



# Géotechnique Letters

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## **Predicting the time-swell relationship of an unsaturated highly expansive clay using the heat equation**

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**Predicting the time–swell relationship of an unsaturated highly expansive clay using the heat equation**

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Cnr. Lynnwood Rd and University Rd, Hatfield 0028, South Africa**Submission date:** Original – 18/10/2024, R1 – 11/08/2025, For production – 28/10/2025**Figure count:** 9**Table count:** 3

## Abstract

Although extensive research and practical experience have established a good understanding of the magnitude of swell in unsaturated expansive clays due to wetting, significantly less is known regarding the time taken for the swelling strains to develop. This paper describes a mathematical model, based on analogies with conventional consolidation theory and the heat equation, to predict the time-dependent volume increase during swelling of an initially unsaturated expansive clay. A series of oedometer tests on a highly expansive clay was performed to determine the coefficients of swell used in the proposed model. The observed oedometer swelling curves showed good agreement with the theoretical relationship. The results were then used to predict swell over time in a layered centrifuge model constructed from the same clay. The predicted heave closely matched the observed result at any time over an eleven-year swelling period in prototype time.

## Keywords chosen from ICE Publishing list:

expansive soils; unsaturated soils; centrifuge modelling.

## List of notations

$c_s$ :	coefficient of swell	$t$ :	time
$d$ :	drainage/infiltration path length	$t_{50}$ :	time for 50% swell
$d_{avg}$ :	average drainage/infiltration path length	$t_{90}$ :	time for 90% swell
$e$ :	void ratio	$t_m$ :	centrifuge model time
$e_0$ :	initial void ratio	$t_p$ :	centrifuge prototype time
$h_0$ :	initial layer height	$T_s$ :	dimensionless time factor
$i, m, M$ :	summation indices	$U_s$ :	degree of swell
$\sqrt{\text{MSE}}$ :	root of mean squared error	$w$ :	gravimetric water content
$n$ :	number of layers	$w_0$ :	initial water content
$N$ :	centrifuge model scaling factor	$z$ :	depth
$R^2$ :	coefficient of determination	$\Delta H$ :	total heave
$s$ :	suction	$\varepsilon_v$ :	volumetric strain
$s_0$ :	initial suction	$\varepsilon_{v,min}$ :	ultimate swelling strain
$S$ :	degree of saturation	$\bar{\sigma}_v$ :	net vertical stress
$S_0$ :	initial degree of saturation	$\bar{\sigma}_{v,0}$ :	initial net vertical stress

## Acknowledgements

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## 1 1. Introduction

2 In engineering problems where significant volume changes occur, two aspects are typically of  
3 importance to the geotechnical practitioner: the magnitude of deformation, and how long it will  
4 take to occur. Terzaghi's theory of one-dimensional consolidation (Terzaghi, 1923; 1943) has long  
5 been used to address both aspects for fully saturated clays.

6 Unsaturated expansive clays exhibit large volume changes with water content variations during  
7 seasonal wetting and drying. The expected magnitude of swell has received considerable  
8 attention in research and practice. Empirical heave prediction methods based on field monitoring  
9 or laboratory tests have been developed for practitioners (e.g. Van der Merwe, 1964; Brackley,  
10 1980; Jones, 2017), and standard test methods for swelling potential are routinely implemented  
11 (ASTM D4546-21, BS 1377-5:1990). Suction-controlled testing has allowed for the investigation  
12 of complex stress paths (e.g. Monroy et al., 2015), and comprehensive understanding of the  
13 influences of suction and net stress on volume change behaviour of expansive clays has been  
14 offered through several constitutive models (e.g. Gens & Alonso, 1992; Houston & Zhang, 2023).

15 Despite these advances, the time–heave relationship has not been as extensively addressed and  
16 is not generally considered in practice. The previously discussed methods are either empirical,  
17 consider a worst-case scenario where complete wetting reduces suction to zero, or they require  
18 a reliable measure of the expected change in suction. It is uncommon to predict the time it will  
19 take for the volume change to occur, or the magnitude of volume change anticipated for a partial  
20 wetting event over a given timeframe (e.g. design storms/flood events). This paper proposes a  
21 simple model that describes the dissipation of suction and associated one-dimensional swelling  
22 over time using the heat equation, with solutions drawn from conventional consolidation theory  
23 for saturated soils. Researchers have long recognised the potential value of extending established  
24 concepts (e.g. effective stress) and techniques (e.g. measurement of volumetric changes and  
25 porewater pressures) from saturated to unsaturated soil mechanics. However, experimental  
26 efforts to validate such extensions often reveal poor agreement with predictions (Jennings and  
27 Burland, 1962; Fredlund and Morgenstern, 1977). Historically, in-situ measurements of time–  
28 swell relationships in high-plasticity clays are scarce because their extremely low hydraulic  
29 conductivities make the required monitoring periods impractically long. In one extreme case,  
30 Williams (1991) reported swell continuing for over 10 years. Even when such measurements exist,

31 they are rarely adequate for validating prediction models because the boundary conditions of  
32 heterogeneous profiles in situ are poorly defined. To address this, the proposed model was  
33 validated against results from a centrifuge test that simulated the swell of an expansive clay profile  
34 over a prototype timeframe of nearly 12 years – providing a well-controlled, long-duration dataset  
35 that would be almost impossible to obtain in the field.

