasts relative to the univariate and unrestricted Classical VAR models. Moreover, the BVAR models are also capable of correctly predicting the direction of change of the macroeconomic variables. In such a backdrop, this paper tries to analyze the capability of a BVAR model in forecasting the South African economy relative to alternative 'popular' methods of forecasting. The result of such an analysis would be to obtain a BVAR model, in case it is relatively superior to the other standard forecasting methods, which, in turn, can then be used to forecast the

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[1] For example, Amirizadeh and Todd (1984), Kuprianov and Lupoletti (1984), Hoehn *et al.* (1984), Hoehn and Balazsy (1985), Kinal and Ratner (1986), Gruben and Long (1988a, b), Lesage (1990), Gruben and Hayes (1991), Shoesmith (1992), Dua and Ray (1995), Dua and Smyth (1995), Dua and Miller (1996), Dua *et al.* (1999) and Banerji *et al.* (2006).

macroeconomy. To the best of our knowledge this is the first attempt to forecast the South African economy using BVAR models.[2]

Besides the introduction and the conclusion, the paper is structured in the following fashion: section 2 discusses the advantages of using a VAR model *versus* a structural model,[3] and also describes the parameters required to specify a BVAR model. Section 3 lays out the model for the South African Economy, while, section 4 compares the accuracy of the out-of-sample forecasts generated from alternative models. Section 5 discusses, in detail, the performance of the alternative models used for forecasting, in terms of their ability to predict the turning points in the economy, if any.

2. ADVANTAGES OF USING VAR OVER STRUCTURAL MODELS

Generally, economy-wide forecasting models are in the form of simultaneous-equations structural models. However, two problems, often, encountered with such models are as follows: *(i)* Correct number of variables needs to excluded, for proper identification of individual equations in the system, which are, however, often based on little theoretical justification (Cooley and LeRoy (1985)), and; *(ii)* Given that, projected future values are required for the exogenous variables in the system, structural models are poorly suited to forecasting.

The Vector Autoregressive (VAR) model, though 'atheoretical' is particularly useful for forecasting purposes. Moreover, as shown by Zellner (1979) and Zellner and Palm (1974) any structural linear model can be expressed as a VAR moving average (VARMA) model, with the coefficients of the VARMA model being combinations of the structural coefficients. Under certain conditions, a VARMA model can be expressed as a VAR and a VMA model. Thus, a VAR model can be visualized as an approximation of the reduced-form simultaneous equation structural model.

Note an unrestricted VAR model, as suggested by Sims (1980), can be written as follows:

$$y_t = C + A(L)y_t + \mathbf{e}, \tag{1}$$

where y: $(nX\ 1)$ vector of variables being forecasted; $A(L)$: $(nX\ n)$ polynomial matrix in the backshift operator L with lag length p, *i.e.*, $A(L) = A1L + A_2L^2 +$ .... $+ Apll$; C: $(\,n\,X\ \ 1)$ vector of constant terms, and; $\mathbf{£}$: $(n\,\boldsymbol{X}\ 1)$ vector of white-noise error terms.

The VAR model uses equal lag length for all the variables of the model. One drawback of VAR models is that many parameters are needed to be estimated, some of which may be insignificant. This problem of overparameterization, resulting in multicollinearity and loss of degrees of freedom leads to inefficient estimates and large out-of-sample

[2] However, economy wide forecasting using BVAR models have been widely used in the United States, ever since Litterman (1986). For a detailed review on the use of BVAR models on forecasting, both at country- and regional (state)-levels, as well as, the housing market, see Dua and Ray (1995) and Banerji *et al.* (2006).

[3] This section of the paper relies heavily on the discussion available in Dua and Ray (1995) and Banerji *et al.* (2006).

forecasting errors. One solution, often adapted, is simply to exclude the insignificant lags based on statistical tests. Another approach is to use near VAR, which specifies unequal number of lags for the different equations.

However, an alternative approach to overcome this overparameterization, as described in Litterman (1981), Doan *et al.* (1984), Todd (1984), Litterman (1986), and Spencer (1993), is to use a Bayesian VAR (BVAR) model. Instead of eliminating longer lags, the Bayesian method imposes restrictions on these coefficients by assuming that these are more likely to be near zero than the coefficient on shorter lags. However, if there are strong effects from less important variables, the data can override this assumption. The restrictions are imposed by specifying normal prior distributions with zero means and small standard deviations for all coefficients with the standard deviation decreasing as the lags increases. The exception to this is, however, the coefficient on the first own lag of a variable, which has a mean of unity. Note Litterman (1981) used a diffuse prior for the constant. This is popularly referred to as the 'Minnesota prior' due to its development at the University of Minnesota and the Federal Reserve Bank at Minneapolis.

The standard deviation of the distribution of the prior for lag *m* of variable *j* in equation *i* for all *i, j* and *m,* defined as *S(i, j, m),* can be specified as follows:

$$S(i, j, m) = [w \times g(m) \times [f(i, j)] \frac{\sigma_i}{\sigma_j} \qquad (2)$$

with _/"(/', *j)* = 1, if *i* =*j* and *ky* otherwise, with (0 < *ky* < *1), g(m)* = *nT* , *d* > 0. Note *O,* is the standard error of the univariate autoregression for variable *i.* The ratio *O*/*OJ* scales the variables so as to account for differences in the units of measurement and, hence, causes specification of the prior without consideration of the magnitudes of the variables. The term *w* indicates the overall tightness and is also the standard deviation on the first own lag, with the prior getting tighter as we reduce the value. The parameter *g(m)* measures the tightness on lag *m* with respect to lag 1, and is assumed to have a harmonic shape with a decay factor of *d,* increasing which tightens the prior on increasing lags. The parameter *f(i, j)* represents the tightness of variable *j* in equation *i* relative to variable *i,* and by increasing the interaction, *i.e.,* the value of *ky,* we can loosen the prior.

The BVAR model is estimated using Theil's (1971) mixed estimation technique, which involves supplementing the data with prior information on the distribution of the coefficients. In an artificial way, the number of observations and degrees of freedom are increased by one, for each restriction imposed on the parameter estimates. The loss of degrees of freedom due to over parameterization associated with a VAR model is, therefore, not a concern in the BVAR model.

It must be pointed out that some econometricians, like Lutkepohl (1993, p. 375), has claimed that the Minnesota prior is not a good choice if the variables in the system are believed to be cointegrated. He makes such an argument based on the interpretation of the prior as to suggesting that the variables are roughly random walks. Moreover, Engle and Yoo (1987) argued that with the Minnesota prior, a BVAR model approaches the classical VAR model with differenced data, and, hence, would be misspecified for cointegrated variables without an error correction term.

[4] For an illustration, see Dua and Ray (1995).

