CHAPTER 1

CREDIT RISK AND PORTFOLIO MODELLING

1.1 INTRODUCTION

The core principle for addressing practical questions in credit portfolio management lies in the ability to link the cyclical or systematic components of firm credit risk with the firm's own idiosyncratic credit risk as well as the systematic credit risk component of every other exposure in the portfolio. Simple structural credit portfolio management approaches have opted to represent the general economy or systematic risk by a single risk factor. The systematic component of all exposures, the process generating asset values and therefore the default thresholds are homogeneous across all firms. Indeed, this Asymptotic Single Risk Factor (ASRF) model has been the foundation for Basel II. While the ASRF framework is appealing due to its analytical closed-form properties for regulatory and generally universal application in large portfolios, the single risk factor characteristic is also its major drawback. Essentially it does not allow for enough flexibility in answering real-life questions. Commercially available credit portfolio models make an effort to address this by introducing more systematic factors in the asset- value-generating process but from a practitioner's point of view, these models are often a "black-box" allowing little economic meaning or inference to be attributed to systematic factors.

This study aims to construct a country-specific macroeconometric risk driver engine which is compatible with and could feed into the GVAR model and framework of Pesaran, Schuermann and Weiner (PSW) (2004) using vector error-correcting (VECM) techniques. This will allow conditional loss estimation of a South African-specific credit portfolio but also opens the door for credit portfolio modelling on a global scale because such a model can easily be linked into the GVAR model. Here the set of domestic factors is extended beyond those used in PSW (2004) in such a way that the risk-driver model is applicable for both retail and corporate credit risk. As such, the model can be applied to a total bank balance sheet, incorporating the correlation and diversification between both retail and corporate credit exposures.

1.2 DEFINING CREDIT RISK AND PORTFOLIO MODELLING

During the late 1980s and early 1990s banks across the globe suffered extensive default experiences within their credit portfolios. Driven by intense competition for market share banks allowed credit portfolios to become less diversified (across all dimensions, countries, industries, sectors and size) and were willing to accept lesser quality assets on their books. As a result, even well-capitalised banks came under severe solvency pressure when global economic conditions turned. Banks soon realised the need for more sophisticated loan origination, and credit and capital management practices.

The need to understand portfolio credit risk was reinforced by the Bank of International Settlements' (BIS) minimum regulatory capital adequacy guidelines in 1988 (Basel I). Although a significant step forward, Basel I was not able to give an accurate measure of the risk-reward characteristics of a credit portfolio and more specifically did not allow individual credit exposure characteristic to influence minimum regulatory requirements through diversification and/or borrower characteristics. More recently, reforms by the BIS (2004) through the New Basel Accord (Basel II) aim to include exposure-specific credit risk characteristics within the regulatory capital requirement framework but are still unable to allow diversification and concentration risk to be fully recognised within the credit portfolio.

Leading institutions have seized the opportunity to develop risk-sensitive credit risk management systems and as a result have realised that credit pricing was highly inefficient. New pricing techniques allowed them to price loan portfolios based on the underlying credit risk and they started to move assets which were underperforming on a risk-adjusted basis (e.g. return on risk adjusted capital) from their balance sheets. Consequently, those banks which did not embrace new credit risk developments were faced with adverse selection and failed to provide shareholders with the same return as sophisticated banks seeing that they were more willing to accept under-priced assets (often buying them from the more sophisticated banks) i.e. accepting assets on their balance sheets but not charging a credit risk premium which is reflective of its riskiness.

With credit markets becoming increasingly liquid, new opportunities continuously present themselves for active credit portfolio management. In fact, credit portfolio optimization and analysis have the potential to significantly enhance profitability. "Using only very basic optimization techniques a typical institution might expect to reduce the economic capital consumed by its credit portfolio by 25 per cent to 30 per cent" (Garside, Stott and Stevens (1999)).