36

## 37 2. Theoretical model

38 One of the assumptions listed by Terzaghi (1943) in developing the equation for one-dimensional  
39 consolidation was that the soil is fully saturated. However, Terzaghi (1943) noted that in very low  
40 permeability clays (e.g. smectites) swelling is controlled by infiltration of water through diffusion,  
41 and that the fundamental equations governing consolidation and diffusion over time are identical.  
42 In principle, both processes could thus potentially be described using the same mathematical  
43 framework. Furthermore, it can be reasonably assumed that the relationship between suction ( $s$ ),  
44 depth ( $z$ ), and time ( $t$ ) within an element of unsaturated clay during one-dimensional swelling  
45 follows the form of the heat equation, as expressed in Equation 1. A similar concept was proposed  
46 by researchers at the South African National Building Research Institute (De Wet, 1957; Blight,  
47 1965; Blight & De Wet, 1965) but saw limited adoption in practice due to a lack of comprehensive  
48 experimental validation.

49

$$\frac{\partial s}{\partial t} = c_s \frac{\partial^2 s}{\partial z^2} \quad (1)$$

50

51 The coefficient of swell ( $c_s$ ) is likely a function of the hydraulic conductivity and clay  
52 expansiveness. However, an explicit formulation of such relationships is not necessary for volume  
53 change prediction, since the coefficient can be directly obtained from laboratory swell tests.  
54 Utilising the average degree of suction dissipation over the depth of the element  
55 ( $U_s = \int (s_0 - s) dz / \int s_0 dz$ ) and a dimensionless time factor for swell ( $T_s = c_s t / d^2$ , where  $d$  is the  
56 maximum drainage/infiltration flow path length), the solution for Equation 1 for an open layer and  
57 constant initial suction ( $s_0$ ) with depth is:

58

$$U_s = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} \exp(-M^2 T_s) ; M = \frac{\pi}{2} (2m + 1) \quad (2)$$

59

60 It has been found that the relationship can be approximated nearly exactly by the closed-form  
61 relations in Equation 3 (Taylor, 1948), which is useful for implementation of the method in practice.

62

$$T_s = \begin{cases} \frac{\pi}{4} U_s^2 & U_s < 0.60 \\ -0.933 \log_{10}(1 - U_s) - 0.085 & U_s > 0.60 \end{cases} \quad (3)$$

63

64 It is assumed that the average degree of dissipated suction is also equal to the ratio between  
65 current volumetric strain ( $\varepsilon_v$ ) and the ultimate swelling strain as time approaches infinity ( $\varepsilon_{v,min}$  if  
66 swelling strains are taken as negative). Since mathematically equivalent expressions are used for  
67 the theoretical relationships in consolidation theory, the graphical methods by Taylor (1948) for  
68 determining time to 90% swell ( $t_{90}$  corresponding with  $T_s = 0.848$ ) and by Casagrande & Fadum  
69 (1940) for 50% swell ( $t_{50}$  corresponding with  $T_s = 0.196$ ) may be used to find  $c_s$  values from time–  
70 swell curves measured in oedometer tests. These coefficients and the predicted ultimate swell  
71 (from oedometer tests or empirical/analytical methods) may then be used to convert  $U_s$  and  $T_s$  to  
72 predict swell ( $-\varepsilon_v$ ) over time ( $t$ ) in the field.

73

### 74 3. Oedometer swell tests

75 A suite of *wetting after loading* tests (ASTM D4546-21), as reported by Murison et al. (2025), was  
76 conducted on a compacted black weathered norite clay sampled near Steelpoort, South Africa.  
77 The clay's properties are summarised in Table 1. To show relationships between initial conditions  
78 and suction, Figure 1 presents first drying soil-water retention and shrinkage curves for samples  
79 that were graded and compacted to two initial density conditions, representative of the in-situ  
80 range. Suction was measured using a dewpoint hygrometer (as described by Murison et al.  
81 (2023)) and the ranges of in-situ states determined from intact samples are indicated in Table 1  
82 and Figure 1.