But as LeSage (1990) and Dua and Ray (1995) indicates that the suggestion of the Minnesota prior being inappropriate, when the variables are cointegrated is incorrect. They point out that the prior sets the mean of the first lag of each variable equal to one in its own equation and sets all the other coefficients to be zero, and hence, this implies, if the prior means were indeed the true parameter values, each variable would be a random walk. But at the same time the prior probability that the coefficients are actually at the prior mean is zero. The Minnesota prior, indeed, places high probability on the class of models that are stationary. Alternatively, if a model is specified in levels is equivalent to one in differences, then the sum of the coefficients on the own lags will equal to one, while, the sum of the coefficients on the other variables exactly equals zero. Though this holds for the mean of the Minnesota prior, used in this paper, the prior actually assigns a probability of zero to the class of parameter vectors that satisfy this restriction. Lesage (1990) and Dua and Ray (1995), however, points out that if a very tight prior is specified, the estimated model will be close to a model showing no cointegration. With the Minnesota priors, chosen in practice, being not so tight to produce the forecasts, concerns of mispecification with cointegrated data are, therefore, misplaced.

3. A BVAR MODEL FOR THE SOUTH AFRICAN ECONOMY

Along the lines of Litterman (1986) and Ni and Sun (2005), we estimate a BVAR model of the South African economy for the period of 1970:1 to 2000:4, based on quarterly data. Then we compute the out-of-sample one- through eight-quarters-ahead forecasts for the period of 2001:1 to 2005:4 and compare the forecast accuracy relative to that of the forecasts generated by an unrestricted VAR model and our benchmark univariate models for each variable, as in Dua and Miller (1996). The variables included are real GDP, consumption, investment, 91 days Treasury bill rate, 10 years or older government bond rate, and the CPI. All data are seasonally adjusted in order to, *inter alia*, address the fact that as pointed out by Hamilton (1994:362) the Minnesota prior is not well suited for seasonal data. The data are from the Quarterly Bulletin of the Reserve Bank of South Africa. Note real variables correspond to the values of the variables at year 2000's prices. There are 25 parameters, including the constant, in each equation, given the fact that the model is estimated with four lags of each variable, as in Dua and Ray (1995).[5] All variables, except for the measures of the short- and long-term interest rates, have been measured in logarithms. Note Sims *et al.* (1990) indicates that with the Bayesian approach entirely based on the likelihood function, the associated inference does not need to take special account of nonstationarity, since the likelihood function has the same Gaussian shape regardless of the presence of nonstationarity. Given this, the variables have been specified in levels.

[5] Hafer and Sheehan (1989) find that the accuracy of the forecasts from the VAR is sensitive to the choice of lags. Their results indicated that shorter-lagged models are more accurate, in terms of forecasts, than longer lag models. Therefore, as in Dua and Ray (1996), for a 'fair' comparison with the BVAR models, alternative lag structures for the VAR models were also examined. When we reduce the lag length to 3 and then to 2, we find marginal improvements in the accuracy of all six variables, but the rank of ordering, resulting from the alternative forecasts remained unchanged.

The, so called, ‘optimal’ Bayesian prior is selected on the basis of the Theil *U* values of the out-of-sample forecasts. Specifically, the six-variable BVAR model with four lags is estimated for an initial prior for the period of 1970:1 to 2000:4. Then, forecasts along with the Theil *U* values for up to eight-quarters-ahead are computed. One more observation is added to the sample and new forecasts up to eight-quarters-ahead are generated and so on.[6] We use the Kalman filter algorithm in RATS[7] for this purpose. The average of the Theil *U* statistic values for one- to eight-quarters-ahead forecasts for the period 2001:1 to 2005:4 are then examined. We then change the prior and a new set of Theil *U* values is generated. The combination of the parameter values, in the prior, that produces the lowest average Theil *U* values is selected, as the ‘optimal’ Bayesian prior. Following Doan (1990) and Dua *et al.* (1999), we choose 0.1 and 0.2 for the overall tightness (w) and 1 and 2 for the harmonic lag decay parameter (d). Moreover, as in Dua and Ray (1995), we also report our results for a combination of $w = 0.3$ and $d = 0.5$. Finally, a symmetric interaction function $f(i, j)$ is assumed with $k_{ij} = 0.5$, as in Dua and Smyth (1995).

4. EVALUATION OF FORECAST ACCURACY

To evaluate the accuracy of forecasts generated by the BVAR model, we need alternative forecasts. As in Dua and Miller (1996), the benchmark forecasts are obtained from the univariate models for each variable. Note the accuracy of the out-of-sample forecast of one- to eight-quarters-ahead for the period of 2001:1 to 2005:4 has been evaluated using the Theil *U* statistic. Furthermore, it must be noted that since the *U* statistic measures the ratio of the root mean square error[8] (RMSE) of the forecasts generated by a model to the RMSE of no change (naïve) forecasts, the *U* statistic implicitly incorporates a comparison with the naïve model. Hence, a *U* value equal to one would suggest that the model forecasts are as good as naïve. While, a *U* statistic greater (less) than one shows that the no change forecast is better (worse) than the forecasts from the specific model in concern. The *U* statistic is a relative measure of forecast accuracy, compared to the absolute measure given by the RMSE, and, hence, is unit free. We also report the RMSE for the out-of-sample forecast of one- to eight-quarters-ahead for the period of 2001:1 to 2005:4.[9] The measure is not unit free.

To make the Theil *U*s and the RMSEs comparable with the BVAR model, we report the same set of statistics for the out-of-sample forecasts generated from an unrestricted classical VAR. The unrestricted VAR has been estimated in levels with four lags.

[6] Note that if $A_{t+„}$ denotes the actual value of a specific variable in period $t+n$ and ${}_tF_{t+„}$ is the forecast made in period t for $t+n$, the Theil £/statistic can be defined as p(A +.-»#+.) \ X(4+„-4)². For $n = 1$, the summation runs from 2001:1 to 2005:4, and for $n = 2$, the same covers the period of 2001:2 to 2005:4 and so on.

[7] All statistical analysis was performed using RATS, version 5.0.

[8] The root mean square error for the «-periods-ahead forecast is given by the following formula:

RMSE = J'L(A t*n~'F'*n'>² n \ J\[1 where A7 is the number of observations.

[9] To compute the Theil *U* statistic and the RMSEs, the latest revised estimates of the variables were used. Alternatively, one can use the data available at the time of forecast, which are often revised subsequently.

