Chapter 1

1.1 The objective of building thermal simulation

1.1.3 Optimal design

With the growing environmental consciousness the more efficient use of energy is a priority. International studies have show that 50% of world primary energy consumption is used for building operational costs (Deterding, V.A., Matrosov, Y.A. and Tabandehkar, Y.G.). Clearly any improvement in building energy usage will benefit overall energy consumption.

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According to Hong and Tang there are two reasons to calculate building HVAC system loads. One is to calculate the peak load for design sizing, the other is to calculate the annual energy consumption for energy audit and design of energy efficient building HVAC systems. (Hong, T. and Hang, Y., 1995; Yost, H. and Yost, Y., 1992). This does not include the calculation of passive building inside temperatures.

The idea of passive building design has been used for a long time. Examples are the pueblos of Southwestern United States and the igloos of the Eskimo people. (Moser, W.T., 1983; Ebert, J.P., 1983; Ooster, L.L., 1976). A passive building will be the best design of all, a building that uses no energy at all for climate control.

In many buildings HVAC systems will (and should) be used. The problem is that case studies, such as the six done by Pilk (Plam, C.B., 1990), have shown HVAC systems over-designed by a factor of up to 1.8. This over sizing leads to systems being operated far from the optimal conditions for most of the time, as well as unnecessary initial costs. Furthermore, from the perspective of the building operators, more energy efficient buildings can lead to substantial savings on energy costs.
1.1 The objective of building thermal simulation

1.1.1 Optimal design

With the growing environmental consciousness the more efficient use of energy is a priority. International studies have show that 37% of world primary-energy-consumption is used for building operational costs. (Drozdov, V.A., Matrosov, Y.A. and Tabunschikov, Y.A.) Clearly any improvement in building energy usage will benefit overall energy consumption.

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In many buildings HVAC systems will (and should) be used. The problem is that case studies, such as the six done by Piani (Piani, C.B., 1995), have shown HVAC systems over-designed by a factor of up to 1.8. This over sizing leads to systems being operated far from the optimal conditions for most of the time, as well as unnecessary initial costs. Furthermore, from the perspective of the building operators, more energy efficient buildings can lead to substantial savings on energy costs.
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For this reason we need a tool that can not only calculate the peak load expected on a HVAC system and the annual energy consumption, but also the passive building's temperature to find out if the HVAC system is really needed. This tool can then be used by the engineer when designing a HVAC system, and also by the architect to optimize the building for smaller energy requirements or passive design.

1.1.2 The stochastic answer

It is unrealistic to expect to be able to predict building thermal performance with total accuracy. The weather, one of the most important influences on building thermal performance, behaves in a stochastical manner (Hong, T. and Jiang, Y., 1995). The best way to find the thermal performance would be to find the statistics of the building inside temperature. This brings us to stochastical modeling.

Stochastical modeling is a way to model the statistical parameters of the system rather than the actual response. Stochastic processes has been described thus: 'A stochastic process is one which cannot be modeled deterministically because there are unknown (or unpredictable) factors affecting the variable, which prevent the exact calculation of its future behavior. We can develop stochastic models for these processes which allow us to calculate the expected future value of the process and the probability that a future value will be between two specified limits.' (Hittle, D.C. and Pedersen, C.O., 1981)

Where deterministic methods will have a certain output for a certain input, stochastical methods will have output statistics for input statistics. The question we ask is 'will the HVAC plant be big enough', or for a passive building, 'will the building be comfortable'. Because of the stochastic nature of the problem, the ideal answer is not yes/no, but rather: "yes for 99% or 20% of the time", and we need to know the statistics of the unpredictable input variables, not a single value of the variable.
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There is an easier way around the stochastic model, which is very difficult to solve. If we can generate a large set of input data, with the same statistics as the population, or real data, use all of the data with a deterministic method to generate output data and find the statistics of the output, we will have the same answer. This is known as a Monte Carlo method.