83

84

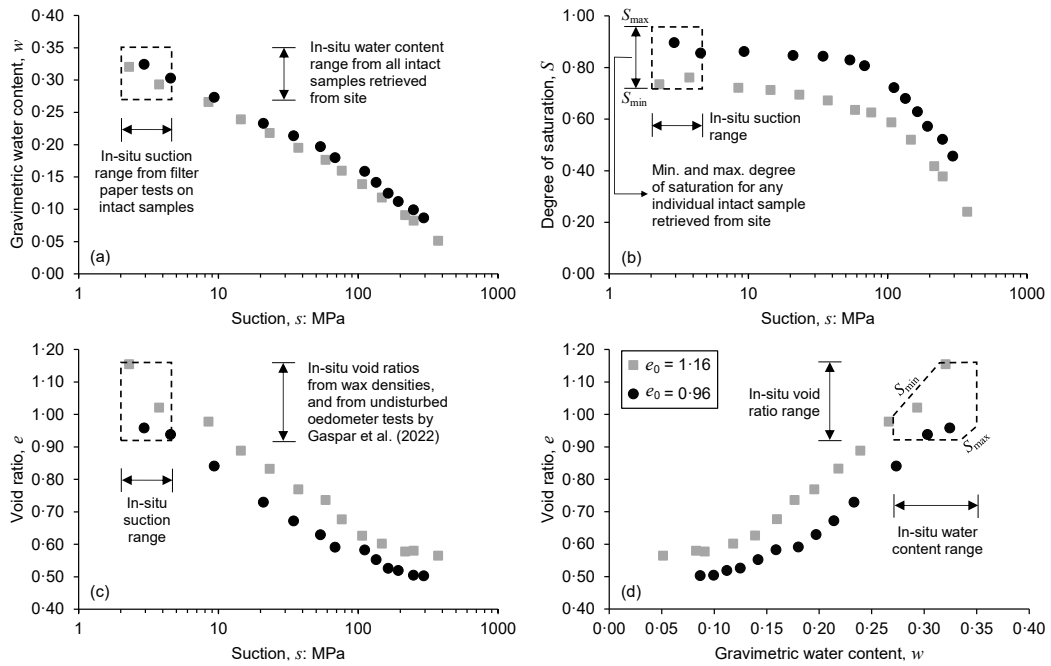
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86 **Table 1:** Properties of the highly expansive clay used in this study (after Murison et al., 2025)

Property	Quantity/description
Description of soil in situ (according to profiling guidelines by Jennings et al., 1973)	Moist, black, stiff, fissured & slightly slickensided, silty clay with scattered calcrete nodules. Alluvium.
Sampling depth: m	0.5 – 1.5
Specific gravity	2.65
Liquid limit	92
Plasticity index	55
Smectite content from X-ray diffraction: %	64
Clay fraction by mass (<2 $\mu\text{m}$ ): %	64
Silt fraction by mass (2-63 $\mu\text{m}$ ): %	19
Sand fraction by mass (63-2000 $\mu\text{m}$ ): %	17
Classification (BS 1377-2:1990)	CV
Activity (Skempton, 1953)	0.86
Potential expansiveness (Van der Merwe, 1975)	Very high
In-situ suction, $s_0$ : MPa <sup>a</sup>	2 – 4.5
In-situ water content, $w_0$ : % <sup>a</sup>	27 – 35
In-situ void ratio, $e_0$ <sup>a</sup>	0.93 – 1.17
In-situ degrees of saturation, $S_0$ : % <sup>a</sup>	72 – 96

87 <sup>a</sup> details on determination of in-situ states provided in Figure 1

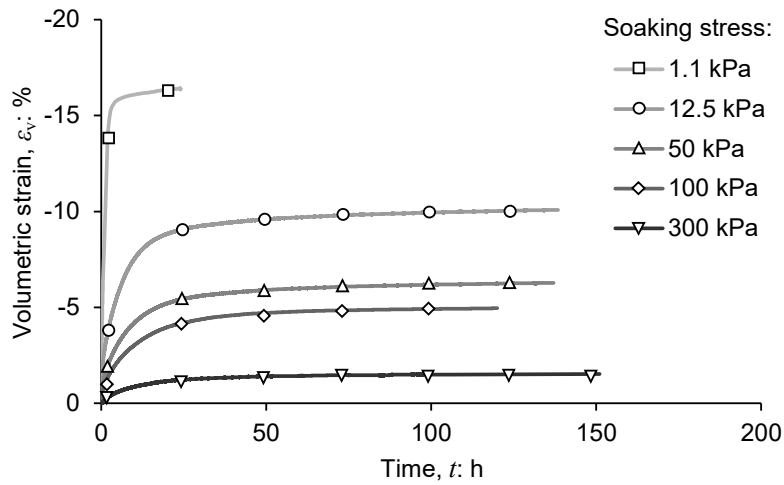
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89

90 **Fig. 1:** First drying soil-water retention and shrinkage curves for samples compacted to two  
 91 initial density conditions, with ranges of in-situ states indicated by the areas enclosed by broken  
 92 lines  
 93

94 The clay was grated and recompacted to the in-situ density using the methodology described by  
 95 Gaspar et al. (2022, 2023) to create a ‘fissured’ macrofabric emulating the in-situ fabric.  
 96 Oedometer samples were loaded unsaturated to the desired net vertical stress ( $\bar{\sigma}_v$ ), termed the  
 97 soaking stress, after which the cell was flooded with water. The initial void ratio ( $e_0$ ), water content  
 98 ( $w_0$ ), degree of saturation ( $S_0$ ), and the soaking stresses ( $\bar{\sigma}_v$ ) are given for each sample in Table 2.  
 99 The development of swelling strains over time is illustrated in Figure 2.