Moreover, unlike in the literature, we also estimate univariate BVAR models for each variable and report the Theil *U*s and the RMSEs for the out-of-sample forecasts generated by these models. Note that this is easily achieved by setting the interaction parameter k_{ij} to a very small value, in our case 0.001.[10] The univariate BVAR model is also estimated for the alternative choices of the overall tightness (*w*) and the decay parameter (*d*), used for the multivariate BVAR model as discussed in the previous section. In Tables 1 to 6, we compare the Theil *U* and the RMSE values of one- to eight-quarters-ahead out-of-sample-forecasts for the period of 2001:1 to 2005:4, generated by the univariate OLS, the unrestricted VAR and the 5 alternative multivariate and univariate BVAR models. The conclusions from these Tables are as follows:

(i) Unlike, the Theil *U* statistics, the RMSEs for all the variables increases with the increase in the forecast horizon. No such specific pattern is observed for the Theil *U* values.
(ii) Comparing the univariate OLS with the univariate BVARs, we find that, except for the case of investment, the univariate BVAR is more accurate than the univariate OLS, in terms of the Theil *U*s. However, for the 10 years and longer government bond rate, the investment and the 91 Days treasury bill rate, the average Theil *U* values for the univariate OLS and the BVAR models producing the minimum avergae Theil *U* values exceed one. This implies that the no change or naïve forecasts are better than the model forecasts for these variables. Moreover for consumption, GDP, long-term interest rate and investment, *i.e.*, 4 out of the 6 variables, the Univariate BVAR model number 5 (BVAR-5-UV, w = 0.3, d = 0.5) produces on average the minimum Theil *U* values. This implies that for these set of variables, at least based on the univariate BVARs, we require a relatively loose prior to obtain lower out-of-sample forecast errors. For the price index and the treasury bill rate, BVAR model number 2 (BVAR-3-UV, w = 0.2, d = 1.0) and BVAR model number 3 (BVAR-2-UV, w = 0.1, d = 1.0), respectively, has the lowest average Theil *U* values.
(iii) Except for the long-term interest rate the BVAR models produces lower out-of-sample forecast errors, in comparison to the univariate models. However, the minimum Theil *U* values obtained from the multivariate BVAR number 5 (BVAR-5-MV, w = 0.3, d = 0.5) is greater than one. But clearly the multivariate BVAR number 4 (BVAR-4-MV, w = 0.1, d = 2) is best suited for forecasting consumption, GDP and investment, given that it has the minimum average Theil *U* values for the eight-quarters-ahead out-of-sample forecasts. So, unlike in the case of the univariate BVARs, the multivariate BVAR tends to suggest that the best forecast can be produced by relatively tighter priors, and as it stands out, in our case, BVAR-4-MV has the the most tight priors. However, we require BVAR-1-MV (w = 0.2, d = 2), to generate the best possible forecasts for the price level, which, in turn, is second in the tightness scale of the priors used. Note the multivariate BVAR-1 has a lower over all tightness parameter but the same decay factor when compared to the multivariate BVAR-4.
(iv) Except for the two interest rate measures, the unrestricted VAR has lower Theil *U* values than the univariate BVARs corresponding to the minimum average *U* statistic. However, the Theil *U* values from the unrestricted VAR is greater than one for investment and the short- and long-term interest rates.

[10] This value is recommended by Doan (2000).

Table 1. Accuracy of out-of-sample forecasts (2001:1-2005:4): Final consumption expenditure by households in logs

Quarter ahead	Forecast statistic	N	OLS (univariate)	U-VAR	BVAR-1 (w = 0.2, d = 2)		BVAR-2 (w = 0.1, d = 1)		BVAR-3 (w = 0.2, d = 1)		BVAR-4 (w = 0.1, d = 2)		BVAR-5 (w = 0.3, d = 0.5)	
					UV	MV	UV	MV	UV	MV	UV	MV	UV	MV
1	U	20	0.586	0.480	0.539	0.370	0.535	0.334	0.527	0.380	0.541	0.321	0.516	0.403
	RMSE		0.007	0.006	0.007	0.005	0.007	0.004	0.007	0.005	0.007	0.004	0.007	0.005
2	U	19	0.600	0.394	0.546	0.363	0.542	0.331	0.536	0.372	0.547	0.320	0.526	0.388
	RMSE		0.015	0.010	0.014	0.009	0.014	0.008	0.014	0.014	0.014	0.008	0.013	0.010
3	U	18	0.615	0.379	0.555	0.354	0.552	0.329	0.548	0.363	0.555	0.319	0.539	0.379
	RMSE		0.023	0.014	0.021	0.013	0.021	0.012	0.021	0.014	0.021	0.012	0.020	0.014
4	U	17	0.634	0.377	0.562	0.346	0.561	0.327	0.558	0.335	0.562	0.320	0.553	0.373
	RMSE		0.032	0.019	0.028	0.017	0.028	0.017	0.028	0.018	0.028	0.016	0.028	0.019
5	U	16	0.647	0.375	0.569	0.343	0.568	0.330	0.567	0.351	0.569	0.326	0.564	0.368
	RMSE		0.041	0.024	0.036	0.022	0.036	0.021	0.036	0.022	0.036	0.021	0.035	0.023
6	U	15	0.654	0.358	0.573	0.337	0.572	0.331	0.572	0.343	0.573	0.329	0.571	0.357
	RMSE		0.049	0.027	0.043	0.025	0.043	0.025	0.043	0.026	0.043	0.025	0.043	0.027
7	U	14	0.657	0.333	0.574	0.324	0.574	0.326	0.574	0.328	0.574	0.326	0.574	0.338
	RMSE		0.057	0.029	0.050	0.028	0.050	0.028	0.050	0.028	0.005	0.028	0.050	0.029
8	U	13	0.657	0.309	0.573	0.311	0.573	0.332	0.573	0.311	0.572	0.324	0.574	0.316
	RMSE		0.065	0.020	0.056	0.031	0.056	0.032	0.056	0.031	0.056	0.032	0.056	0.031
	Average U		0.631	0.376	0.561	0.344	0.560	0.329	0.557	0.350	0.562	0.323	0.552	0.365
	Average RMSE		0.036	0.020	0.032	0.019	0.032	0.018	0.032	0.019	0.032	0.018	0.032	0.020

Note: N is the number of observations; U is the Theil statistics; RMSE is the root mean square error; U-VAR: unrestricted VAR; *MV*: multivariate; UV: univariate.