With the generation of data lies a problem: In general, the input data will have cross-correlations between the various input components. Let $S(x)$ donate the statistics of input $x$, and similar for $y$ and $z$. If the effective input function consists of the sum $x+y+z$, then:


Care will have to be taken that terms such as $S(xy)$, donating the statistics of the cross-correlations between $x$ and $y$, are not neglected. The generated data must have the correct statistics, i.e. reflects the real world. To achieve that the cross-correlations have to be determined.
1.2 Existing methods

We have seen that for optimal design the stochastic answer is needed. Before we reinvent the wheel, we must first study the existing methods, to try and find one that suits our needs. Only if we cannot find one should we develop a new method. Since the weather data format tend to be specific to the simulation model used, the model and the weather are discussed hand-in-hand in the following sections.

1.2.1 Deterministic methods

Because of the difficulty of the stochastic methods, deterministic methods were developed. Probably the most well known methods are those of ASHRAE. ASHRAE (ASHRAE Fundamentals, 1997) discusses two principle methods to calculate building heating or cooling peak-load: TFM and CLTD. The Transfer Function Method (TFM) (Mitalas, G.P. 1972) and the Cooling Load Temperature Differential method (CLTD) (Rudoy, W. and Duran, F., 1975) derived from it, use tabulated factors with equations to take into account the time-delay in the cooling load.

These methods either use a 24 hour weather cycle or, from the data as tabulated in ASHRAE Fundamentals, a daily range and a single design day temperature. Clearly they should be considered highly simplified, and are not to be used if a highly accurate answer is needed. They can also not calculate the passive temperatures inside a building. The heating load is calculated as a simple steady state problem.

Software exists to simulate the annual energy performance of buildings requiring a one-year (8760 h or 365 days) data set of weather conditions. The data represents a typical year from the viewpoint of weather-induced energy loads on a building. (ASHRAE Fundamentals, 1997) These sets of data are then also available from ASHRAE.
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The days are simulated separately, and thus give 365 days of output data. No explicit effort was made to represent extreme conditions, so these files do not represent design conditions, and should not be used for HVAC systems sizing. (ASHRAE Fundamentals, 1997) This method was created to give as single output the amount of energy the building will use over the period of one year.

Britain’s Chartered Institution of Building Services Engineers, CIBSE, (Chartered Institution of Building Services Engineers, 1986) gives weather data in several different formats. For heating load plant sizing a single temperature is given for different number of occurrences per heating season. Thus the designer is able to choose the extremeness of the weather he is designing for. A daily mean outdoor temperature is given to calculate heating load energy consumption over the heating season.

For the cooling season wet and dry bulb temperatures are given again for different amounts of occurrences per cooling season, both correlated to each other and separate. Wet and dry bulb temperatures are also given for 1% and 2.5% of hours of the cooling season, with correction factors for height above sea level.

Annual load data is given as banded weather data, where the proportion of the month is given that the average value falls within a certain range or band. Each band within each month is treated as a separate block of weather data when running an environmental computer program and the results obtained are weighted by the proportion of the month within each band. (CIBSE, 1986)

Load calculations can be done by CIBSE’s admittance procedure (CIBSE, 1986), taking into account both radiant and air temperature. The modifying effect of the surfaces is taken into account, and the procedure is suitable for programming into a PC. Manual calculations can be done by a steady state method for heating load, and for cooling load a manual method with correction for time lag is given.
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The South African CSIR (Wentzel, J.D. et al, 1985) provides weather data as 24-hour extreme days. Data is selected according to temperature, with the average of the days exceeding 10% of the values given as the 10% design day. The corresponding humidity, wind and radiation are calculated as the average of the days with a maximum within about 1 degree of the average maximum of the 10% design day.

The South African data is suitable for the calculation methods of ASHRAE or CIBSE. Its higher accuracy, due to the 24-hour format, also makes it more appropriate for methods like the first-order thermal model of Mathews et al (Mathews et al, 1994). This model has been incorporated into the commercially available program QUICK (MCI(Pty) Ltd., 1998), and extended to the third-order thermal model of Van Heerden et al (Van Heerden, E, et al, 1996).