100

101 **Fig. 2:** Swelling curves for oedometer samples soaked under constant vertical net stress (after  
102 Murison et al., 2025)

103

104 The log-time (Casagrande & Fadum, 1940) and root-time (Taylor, 1948) graphical methods were  
105 used to determine  $t_{50}$  and  $t_{90}$  respectively for each test, which were subsequently used to  
106 calculate the coefficients of swell. These values, as well as the maximum swell ( $-\varepsilon_{v,min}$ ) for each  
107 test, have been included in Table 2. Note that the average drainage/infiltration path length ( $d_{avg}$ ,  
108 i.e. half the average sample height during the swell phase) was used to compute  $c_s$ .

109

110 **Table 2:** Summary of oedometer swell tests

Test no.	$\bar{\sigma}_v$ : <sup>a</sup> kPa	$e_0$ : <sup>a</sup>	$w_0$ : <sup>a</sup> %	$S_0$ : <sup>a</sup> %	$-\varepsilon_{v,min}$ : %	$d_{avg}$ : mm	$t_{50}$ : min	$t_{90}$ : min	$c_s = \frac{0.196d^2}{t_{50}}$ : m <sup>2</sup> /year	$c_s = \frac{0.848d^2}{t_{90}}$ : m <sup>2</sup> /year
1	1.1	1.03	33.7	86.4	16.8	10.820	51	225	$2.35 \times 10^{-1}$	$2.32 \times 10^{-1}$
2	12.5	1.00	34.8	92.2	10.1	9.923	167	790	$6.08 \times 10^{-2}$	$5.56 \times 10^{-2}$
3	50	1.01	34.8	91.6	6.27	9.746	242	1228	$4.04 \times 10^{-2}$	$3.45 \times 10^{-2}$
4	100	1.00	33.0	87.9	4.95	10.248	348	1755	$3.11 \times 10^{-2}$	$2.67 \times 10^{-2}$
5	300	0.99	34.3	91.7	1.40	9.516	397	2037	$2.35 \times 10^{-2}$	$1.98 \times 10^{-2}$

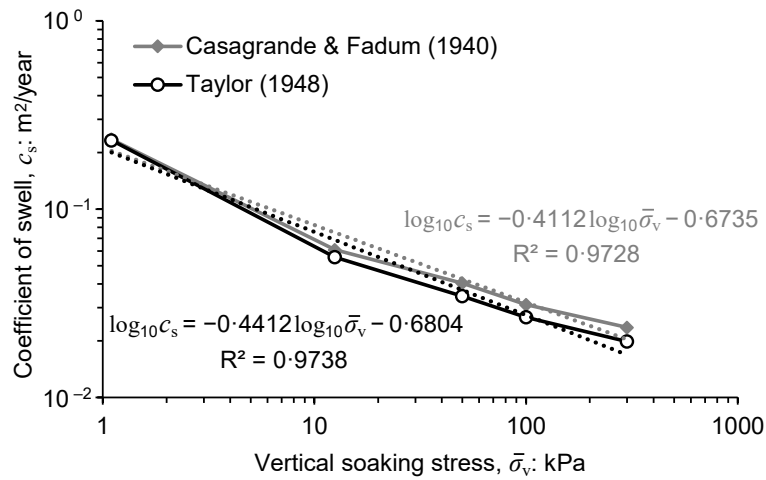
111 <sup>a</sup> reported by Murison et al. (2025)

112

113 The coefficients of swell as functions of soaking stress, determined using each method, are given  
114 in Figure 3. The results using the two methods compared well, both yielding near straight-line  
115 relationships in log-log space with coefficients of determination ( $R^2$ ) exceeding 97%.

116

117



118

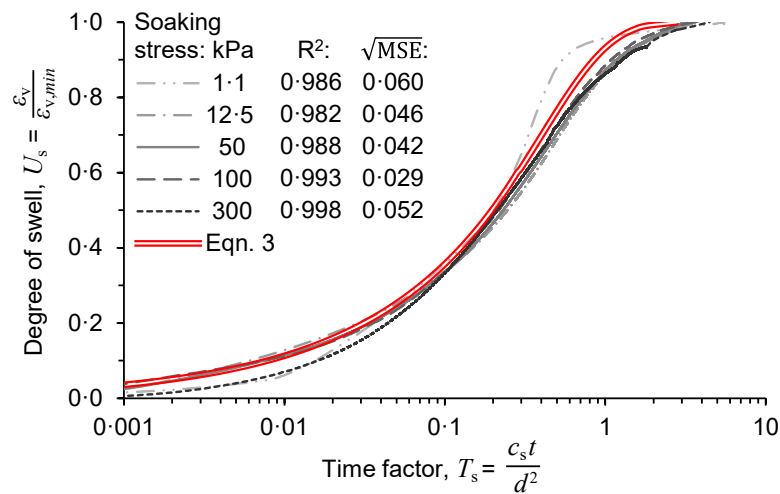
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**Fig. 3:** Coefficients of swell determined from oedometer tests