Table 2. Accuracy of out-of-sample forecasts (2001:1-2005:4): CPI in logs

Quarter ahead	Forecast statistic	N	OLS(univariate)	U-VAR	BVAR-1 (w = 0.2, d = 2)		BVAR-2 (w = 0.1, d = 1)		BVAR-3 (w = 0.2, d = 1)		BVAR-4 (w = 0.1, d = 2)		BVAR-5 (w = 0.3, d = 0.5)	
					UV	MV	UV	MV	UV	MV	UV	MV	UV	MV
1	U	20	0.632	0.626	0.714	0.602	0.686	0.629	0.634	0.597	0.819	0.652	0.631	0.060
	RMSE		0.010	0.010	0.011	0.009	0.011	0.010	0.010	0.009	0.013	0.010	0.010	0.009
2	U	19	0.674	0.673	0.741	0.599	0.706	0.637	0.660	0.611	0.827	0.650	0.664	0.636
	RMSE		0.020	0.020	0.022	0.018	0.021	0.019	0.020	0.018	0.025	0.020	0.020	0.019
3	U	18	0.707	0.700	0.769	0.609	0.736	0.656	0.697	0.635	0.835	0.659	0.700	0.668
	RMSE		0.031	0.031	0.045	0.027	0.033	0.029	0.031	0.028	0.037	0.029	0.031	0.030
4	U	17	0.728	0.708	0.769	0.600	0.741	0.654	0.712	0.635	0.819	0.651	0.719	0.674
	RMSE		0.042	0.041	0.045	0.035	0.043	0.038	0.042	0.037	0.048	0.038	0.042	0.039
5	U	16	0.714	0.684	0.749	0.571	0.723	0.631	0.697	0.609	0.792	0.628	0.705	0.649
	RMSE		0.051	0.049	0.054	0.041	0.052	0.045	0.050	0.044	0.057	0.045	0.051	0.047
6	U	15	0.706	0.656	0.749	0.551	0.722	0.621	0.692	0.588	0.787	0.620	0.696	0.625
	RMSE		0.060	0.056	0.063	0.047	0.061	0.053	0.059	0.050	0.067	0.052	0.059	0.053
7	U	14	0.707	0.637	0.761	0.542	0.733	0.625	0.698	0.577	0.797	0.627	0.696	0.610
	RMSE		0.068	0.061	0.073	0.052	0.071	0.060	0.067	0.056	0.077	0.060	0.067	0.059
8	U	13	0.713	0.630	0.783	0.555	0.756	0.650	0.714	0.583	0.819	0.657	0.704	0.609
	RMSE		0.076	0.067	0.084	0.059	0.081	0.069	0.076	0.062	0.087	0.070	0.075	0.065
	Average U		0.698	0.664	0.754	0.579	0.725	0.638	0.688	0.604	0.812	0.643	0.690	0.635
	Average RMSE		0.045	0.042	0.048	0.036	0.047	0.040	0.044	0.038	0.051	0.041	0.044	0.040

Note: N is the number of observations; U is the Theil statistics; RMSE is the root mean square error; U-VAR: unrestricted VAR; MV: multivariate; UV: univariate.

Table 3. Accuracy of out-of-sample forecasts (2001:1-2005:4): Real GDP in logs

Quarter ahead	Forecast statistic	N	OLS(univariate)	U-VAR	BVAR-1 (w = 0.2, d = 2)		BVAR-2 (w = 0.1, d = 1)		BVAR-3 (w = 0.2, d = 1)		BVAR-4 (w = 0.1, d = 2)		BVAR-5 (w = 0.3, d = 0.5)	
					UV	MV	UV	MV	UV	MV	UV	MV	UV	MV
1	U	20	0.497	0.303	0.538	0.332	0.528	0.330	0.476	0.327	0.576	0.334	0.452	0.317
	RMSE		0.006	0.003	0.005	0.003	0.005	0.003	0.005	0.003	0.006	0.003	0.004	0.006
2	U	19	0.552	0.296	0.562	0.300	0.551	0.295	0.510	0.304	0.587	0.289	0.490	0.306
	RMSE		0.018	0.006	0.011	0.006	0.011	0.006	0.010	0.006	0.012	0.006	0.010	0.006
3	U	18	0.591	0.283	0.577	0.262	0.569	0.253	0.537	0.270	0.593	0.245	0.518	0.280
	RMSE		0.018	0.008	0.017	0.008	0.017	0.008	0.016	0.008	0.018	0.007	0.015	0.010
4	U	17	0.627	0.279	0.586	0.228	0.582	0.211	0.556	0.234	0.597	0.206	0.541	0.253
	RMSE		0.025	0.011	0.023	0.009	0.023	0.008	0.022	0.009	0.024	0.008	0.021	0.010
5	U	16	0.657	0.250	0.594	0.200	0.592	0.183	0.572	0.201	0.601	0.182	0.560	0.218
	RMSE		0.033	0.012	0.029	0.010	0.029	0.009	0.028	0.010	0.030	0.009	0.028	0.011
6	U	15	0.685	0.216	0.601	0.183	0.602	0.171	0.558	0.177	0.606	0.175	0.578	0.186
	RMSE		0.041	0.013	0.036	0.011	0.036	0.010	0.035	0.010	0.036	0.010	0.034	0.011
7	U	14	0.705	0.206	0.605	0.191	0.608	0.187	0.597	0.181	0.608	0.191	0.591	0.180
	RMSE		0.048	0.010	0.041	0.013	0.042	0.013	0.041	0.012	0.042	0.013	0.040	0.012
8	U	13	0.716	0.203	0.605	0.212	0.609	0.215	0.600	0.200	0.606	0.219	0.596	0.189
	RMSE		0.055	0.016	0.046	0.016	0.047	0.017	0.046	0.015	0.047	0.017	0.046	0.015
	Average U		0.629	0.254	0.584	0.239	0.580	0.231	0.555	0.237	0.597	0.230	0.541	0.241
	Average RMSE		0.029	0.100	0.026	0.010	0.026	0.009	0.025	0.009	0.027	0.009	0.025	0.010

Note: N is the number of observations; U is the Theil statistics; RMSE is the root mean square error; U-VAR: unrestricted VAR; MV: multivariate; UV: univariate.

Table 4. Accuracy of out-of-sample forecasts (2001:1-2005:4): 10 years and longer government bond rate