1.2.2 Stochastic and Monte Carlo methods

Recently, with advances in computers and the drive for more energy efficient buildings, several models to account for the random nature of weather have been put forward. These range from creating synthetical weather data that approximate the statistics of real weather over a long period of time, and using it similar to a Reference Year, to breaking the weather up into several components that can be used together to give a stochastic model with statistics similar to that of the real weather.

Van Paasen and De Jong created a method to obtain the ‘Synthetical Reference Outdoor Climate’. The synthetical data are generated by a mathematical model of the outdoor climate, in which all the linear and nonlinear correlations between the weather variables and also their probabilities and auto correlations are formulated. (Van Paasen, A.H. and De Jong, A.G., 1979)
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Later, Hittle and Pedersen expected that weather data could be modeled as the result of both deterministic and stochastic processes. They did a Fourier transform on several years of weather data, and found that ‘true periodicity in the data exists only at the annual cycle and at the diurnal cycle plus its harmonics.’ (Hittle, D.C. and Pedersen, C.O., 1981) This they used to model what they consider the deterministic part of the model. (There must be some deterministic part since we have night, day and seasons. This can be calculated from basic astronomy.) They then used an Auto Regressive Moving Average (ARMA) model to account for the stochastic part. The final model is then the sum of the two parts.

They concluded: ‘Generally, the combined model is more satisfactory than ARMA-only models for characterizing weather data time series, since the deterministic periodic behavior of the data is separated from the stochastic auto regressive behavior.’ (Hittle, D.C. and Pedersen, C.O., 1981)

Hokoi et al used ARMA models for solar radiation, and ARMAX models for air temperature. (Hokoi, S. et al, 1990) Yoshida and Terai proposes a method divided into three components: A Trend component expressing climatic change with time, a deterministic component accounting for the annual and diurnal components modeled by Fourier series, and a stochastic component that they again break up into three components, namely a gradual, a moderate and rapid change random component, with some further refinements. (Yoshida, H. and Terai, T., 1992)

The stochastic weather model (Hong, T. and Jiang, Y., 1995) (Jiang, Y. 1981, J. Sino Refrigeration) (Jiang, Y. 1981, ASHRAE) and building simulation model (Jiang, Y. and Hong, T., 1993) used by Hong and Jiang will now be given, in order that the reader can get a clearer picture of the mathematics involved in stochastic processes.

Hong and Jiang proposed a Vector Auto-Regressive (VAR) time series weather model. They give this as:
$W_r = M_r + DX_r \quad \text{Eq 1.2.2-1}$

where $\tau$ indicates the day number, $W_r$ is a column vector with daily mean and range of the outdoor temperature, daily solar radiation coefficient $Kt$, daily mean and range of the outdoor air humidity respectively. $M_r$ stands for the deterministic part of $W_r$ and $DX_r$ for the random part. With the matrix $D$, $X_r$ becomes a five-dimensional stochastic process and can be modeled by:

$X_r = \Phi X_{r-1} + \Theta A_r \quad \text{Eq 1.2.2-2}$

where $\Phi$ and $\Theta$ describe the auto and cross correlation between each variable. $A_r$ becomes five-dimensional independent random process with standard normal distribution.

They consider the detailed hourly weather data not as important as the daily data because of the significant thermal mass of buildings, and give the following interpolation model to transfer the daily data into hourly data:

$U_r = (u_{1r}, u_{2r}, u_{3r}, \ldots, u_{24r})^T = S(W_r, Q_r) \quad \text{Eq 1.2.2-3}$

Where $Q_r$ is the internal casual gain on the $r$th day, $u_{jr}$ includes temperature, direct and diffuse solar radiation and casual gain at the $j$th hour on the $r$th day, $U_r$ stands here for 24 groups of hourly external and internal disturbance data for the whole day, $S$ is a shape matrix that transfers each variable from daily data into hourly data.