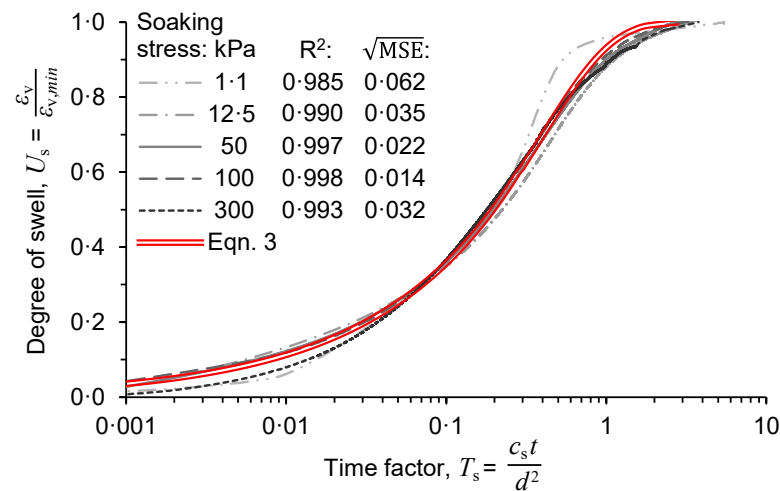
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121 Using the determined  $c_s$  and  $-\varepsilon_{v,min}$  values, the measured swelling curves could be expressed in  
 122 terms of degree of swell and the dimensionless time factor. The curves resulting from the two  
 123 methods are compared with Equation 3 in Figure 4a and 4b, respectively. The square-root of the  
 124 mean squared error ( $\sqrt{\text{MSE}}$ ) and  $R^2$  have been included as statistical descriptors of the goodness-  
 125 of-fit between each test and Equation 3. The normalised swell curves were well predicted by  
 126 Equation 3, illustrating the potential viability of the approach to predict swell over time of an  
 127 expansive clay layer. Given the marginally superior fit obtained using the root-time method  
 128 (Taylor, 1948), the  $c_s$  function determined using this method was used for the centrifuge swell  
 129 predictions.

130



(a)



(b)

131

132 **Fig. 4:** Normalised swell curves resulting from  $c_s$  values determined using (a) the log-time

133

method (Casagrande &amp; Fadum, 1940); (b) the root-time method (Taylor, 1948)

134

135 **4. Centrifuge modelling**

136 Gaspar et al. (2023) reported results from a greenfield swell test on a layered expansive clay

137 model in the geotechnical centrifuge. Clay sampled from the same site as that of the element

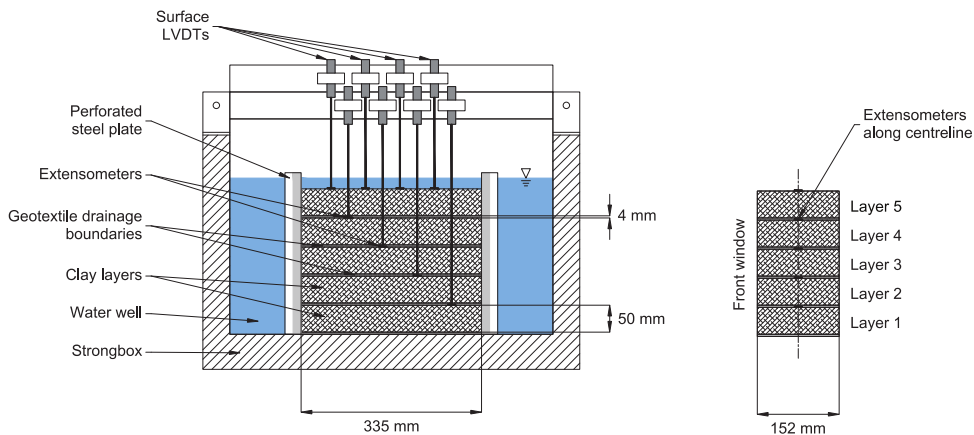
138 tests was grated and compacted into 50 mm thick layers. Gaspar et al. (2023) proposed that the

139 artificially fissured macrofabric facilitated accelerated in-flight swelling of the profile. Importantly,

140 the same fabric was created for the centrifuge model and the oedometer samples tested in this

141 study. The centrifuge model and applicable instrumentation are depicted in Figure 5. Geotextile

142 drainage layers were provided above and below each clay layer, and access to water was  
 143 maintained using water wells. Linear variable differential transformers (LVDTs) were used to  
 144 monitor deformation at the model surface, and extensometers at the top of each layer monitored  
 145 subsurface deformations.  
 146



147

148 **Fig. 5:** Centrifuge model layout (after Gaspar et al., 2023)

149

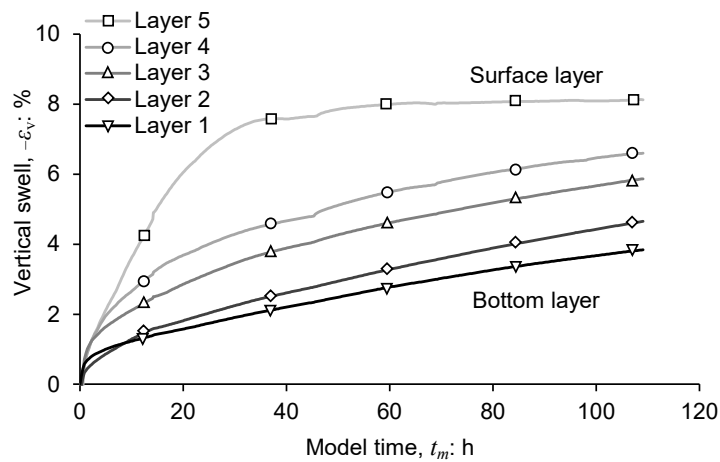
150 The model was accelerated to  $30g$ , i.e. the model scaling factor  $N = 30$ . The water wells were  
 151 filled in flight, and the water level was maintained for 109 hours in model time ( $t_m$ ). Caicedo et al.  
 152 (2006) proposed that since swelling of an unsaturated clay is a diffusive process, the applicable  
 153 scaling law for prototype time ( $t_p$ ) is  $t_p = t_m \times N^2$ . Using this law, the 7.5 m thick prototype profile  
 154 was allowed to swell for approximately 11.2 years in prototype time. Initial conditions of the layers  
 155 are given in Table 3. The model swell–time relationship for each layer is depicted in Figure 6.