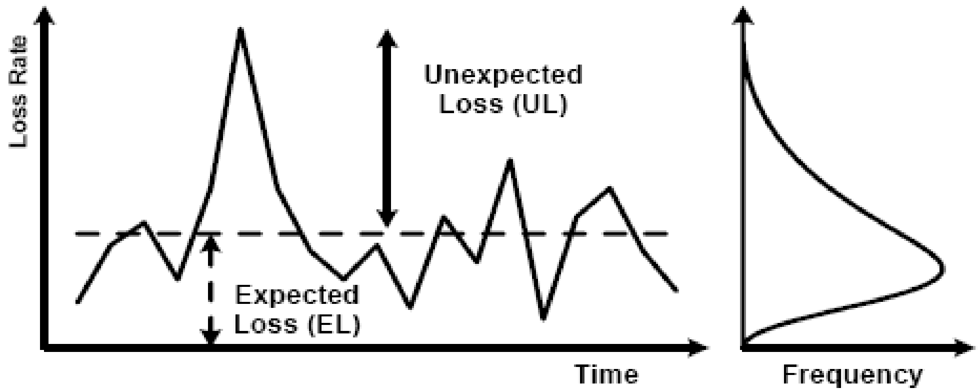
Defining credit risk for regulatory and economic capital purposes is done through the concepts of expected (EL) and unexpected (UL) losses (figure 1.1). Expected loss is the statistical expected cost of doing business over a normal credit cycle and represents the loss that one would expect to incur due to credit-related risk. Unexpected losses, on the other hand, are the risk (volatility) of the underlying portfolio (expected) loss, for example, a loss event which one would expect to incur once every 1000 years on a particular exposure or portfolio. Since such an event will threaten the solvency of an institution and could lead to instability in the financial system as a whole, portfolio- and regulatory-capital models aim to quantify this "loss volatility" i.e. the unexpected events. While expected loss should be priced into the underlying exposure at initiation, unexpected loss needs to be addressed by holding economic capital in large enough sums so that this extreme loss amount is sufficiently covered.

Quantifying credit risk on the individual exposure and portfolio level is usually done through estimating two credit parameters:

- The probability of default (PD) represents the expectation that an individual exposure will not be able to fulfil its obligations on the underlying loan or bond over the lifetime of the agreement.
- The loss given default (LGD) is the percentage of notional exposure which will be unrecoverable in the event of default.

Multiplying these two parameters together will provide the statistical expected loss assumed for a particular exposure and is priced into the exposure at inception.

Figure 1.1 Credit risk: expected and unexpected loss



Source: BIS (2004)

In order to aggregate total portfolio credit risk, the individual credit risk is combined within a credit portfolio model using one of two approaches:

- The ***loss-based*** approach assumes that the exposure is held until maturity and that at the maturity date the exposure has either defaulted or not. In the event of default, the exposure will be worth the recoverable amount, which includes the value of the underlying collateral. In the non-defaulted state the exposure is repaid at par. It is clear that under the loss-based method rating changes do not take place over the horizon up to maturity and that the changes in the underlying value of non-defaulted exposures are not included in the quantification of credit losses.
- The shortcomings of the loss-based approach are addressed through the ***net present-value*** approach. Here the realisable value or change in market value (i.e. a market-to-market adjustment) of non-defaulted exposures is included with the recoverable amount of defaulted exposures in the estimate of credit losses. Although theoretically more correct, calculating a realisable value of non-defaulted exposures can become very complex and requires meaningful market data on the underlying exposure prices. As such, a trade-off exists between data availability and more accurate estimates of portfolio credit risk.

Notwithstanding the approach followed, the extremity of portfolio losses (thus the unexpected loss) is driven mainly by two factors: concentration and correlation.

Similar to equity portfolios, credit and bond portfolios also suffer from diversification risks. As such, lumpy or over-weighted portfolios to an individual obligor, industry or country can significantly increase the losses experienced in the portfolio in adverse conditions. In fact, the BIS (2006) estimates that single name concentration risk i.e. undiversified risk to a single counter party can increase economic capital by between 2 and 8 per cent of credit value at risk while sector concentration i.e. undiversified risk due to a group of counterparties concentrated in one industry can increase economic capital between 20 and 40 per cent.

Correlation refers to the co-movement of individual exposures' credit risk over the business cycle. Conceptually one would expect that the correlation of individual exposures with the business cycle would imply that in downturn economic conditions portfolio credit risk is enhanced by the simultaneous increase in risk of exposures which are sensitive to the same macroeconomic variables. In fact, much of the discussion taking place since the introduction of Basel II has centred on the effects of business cycles on portfolio credit risk and economic capital (see for example Carpenter, Whitesell and Zakrajsek (2001), Carey (2002), Allen and Saunders (2004), Jarrow and Van Deventer (2004) and Elizalde (2005)).