Quarter ahead	Forecast statistic	N	OLS(univariate)	U-VAR	BVAR-1 (w = 0.2, d = 2)		BVAR-2 (w = 0.1, d = 1)		BVAR-3 (w = 0.2, d = 1)		BVAR-4 (w = 0.1, d = 2)		BVAR-5 (w = 0.3, d = 0.5)	
					UV	MV	UV	MV	UV	MV	UV	MV	UV	MV
1	U	20	1.085	1.124	1.090	1.240	1.089	1.208	1.097	1.205	1.087	1.216	1.097	1.165
	RMSE		0.714	0.739	0.717	0.815	0.717	0.795	0.721	0.793	0.715	0.800	0.722	0.766
2	U	19	1.146	1.266	1.144	1.375	1.142	1.327	1.157	1.335	1.136	1.335	1.166	1.298
	RMSE		1.148	1.269	1.146	1.378	1.144	1.330	1.159	1.338	1.138	1.337	1.168	1.301
3	U	18	1.156	1.354	1.191	1.503	1.186	1.436	1.195	1.440	1.182	1.450	1.192	1.384
	RMSE		1.433	1.678	1.476	1.864	1.471	1.781	1.482	1.785	1.465	1.798	1.478	1.716
4	U	17	1.165	1.494	1.271	1.680	1.261	1.585	1.262	1.579	1.260	1.608	1.237	1.496
	RMSE		1.578	2.024	1.721	2.275	1.708	2.147	1.709	2.139	1.706	2.178	1.676	2.026
5	U	16	1.213	1.598	1.340	1.864	1.327	1.748	1.327	1.737	1.326	1.774	1.295	1.618
	RMSE		1.794	2.364	1.982	2.758	1.964	2.586	1.963	2.569	1.962	2.625	1.916	2.393
6	U	15	1.306	1.752	1.405	2.067	1.391	1.933	1.396	1.924	1.388	1.957	1.371	1.780
	RMSE		2.083	2.794	2.242	3.298	2.219	3.083	2.227	3.069	2.214	3.121	2.187	2.840
7	U	14	1.393	1.913	1.470	2.252	1.455	2.102	1.465	2.098	1.450	2.120	1.446	1.935
	RMSE		2.332	3.202	2.461	3.769	2.435	3.518	2.452	3.512	2.428	3.549	2.421	3.240
8	U	13	1.427	1.968	1.458	2.292	1.443	2.143	1.456	2.147	1.438	2.156	1.447	1.984
	RMSE		2.723	3.755	2.783	4.374	2.754	4.088	2.779	4.096	2.744	4.113	2.761	3.787
	Average U		1.236	1.558	1.296	1.784	1.287	1.685	1.294	1.683	1.284	1.702	1.281	1.583
	Average RMSE		1.726	2.228	1.816	2.566	1.801	2.416	1.811	2.413	1.284	2.440	1.791	2.259

Note: N is the number of observations; U is the Theil statistics; RMSE is the root mean square error; U-VAR: unrestricted VAR; MV: multivariate; UV: univariate.

Table 5. Accuracy of out-of-sample forecasts (2001:1-2005:4): Investment expenditure in logs

Quarter ahead	Forecast statistic	N	OLS(univariate)	U-VAR	BVAR-1 (w = 0.2, d = 2)		BVAR-2 (w= 0.1, d = 1)		BVAR-3 (w = 0.2, d = 1)		BVAR-4 (w= 0.1, d = 2)		BVAR-5 (w = 0.3, d = 0.5)	
					UV	MV	UV	MV	UV	MV	UV	MV	UV	MV
1	U	20	1.415	2.005	1.633	1.640	1.528	1.253	1.561	1.674	1.587	1.202	1.535	1.851
	RMSE		0.045	0.064	0.052	0.053	0.049	0.040	0.050	0.054	0.051	0.038	0.049	0.059
2	U	19	1.521	2.116	1.927	1.686	1.807	1.249	1.789	1.742	1.906	1.153	1.713	1.930
	RMSE		0.072	0.101	0.092	0.080	0.086	0.060	0.085	0.083	0.091	0.055	0.082	0.092
3	U	18	1.651	2.085	2.012	1.534	1.900	1.143	1.885	1.644	2.000	1.028	1.844	1.904
	RMSE		0.107	0.135	0.130	0.099	0.123	0.074	0.122	0.106	0.129	0.066	0.119	0.123
4	U	17	1.671	1.992	2.048	1.329	1.942	0.988	1.923	1.475	2.040	0.863	1.879	1.768
	RMSE		0.136	0.162	0.167	0.108	0.158	0.080	0.156	0.120	0.166	0.070	0.153	0.144
5	U	16	1.678	1.850	2.019	1.101	1.925	0.816	1.907	1.257	2.014	0.703	1.873	1.570
	RMSE		0.168	0.185	0.202	0.110	0.193	0.082	0.191	0.126	0.202	0.070	0.188	0.157
6	U	15	1.638	1.603	1.954	0.884	1.873	0.654	1.857	1.035	1.951	0.560	1.829	1.333
	RMSE		0.199	0.194	0.237	0.107	0.227	0.079	0.225	0.125	0.236	0.068	0.222	0.162
7	U	14	1.599	1.352	1.881	0.655	1.813	0.474	1.798	0.793	1.879	0.406	1.776	1.076
	RMSE		0.231	0.195	0.272	0.095	0.262	0.068	0.260	0.115	0.271	0.059	0.256	0.155
8	U	13	1.553	1.133	1.813	0.552	1.755	0.398	1.741	0.640	1.812	0.362	1.721	0.897
	RMSE		0.258	0.188	0.301	0.087	0.291	0.066	0.289	0.106	0.301	0.060	0.286	0.149
	Average U		1.591	1.761	1.911	1.169	1.818	0.872	1.808	1.283	1.889	0.785	1.772	1.541
	Average RMSE		0.152	0.153	0.182	0.092	0.174	0.069	0.172	0.104	0.181	0.061	0.169	0.130

Note: N is the number of observations; U is the Theil statistics; RMSE is the root mean square error; U-VAR: unrestricted VAR; MV: multivariate; UV: univariate.

openUP (June 2007)

Table 6. Accuracy of out-of-sample forecasts (2001:1-2005:4): 91 days Treasury Bill rate

Quarter ahead	Forecast statistic	N	OLS(univariate)	U-VAR	BVAR-1 (w = 0.2, d = 2)		BVAR-2 (w = 0.1, d = 1)		BVAR-3 (w = 0.2, d = 1)		BVAR-4 (w = 0.1, d = 2)		BVAR-5 (w = 0.3, d = 0.5)	
					UV	MV	UV	MV	UV	MV	UV	MV	UV	MV
1	U	20	0.986	0.957	0.945	0.978	0.945	1.091	0.906	0.945	0.993	1.137	0.888	0.925
	RMSE		0.842	0.817	0.807	0.835	0.807	0.931	0.774	0.807	0.848	0.971	0.758	0.790
2	U	19	1.165	1.060	0.984	1.058	0.980	1.201	0.970	1.055	1.005	1.212	0.978	1.042
	RMSE		1.806	1.642	1.525	1.640	1.518	1.861	1.503	1.635	1.557	1.878	1.516	1.615
3	U	18	1.306	1.158	1.002	1.155	0.999	1.322	1.005	1.175	1.009	1.302	1.031	1.157
	RMSE		2.720	2.411	2.087	2.406	2.080	2.753	2.093	2.446	2.102	2.712	2.148	2.409
4	U	17	1.406	1.226	1.008	1.252	1.006	1.430	1.021	1.284	1.009	1.388	1.058	1.251
	RMSE		3.498	3.051	2.508	3.116	2.503	3.557	2.540	3.194	2.509	3.454	2.632	3.112
5	U	16	1.496	1.289	1.014	1.351	1.011	1.532	1.034	1.386	1.009	1.476	1.083	1.331
	RMSE		4.189	3.610	2.838	3.782	2.832	4.289	2.896	3.881	2.824	4.134	3.033	3.727
6	U	15	1.587	1.362	1.021	1.467	1.016	1.646	1.043	1.498	1.014	1.583	1.105	1.418
	RMSE		4.801	4.120	3.089	4.439	3.075	4.981	2.896	4.533	3.067	4.790	3.344	4.290
7	U	14	1.689	1.498	1.033	1.632	1.025	1.802	1.051	1.656	1.026	1.734	1.125	1.555
	RMSE		5.357	4.752	3.277	5.175	3.250	5.716	3.334	5.253	3.255	5.500	3.567	4.932
8	U	13	1.784	1.672	1.050	1.824	1.039	1.978	1.066	1.844	1.043	1.905	1.149	1.729
	RMSE		5.931	5.560	3.490	6.065	3.453	6.576	3.543	6.130	3.467	6.334	3.820	5.747
	Average U		1.427	1.278	1.007	1.340	1.003	1.500	1.012	1.355	1.013	1.467	1.052	1.301
	Average RMSE		3.643	3.245	2.453	3.432	2.440	3.833	2.480	3.485	2.454	3.722	2.602	3.328