156

157 After  $t_p = 11.2$  years of constant flooding, the swelling of the profile made up of 1.5 m thick  
 158 expansive clay layers was evidently not complete. To predict swell over time for the centrifuge  
 159 prototype, coefficients of swell for each layer given the initial average net vertical stress ( $\bar{\sigma}_{v,0}$  at  
 160 mid-height of the layer) were determined using the relationship in Figure 3 for the root-time  
 161 method (Taylor, 1948). These coefficients were used to transform the time factor in Equation 3 to  
 162 prototype time for each layer. Predicted swell at a given time can be obtained by multiplying the  
 163 degree of swell ( $\varepsilon_v/\varepsilon_{v,min}$ ) by the ultimate swell ( $\varepsilon_{v,min}$ ), which occurs when suction is reduced to

164 zero for the relevant clay element. Since the ultimate swell at  $t_p = \infty$  was not known, values were  
 165 chosen to minimise the mean squared error between the predicted and measured time–swell  
 166 relationships. The results in Figure 7 therefore do not illustrate the accuracy with which the  
 167 proposed model predicts ultimate swell. Instead, the results highlight how the time–swell  
 168 relationship (i.e. shape of the curve) is closely matched. The calculated best-fit ultimate strains,  
 169 as well as the statistical descriptors of the goodness-of-fit of the time–swell relationships, are  
 170 included in Table 3.

171



172

173 **Fig. 6:** Centrifuge model swelling curves in model scale (after Gaspar et al., 2023)