They arrive at the building model by stating that according to the State Space method the thermal processes within a building can be described by

$$\frac{dT}{d\tau} = KT + PU(\tau) \quad \text{Eq 1.2.2-4}$$

where $T$ is a vector consisting of all the node temperatures within the building, $U$ is a vector consisting of all the external and internal disturbances such as the outdoor air temperature, the solar radiation and the casual gains, matrix $K$ and $P$ depend upon the construction of the building and the thermal properties of the building materials and the convective and radiative surface coefficients at each surface.
The zone temperature $t_z$ in the building is obtained from the state variable $T$:

$$t_z = VT \quad \text{Eq 1.2.2-5}$$

where $V$ is a vector. They give the solution of this equation in the form of

$$t_{x \eta} = \sum_{i=0}^{\infty} Z_{ij} u_{\tau-i} \quad \text{Eq 1.2.2-6}$$

where $Z_{ij}$ are calculated from $K$, $P$ and $V$ in Eq 4 and 5, $i$ indicates different coefficients and $j$ indicates different hourly values of the zone temperature. The 24 hourly values of $t_z$ can be written as $T_{x \tau}$, a vector of the dialy zone temperature:

$$T_{x \tau} = (t_{x1}, t_{x2}, ..., t_{x24})^T = \sum_{i=0}^{\infty} Z_i U_{\tau-i} \quad \text{Eq 1.2.2-7}$$

where $Z_i$ consists of $Z_{ij}$ in equation 6. By substituting Eq 3 into 7, the zone temperature can be obtained as:

$$T_{x \tau} = \sum_{i=0}^{\infty} (H_{li} W_{\tau-i} + H_{se} Q_{\tau-i}) \quad \text{Eq 1.2.2-8}$$

where $H_{li}$ and $H_{se}$ are calculated by matrix $Z_i$ and $S$. To simplify the rest of the analysis the influence of the casual gain, $Q_i$ is left out at this stage. As the disturbances $W_z$ consist of the deterministic part and the random part, $T_{x \tau}$, the zone temperature can be written as

$$T_{x \tau} = \sum_{i=0}^{\infty} H_i M_{\tau-i} + \sum_{i=0}^{\infty} H_i DX_{\tau-i} \quad \text{Eq 1.2.2-9}$$

They then divide the zone temperature two parts, the deterministic part $T_{dz}$ and the random part $T_r$:

$$T_{x \tau} = T_{dz} + T_r \quad \text{Eq 1.2.2-10}$$

$$T_{dz} = \sum_{i=0}^{\infty} H_i M_{\tau-i} \quad \text{Eq 1.2.2-11}$$

$$T_r = \sum_{i=0}^{\infty} H_i DX_{\tau-i} \quad \text{Eq 1.2.2-12}$$

From equation 2, the stochastic process $X_\tau$ can then be expressed as...
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\[ X_t = \sum_{i=0}^{\infty} \Psi_i A_{\tau-i} \]  

Eq 1.2.2-13

where

\[ \Psi_i = \Phi_i \Theta \]

Then the stochastic model of the zone temperature can be obtained as

\[ T_{\tau} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} H_i D_j \Psi_j A_{\tau-i-j} = \sum_{i=0}^{\infty} \Omega_i A_{\tau-i} \]  

Eq 1.2.2-14

where

\[ \Omega_i = \sum_{j=0}^{\infty} H_j D_i \Psi_{j-i} \]

As \( A_{\tau} \) are standard normal white noise, \( T_{\tau} \) is also a normally distributed multi-dimensional stochastic process of which the expectation value is zero. As the deterministic part of the zone temperature \( T_{\tau} \) varies with time, \( T_{\tau} \) is a non-stationary process. However, as the probability distribution of the \( T_{\tau} \) at every profile is a normal one, it can be described clearly with its first-order correlation matrix \( R(T_{\tau0}, T_{\tau+k}) \) that can be calculated from equation 14 as

\[ R(T_{\tau0}, T_{\tau+k}) = R(T_{\tau0}, T_{\tau+k}) = R_{\tau\tau}(k) = \sum_{i=0}^{\infty} \Omega_i \Omega_i^T \]  

Eq 1.2.2-15

For \( K = 0 \), the \( R_{\tau\tau}(0) \) is

\[ R_{\tau\tau}(0) = \sum_{i=0}^{\infty} \Omega_i \Omega_i^T \]  