Note: N is the number of observations; U is the Theil statistics; RMSE is the root mean square error; U-VAR: unrestricted VAR; *MV*: multivariate; UV: univariate.

(v) The multivariate BVAR number 4 (BVAR-4-MV, $w = 0.1$, $d = 2$) fairs relatively better than the unrestricted VAR for consumption, GDP and investment, while, multivariate BVAR number 1 (BVAR-1-MV, $w = 0.2$, $d = 2$) generates the best possible forecasts for the price index. However, as with the univariate BVARs, the unrestricted VAR has lower, but Theil *U* values of greater than one, for the 91 days treasury bill rate and the long-term government bond rate, when compared to the multivariate BVAR.
(vi) Clearly, from (ii) and (iii), the univariate and the multivariate BVARs reach opposite conclusions in terms of the degree of tightness of the priors, required to produce the best out-of-sample forecasts. While, the univariate BVAR suggests towards using the most loose prior, the multivariate version of the same requires the most tight priors for three of the four variables, *i.e.,* consumption, GDP and investment, it forecasts the best. However, except for the two interest rate measures, the multivariate BVAR for the other four variables outperforms the univariate BVARs by a significant margin. As noted before, in the two cases where the univariate BVAR does better than the multivariate version, the naïve forecasts are better than the model forecasts anyways, with the Theil *U* values exceeding one.

From the above discussion, we observe that the multivariate BVAR model clearly outperforms the other models in terms of forecasting at least consumption, CPI, GDP and investment. Amongst the multivariate BVAR models, the ones with relatively tight priors are found to produce the minimum out-of-sample forecast errors. This result is in sharp contrast with that of Dua and Ray (1996), since they find that a loose prior generally produces more accurate forecasts, but is in line with the findings of Ni and Sun (2005).[11] Interestingly, for the interest rates, the naïve forecasts are always better than any of the model forecasts. In our opinion, this is probably because of the 'atheoretical' aspect of the models used. The interest rates, at least the short-term rates, and in our case the 91 days treasury bill, is likely to move very closely with the specific interest rate variable used as a monetary policy instrument,[12] which, in turn, is adjusted according to perceived or prevailing economic conditions. Hence, the large out-of-sample forecast errors are, perhaps, indicative of the need to have proper theoretical foundations to explain and forecast interest rate movements.

At this stage, it must be pointed out that there are at least two limitations to using a BVAR model for forecasting. Firstly, as it is clear from Tables 1 to 6, the accuracy of the forecasts is sensitive to the choice of the priors. Clearly, then, if the prior is not well specified, an alternative model used for forecasting may perform better. Secondly, in case of the BVAR model, one requires to specify an objective function, for example the Theil *U*s, to search for the 'optimal' priors, which, in turn, needs to be optimized over the period for which we compute the out-of-sample forecasts. However, there is no guarantee that the chosen parameter values specifying the prior will also be 'optimal' beyond the period for which it was selected.

11 Our choice of the priors is in line with the suggestions of Doan (2000).
12 See Sichei (2005) and Ground and Ludi (2006) for detailed reviews on the history of monetary policy in South Africa.

5. TURNING POINTS: THE PERFORMANCE OF ALTERNATIVE MODELS

While, in general, the multivariate BVAR models produces the most accurate forecasts, a different way to evaluate the performance of the alternative models can be based on their ability to predict turning point(s) in the chosen variables. In this regard, we compare the performance of the optimal BVAR models, the univariate and the multivariate, the (benchmark) univariate OLS and the unrestricted VAR with respect to actual data. Note based on the minimum average Theil *U* values for the one- to eight-quarters-ahead out-of-sample forecasts over the period of 2001:1 to 2005:4, the optimal BVAR models that can be identified, from Tables 1 to 6, are:

For the univariate case: The univariate BVAR number 2 (BVAR-2-UV, $w = 0.1$, $d = 1$) is optimal for the 91 days treasury bill rate, and the univariate BVAR number 5 (BVAR-5-UV, $w = 0.3$, $d = 0.5$) is found to produce the minimum average Theil *U* values for Consumption, GDP, investement and the 10 years and older government bond rate. Finally, the univariate BVAR model number 3 (BVAR-3-UV, $w = 0.2$, $d = 1.0$) is the optimal one with respect to the CPI.

For the multivariate case: The multivariate BVAR model number 4 (BVAR-4-MV, $w = 0.1$, $d = 2$) for consumption, GDP and investment, the multivariate BVAR model number 1 (BVAR-1-MV, $w = 0.2$, $d = 2$) for the CPI, and the multivariate BVAR model number 5 (BVAR-5-MV, $w = 0.3$, $d = 0.5$) for the short- and long-term interest rate measures.

As is indicated by Fig. 1 though 6, except for the short-term interest rate measure, the unrestricted VAR and the multivariate 'optimal' BVAR models, generally, correctly predict the direction of change the best, over the period of 2004:1 to 2005:4. The 'optimal' univariate BVAR clearly predicts opposite movements of the interest rates. However, like the 'optimal' univariate BVAR, as can be seen from Fig. 6, the alternative