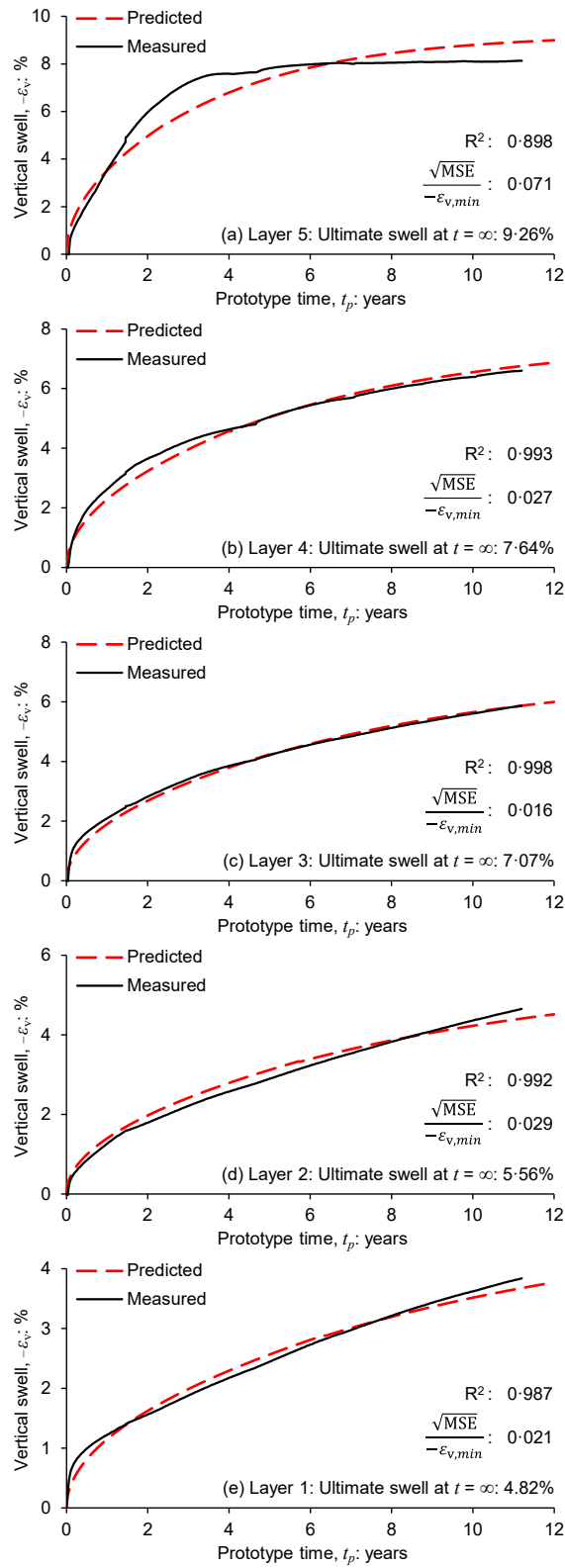
174

175 **Table 3:** Summary of centrifuge test initial conditions and results

Layer no.	$e_0^a$	$w_0^a$ %	$S_0^a$ %	$\bar{\sigma}_{v,0}$ kPa	$c_s$ m <sup>2</sup> /year	$-\varepsilon_v$ at end of test: %	Calculated $-\varepsilon_{v,min}$ at $t_p = \infty$ : %	R <sup>2</sup> of prediction	$\frac{\sqrt{MSE}}{-\varepsilon_{v,min}}$ of prediction
5	1.09	28.9	72.6	12.1	$6.94 \times 10^{-2}$	8.13	9.26	0.898	0.071
4	1.07	29.7	73.8	36.5	$4.27 \times 10^{-2}$	6.60	7.64	0.993	0.027
3	1.09	29.7	72.5	60.8	$3.41 \times 10^{-2}$	5.87	7.07	0.998	0.016
2	1.06	30.1	75.2	85.3	$2.94 \times 10^{-2}$	4.65	5.56	0.992	0.029
1	1.06	30.1	75.0	109.9	$2.62 \times 10^{-2}$	3.84	4.82	0.987	0.021

176 <sup>a</sup> reported by Gaspar et al. (2023)

177

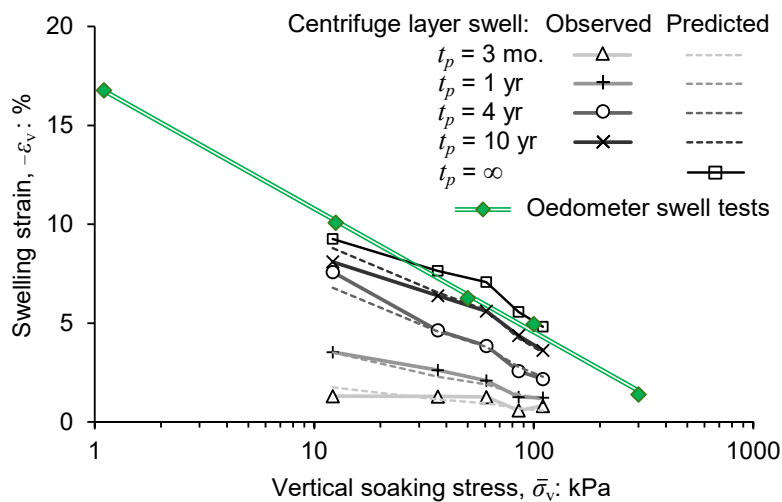


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179

Fig. 7: Centrifuge layer swell prediction results (prototype scale)

180 Figure 8 summarises measurements and predictions of swell at different instances in time. For  
 181 comparison, ultimate swell values as determined from oedometer tests have also been included  
 182 as a soaking-under-load curve. The calculated best-fit ultimate swell for the centrifuge layers (from  
 183 Figure 7) was well predicted by the soaking-under-load curve, with a maximum swell difference  
 184 of 1.17%. This illustrates that a practitioner can use the oedometer tests to determine both the  
 185 ultimate swell at zero suction, and coefficients of swell as model inputs. A common criticism of  
 186 swell magnitude predictions from conventional oedometer tests is that such tests allow samples  
 187 to reach a state of zero suction (Schriener, 1988). From Figure 8, it is clear that such a scenario  
 188 would only be approached if the prototype profile had continuous access to water for an unrealistic  
 189 period of time (over 10 years in the model considered). The advantage of the proposed prediction  
 190 model is that it allows consideration of realistic wetting durations to estimate swell magnitude. For  
 191 example, if the clay profile were flooded for one year (a highly conservative assumption), the swell  
 192 would reach only 23–35% of the magnitude measured in oedometer tests. This highlights the  
 193 value of a simple method to obtain a more rational swelling profile for most practical scenarios.  
 194



195  
 196 **Fig. 8:** Swelling strains as a function of overburden stress and prototype time, compared to the  
 197 oedometer soaking-under-load curve

198  
 199 For a practitioner, prediction of total heave ( $\Delta H$ ) of the profile at any given time can be computed  
 200 by determining the degree of swell of each layer  $i$  (using Equation 3 and  $T_s$  for layer  $i$ ), multiplying