Eq 1.2.2-16

the main diagonal elements in \( R_{\tau\tau}(0) \) will be the deviations of the zone temperature at each hour during a day and the other elements show the relations of the zone temperature between different times within a day. The expected process \( T_{\tau} \) and the first-order correlation matrix \( R_{\tau\tau}(k) \) give the whole characteristics of the zone temperature process.
1.3 Shortcomings of present methods

1.3.1 Deterministic methods

The stochastical problem is very difficult to solve. Not only does it require stochastic models for both buildings and weather, but also the computing power required is often not available to practicing architects and engineers. Since stochastic models were not available, deterministic models were developed to be used in the design of buildings and the sizing of HVAC systems. These deterministic models were not as accurate or complete as the stochastic models would have been, but for lack of better techniques the approximations they supplied help considerably.

These methods were created by organizations such as ASHRAE and CIBSE for use by their members. The problem was approached deterministically from the other angle - the question was if the building would be comfortable with the weather occurring only 1 or 2% of the time, and the answer was a simple yes/no. No or little computing power was required.

Input weather data was chosen as either a mean value or a 24 hour cycle, and well known heat transfer principles was used to calculate the deterministic HVAC load, or inside temperatures.

For the drive to improve the energy efficiency of buildings and their HVAC systems, we need much more sophisticated design measures. The common measures have already been applied. (Yoshida, H. and Terai, T., 1992) The first deterministic methods were developed when no or little computers were available to assist people. Those that were powerful enough were prohibitively expensive. Now everybody has a computer on her or his desk. This lead to the development of stochastic methods.
1.3.2 Stochastic and Monte Carlo methods

The Synthetical Reference Outdoor Climate as developed by Van Paaassen and De Jong is method independent, as it is used to generate a large number of days for an outdoor climate that will conform to the real weather statistics. Although care is taken to account for all auto and cross correlations, it still uses generated data.

Any deterministic simulation method can be used to simulate all of the weather days generated. It can thus be used to generate data for a Monte Carlo method. It would, however, result in a large amount of simulations, which would still be time-consuming.

Hittle and Pedersen use their FFT plus ARMA model to be able to generate large amounts of weather data that was then simulated with Building Loads Analysis and System Thermodynamics (Blast) (Hittle, D.C., 1979). No attempt is made to reduce the amount of data needed. Also cross correlations are not considered. (Hokoi, S. and Matsumoto, M 1993)

Others, such as Hokoi et al and Yoshida et al, give stochastic building simulation models with their stochastical weather models. They base their weather models on the work of Hittle and Pedersen, with certain improvements. Most notably of these are the cross-correlation between Dry-bulb temperature and global radiation.

As can be seen from the discussion on the methods of Jiang and Hong given, these methods are either quite complex, or require simulations to be done for a large number of days. They are not accessible to most people who design buildings and install HVAC systems. This is a serious drawback.
1.4 Overview of Quick

1.4.1 Introduction

As an example of a deterministic model, Quick will be discussed in more detail. Van Heerden (Van Heerden, E et al, 1996) (Van Heerden, E 1997) used the first-order building model of Mathews, Richards and Lombard (Mathews, E.H. et al 1994) (Richards, P.G. 1992) as a point of departure, and continued with a similar philosophy.

The first order model was adequate for passive buildings, but not for buildings with HVAC systems installed. (Van Heerden, E et al, 1996) For this reason Van Heerden created a third order model. One of the very important philosophical ideas behind the first model was to obtain a physically interpretable thermal model. This was continued with the work of Van Heerden (Van Heerden, E. 1997).