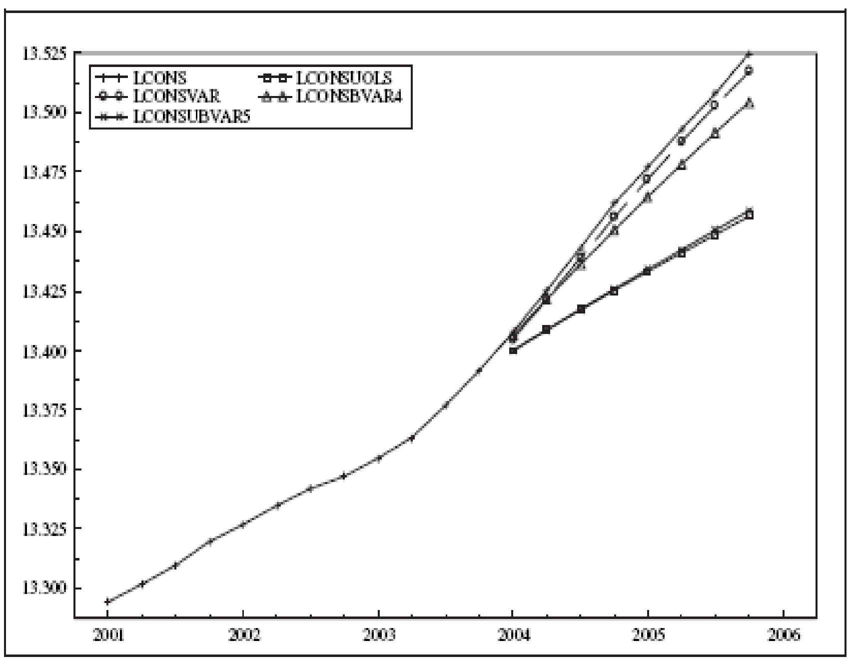

Figure 1. Household consumption, forecasts for 2004:1-2005:4

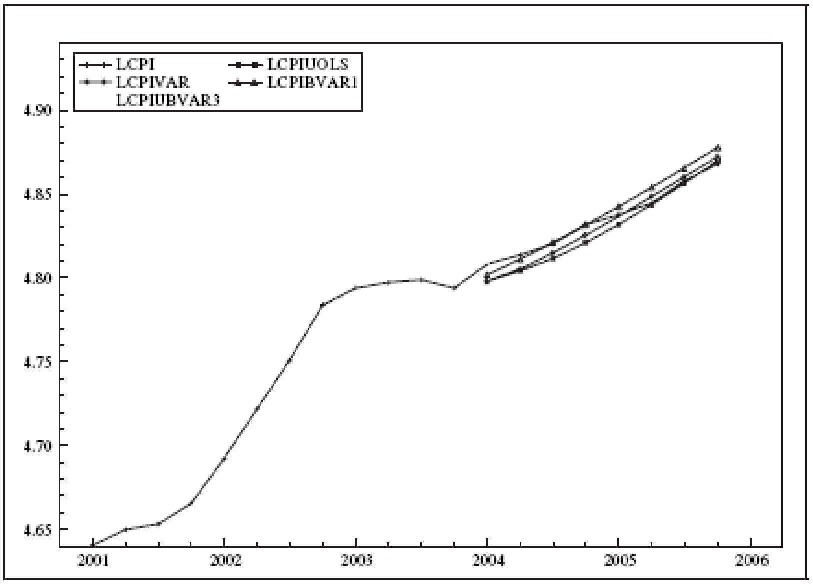

Figure 2. Consumer Price Index, forecasts for 2004:1-2005:4

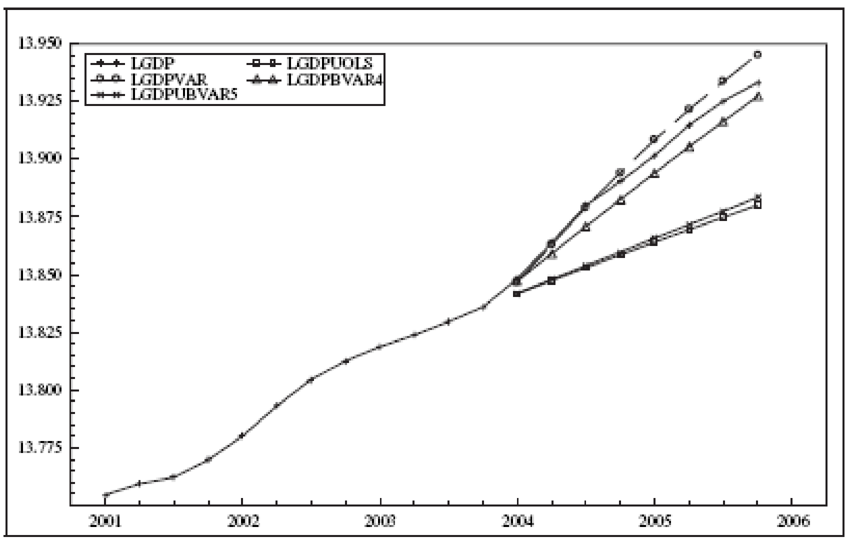

Figure 3. Real gross domestic product, forecasts for 2004:1-2005:4

forecasting models predict an increase in the treasury bill rate when it has actually declined over the period concerned. Note, in terms of predicting the direction of movement for the CPI and the 10 years and longer government bond rate, the univariate OLS fairs nearly as well as the unrestricted VAR and the 'optimal' multivariate BVAR. However, for the other variables, it is clear from the figures, as to why the univariate OLS has such large out-of-sample forecast errors, as observed in the Tables above.