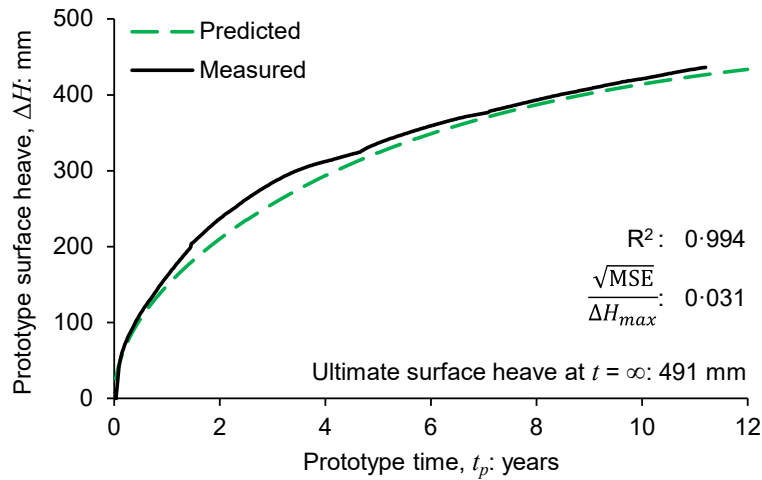
201 by the ultimate swell of the layer and its initial thickness ( $h_0$ ), and summing the resulting heave  
 202 values for all  $n$  layers as illustrated in Equation 4. In this calculation, the coefficient of swell is  
 203 obtained from oedometer swell tests, while ultimate swell can be determined using the engineer's  
 204 preferred method (e.g. oedometer swell tests, empirical or analytical predictions).  
 205

$$\Delta H(t_p) = \sum_{i=1}^{i=n} U_{s_i} \times (-\varepsilon_{v, min_i}) \times h_{0_i}; \quad U_{s_i} = f\left(\frac{c_{s_i} \times t_p}{d_i^2}\right) \quad (4)$$

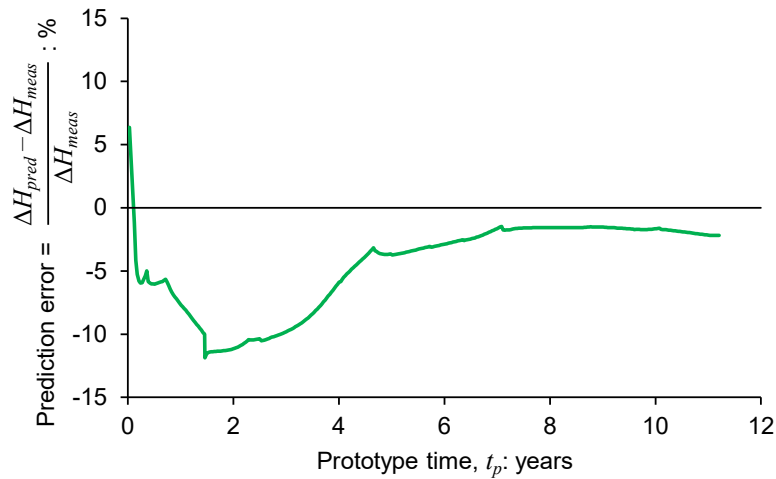
206  
 207 To illustrate the performance of this approach, Figure 9a illustrates the prediction of total heave  
 208 throughout the swell process compared to the measured surface heave of the centrifuge model.  
 209 For the purposes of this illustration, the ultimate swell of each layer was inferred from the  
 210 oedometer soaking-under-load curve. The predictions were in good agreement with the measured  
 211 result, with  $R^2$  exceeding 99% and a mean error ( $\sqrt{MSE}$ ) of 14.98 mm (3.1% of the total heave)  
 212 over the 11.2-year period. The prediction remained within 12% of the measured result over the  
 213 entire test duration, as shown in Figure 9b. This shows that the proposed method was used to  
 214 successfully predict the time–swell behaviour of a highly expansive clay profile during wetting for  
 215 a boundary value problem, using coefficients of swell and ultimate swell values obtained from the  
 216 same set of element tests.

217 Through validation against a one-dimensional boundary value problem, this study highlights that  
 218 concepts borrowed from consolidation theory for saturated soils can be applied to predict the  
 219 time–swell behaviour of an unsaturated highly expansive clay. If drainage boundaries and  
 220 geometry can be well defined, this may be directly applied for simple prediction of one-  
 221 dimensional swell over time in the field. The finding may also be directly extended to the prediction  
 222 of three-dimensional temporal swell-shrink behaviour and for more complex moisture regimes,  
 223 geometries, and stress conditions using the finite difference method and equations akin to Biot  
 224 (1941) three-dimensional consolidation.

225



(a)



(b)

226

227 **Fig. 9:** (a) Prediction results for total surface heave of the centrifuge prototype profile; (b)

228

prediction error over the duration of the test

229

230 **5. Conclusions**

231

A mathematical model based on consolidation theory was used to predict swell with time of an

232

unsaturated expansive clay. Coefficients of swell determined from oedometer swell tests exhibited

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a decreasing trend with increasing vertical net stress for a grated and compacted highly expansive

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clay. The swelling curves measured during the oedometer tests showed good agreement with the

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theoretical relationship. The time–swell relationships for layers of the same expansive clay in a

236 centrifuge model profile were successfully predicted using the proposed mathematical  
237 relationship and oedometer test results. The total heave of the centrifuge profile was predicted  
238 with less than 12% error at any prototype time over the 11.2-year swelling period, with a mean  
239 error equal to 3.1% of the ultimate heave, and an  $R^2$  exceeding 99%. These results support the  
240 use of the proposed model to predict in-situ swelling over time in an expansive clay profile, and  
241 demonstrate the usefulness of a simple approach for obtaining a more realistic swell profile under  
242 partial wetting conditions.

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**310 Table list**

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