Quick uses four climatic variables. Firstly outside dry-bulb temperature, measured in degrees Celsius. Secondly, global radiation on a horizontal surface, measured in kW/m². Global radiation is made up of beam, or direct radiation (i.e. direct sunlight) and diffuse radiation (the radiation that falls on surfaces in the shadow). The third variable is the diffuse radiation on a horizontal surface. Lastly is Relative Humidity, RH. This is the ratio between the mass of moisture in the air and the mass of moisture the air can absorb. (Van Heerden, E. 1997)

Van Heerden recommends the following procedure:
- The air node is treated as a separate node; its capacitance is simply $C_t = V o l c_p$
- The ventilation, infiltration and environmental control are treated separately, i.e. the resistance is given as $1/\text{air changes per second}$.
- Any internal masses are combined and treated as one capacitor.
- Provision is made for two structural heat flow paths:
  1. Glass, fenestration in general, and other low mass structures (low mass path, single node)
  2. A heavy mass path (triple node).
- The heat gains through the floor are treated separately, or incorporated into the internal mass.
- Convective heat gains act directly on the air node.
- Radiative heat gains are weighed according to surface area and act directly on the surface.

1.4.2 The Electrical analogy

The building model can be visualized by the electrical analogy given in Figure 1.4.2-1

![Electrical analogy diagram](image)

Figure 1.4.2-1

Physical restrictions on $C_i$ and $R_i$ are that the steady state thermal resistance used in the model must be equal to the total thermal resistance of the building, and the total capacity used in the model must be equal to the thermal capacity of the building.
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The final model makes provision for all structural heat flow paths, i.e. a high mass flow path, a low mass flow path, internal mass, floors in ground contact, and convective heat flow by ventilation. It is solved using a forward-in-time differencing scheme.

1.5.1 Why not implement an available method?

Since there are stochastic methods available, we have to ask why do we not simply use one of them? The reason is that the stochastic methods available up to now are complex. The weather normally has to be modeled by at least an FFT plus ARMA model, and the correlations between temperature and radiation have to be considered. As can be seen from the equations of Jones and Haug (1988), a complex simulation model cannot be set up. It is not whether they are not freely available to people in the building industry.

The fact that are easy to use, physically conservative methods, such as the one used by Quick, are still in use may be questioned. What if it is simply using a very simple model, combined with a Monte Carlo method, an approximation of the stochastic answer, can be found? This answer would greatly increase the information available and assist in the process of design as well as the setting of HVAC equipment.

Furthermore, it would be far easier to implement, it would be less trouble than to develop a stochastic method or implement one of the highly complex methods actually available.

This way, it will have an impact long before other methods, and it will move from the domain of the academic to be a practical tool.

1.5.2 Closure

The main aim of this study is to develop a Monte Carlo method to calculate the indoor temperature statistics of the building heat transfer problem. This must be practical, available immediately and easy to use. It was decided to stuck to the passive problem for now, so that all effort could be put into the ideas behind the method. Once the method is
1.5 Objective of this study

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Since there are stochastic methods available, we have to ask why do we not simply use one of them? The reason is that the stochastic methods available up to now are complex. The weather normally has to be modeled by at least an FFT plus ARMA model, and cross correlations between temperature and radiation have to be considered. As can be seen from the methods of Jiang and Hong given, a complex simulation model must be set up. It is no wonder they are not freely available to people in the building industry.

The fact that an easy to use, physically interpretable method, such as the one used by Quick is available leads to the question: What if, by simply using a deterministic method together with a Monte Carlo method, an approximation of the stochastical answer could be found? This answer would greatly increase the information available, and assist in the process of design as well as the sizing of HVAC equipment.

Furthermore it would be far easier to implement. It would be less trouble than to develop a stochastic method or implement one of the highly complex methods already available. This way it will have an impact long before other methods, and it will move from the domain of the academic to be a practical tool.

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developed, and all the problems are better understood, it can easily be expanded to include all stochastic aspects.

For the same reasons the input variables was restricted to the weather. Apart from the building envelope, the weather has the most influence on the inside temperatures, more specifically dry-bulb temperature and radiation. (Hittle, D.C. and Pedersen, C.O., 1981) (Hong, T. and Jiang, Y., 1995) Relative humidity will for this reason not be considered a stochastic variable.

Since Quick uses global radiation, made up of diffuse radiation and direct radiation, global radiation cannot be considered apart from diffuse radiation. They will be considered as a single variable for the purpose of the Monte Carlo simulation. Temperature and radiation (combining the input global and diffuse radiation) are thus the chosen stochastic variables.
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