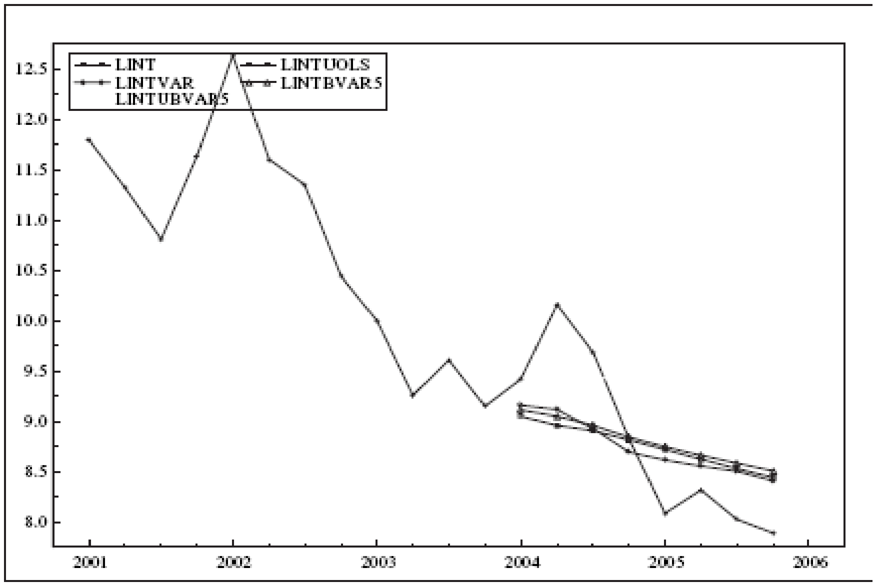

Figure 4. 10 years and longer government bond rate, forecasts for 2004:1-2005:4

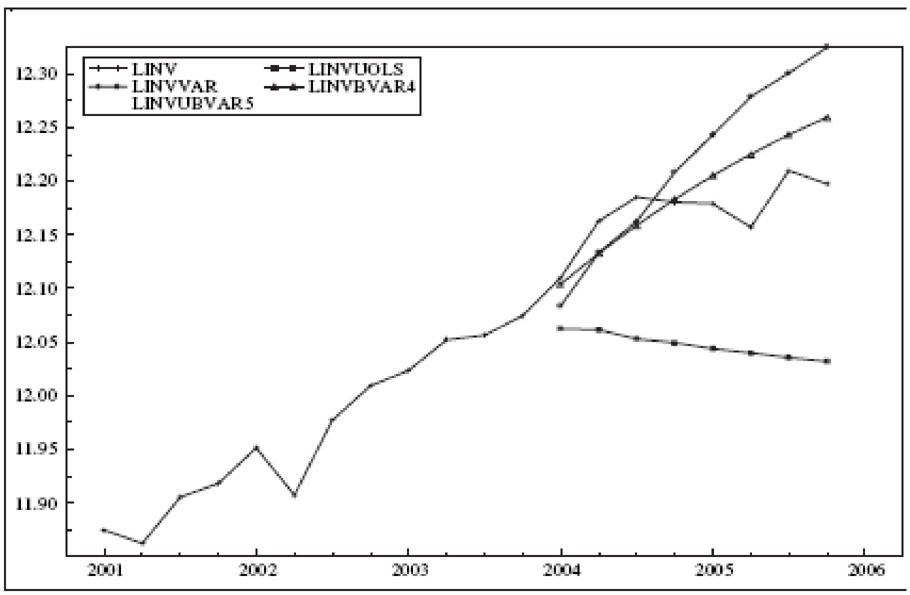

Figure 5. Investment expenditure, forecasts for 2004:1-2005:4

6. CONCLUSIONS AND AREAS OF FURTHER RESEARCH

This paper develops Bayesian vector autoregressive models (BVAR) for the South African economy to forecast real Gross Domestic Product (GDP), consumption, investment, short-term (91 days Treasury Bill rate) and long term interest rates (10 years or longer government bond rate), and the Consumer Price Index (CPI), using quarterly data over the period of 1970:1 to 2000:4. The accuracy of the out-of-sample forecasts from the

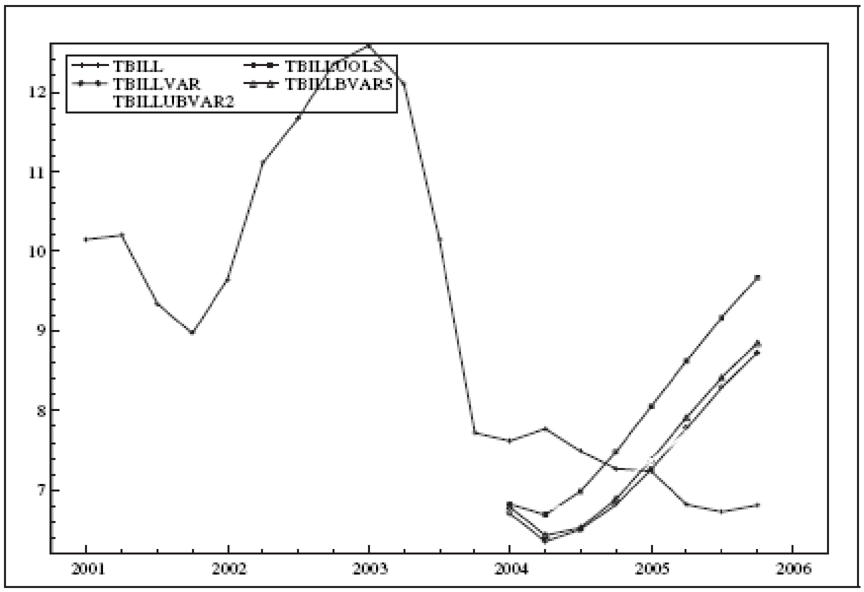

Figure 6. 91 days Treasury Bill rate, forecasts for 2004:1-2005:4

alternative BVAR models, based on the choice of the parameters defining the priors, over the period of 2001:1 to 2005:4, is compared with that of the same generated by the (benchmark) univariate models for each variable, an unrestricted VAR model and univariate versions of the alternative BVAR models.

The multivariate BVAR model, in general, except for the short- and long-term interest rate measures, produces the most accurate forecasts relative to the alternative models. Within the class of the multivariate BVAR models, the models with tighter prior (BVAR-1-MV and BVAR-4-MV) outperform the other models, in terms of forecasting consumption, CPI, GDP and investment. Although, BVAR-1-MV produces the lowest out of sample forecast errors for the price index, note that if we would choose BVAR-4-MV as our model to forecast the South African economy, it would still fair relatively better compared to the univariate and the unrestricted VAR models with respect to the same. The 'optimal' BVAR models, for the exception of the 91 Days Treasury Bill rate, also predicts the direction of change for the chosen variables correctly. In terms of the minimum average Theil *U* values for the out-of-sample forecasts, the multivariate BVAR model number 4 does the best for 3 of the six variables chosen. Based on our study, it seems that a multivariate BVAR model with relatively tight priors is best suited for forecasting the South African economy.

There are, however, as noted earlier, limitations to using the BVAR approach. Firstly, the forecast accuracy depends critically on the specification of the prior, and secondly, the selection of the prior based on some objective function for the out-of-sample forecasts may not be 'optimal' for the time period beyond the period chosen to produce the out-of-sample forecasts.

Besides these, there are two other major concerns, which are, however, general to any traditional statistically estimated models, for example the univariate OLS and the VARS – both Classical and Bayesian, for forecasting at the business cycle frequencies. First, such procedures perform reasonably well as long there are no structural changes experienced in the economy. Such changes, whether in or out of the sample, would then render the models inappropriate. Alternatively, these models are not immune to the 'Lucas Critique'.[13] Furthermore, the estimation procedures used here are linear in nature, and, hence, they fail to take into account of the nonlinearities in the data. One, and, perhaps, the best response to these objections has been the development of micro-founded Dynamic Stochastic General Equilibrium (DSGE) models, which are capable of handling both the problems arising out of the structural changes and the issues of nonlinearities.[14] The current trend in the forecasting-literature is clearly dominated by the use of DSGE models, which, in turn, have also been found to produce better forecasts relative to the traditional forecasting models. In this regard, some studies worth mentioning are: Hansen and Prescott (1993), Ingram and Whiteman (1994), Rotemberg and Woodford (1995), Ireland (2001), and Zimmermann (2001), to name a few. Future research involving DSGE models to forecast the South African economy is, hence, clearly an area to delve into.

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[13] See Lucas (1976) for details.
[14] For a detailed review of the literature on the use of DSGE models for forecasting, see Zimmermann (2001).