Due to the non-symmetrical or non-normal behaviour of individual exposure loss distributions, diversification in credit portfolios is not achieved as easily as in equity portfolios. Normally diversification in equity portfolios can easily be achieved by including as little as 30 individual exposures in a portfolio. However, larger institutions usually possess a sufficient number of individual exposures so that diversification is reached. Measures such as large exposure limits are also put into place which address the question of lumpiness with respect to individual loans comprising too much of a portfolio. As such, correlation ends up being the major source of risk in most credit portfolios and many portfolio managers struggle to find ways to answer questions such as "How would my economic capital change if economic conditions change, e.g. a sudden increase in oil prices, an increase in interest rates, a currency fall-out or an economic recession?".

Although the literature has been able to provide significant guidance as to possible frameworks for such scenario-based analyses, no such framework has been applied in the South African context. Although commercially available credit portfolio models such as Moody's KMV Portfolio Manager™ are applied at various banking institutions, the framework is not flexible enough to answer practical questions easily.

1.3 STRUCTURAL DEFAULT MODELS

In general the literature has attempted to address practical questions to credit risk modelling through the traditional structural approach of credit risk models (although the reduced-form model pioneered by the hazard rate model of Jarrow and Turnbull (1995) has made huge progress in recent years, it is still not used extensively in practical application). The structural approach is based on the option theory of Merton (1974) and was pioneered by Vasicek (1987). "The basic fundamentals of the structural approach are that a company's market value, market value volatility and liability structure are modelled using contingent claims analysis" (Van Deventer (2005)). Once a company's asset value falls below its default threshold level a default will occur. The equity of a company can therefore be seen as a call option while the debt holders conversely hold a put option on the underlying assets of the company. Conceptually, the strike price of the call option is equal to the value of the underlying debt and the value of the call option will be determined by the asset value of the company. Asset values above the strike (threshold) value will place the call option in the money while any asset value below the threshold level will render the call option worthless. Using the normal Merton option theory assumptions and framework, the structural model attempts to value this call option. Various extensions and adjustments to the structural approach have since followed (see for example Vasicek (1991, 2002), Shimko, Tekima and Van Deventer (1993) and Gordy (2003)).

1.4 POSITIONING THE STUDY WITHIN THE LITERATURE

The purpose of the study is to provide a framework for South African specific credit risk portfolios through which quantitative answers can be obtained for questions such as those listed above. Using macroeconometric models and linking them to the return process of individual firms, the framework will be applied in the standard Merton-type credit portfolio default model context. Based on the methodology proposed by Pesaran, Schuermann, Treutler and Weiner (PSTW) (2006) a model for conditional credit losses which combines the systematic risk with the idiosyncratic component of each exposure and also includes an explicit channel for default correlation, will be provided. The methodology is particularly appealing in that it is not only flexible in answering practical portfolio questions (through scenario analysis) but it also steers away from the data confidentiality problem that most practitioners face when using commercially-available credit portfolio models.

Although South African banks are mostly limited to the domestic credit market in terms of exposures on their balance sheets, the framework will be positioned in such a way that it is able to easily link into similar frameworks for other countries and regions. As such, the model can be used for global credit portfolio contents as well. Ultimately the approach can then be used in an enterprise-wide credit risk portfolio setting. In principle the approach is flexible enough, that it can be adapted to a reduced-form credit risk model as well.

1.5 METHODOLOGY

The core principle for addressing practical questions lies in the ability to link the cyclical or systematic components of firm credit risk with each firm's own idiosyncratic credit risk as well as the systematic credit risk component of every other exposure in the portfolio. Most structural approaches have opted for representing the general economy or systematic risk by a single risk factor. The systematic component of all exposures, the process-generating asset values and therefore the default thresholds are homogeneous across all firms. Indeed this Asymptotic Single Risk Factor (ASRF) model has been the foundation for the New Capital Accord under Basel II. While the ASRF framework is appealing due to its analytical closed-form properties for regulatory and general universal application in large portfolios, the single risk factor characteristic is also its major drawback. Essentially it does not allow for enough flexibility in answering real-life questions. Commercially available credit portfolio models have made an effort to address this by introducing more systematic factors in the asset-value generating process.

The proposed methodology departs from the usual literature in two aspects. Firstly, individual firm asset values and returns are modelled in terms of a set of domestic macroeconomic factors such as inflation, interest rates, money supply, exchange rates and GDP as well as explicit linkages to foreign country macroeconomic factors (in order to provide linkages for use in a global portfolio context) and global exogenous variables such as oil prices. Secondly, an assumption is made that two firms that have a similar credit rating at initiation will have the same default equity threshold ratios over the forecast period. Therefore, by using historically observable mean returns, the volatility and default frequencies within a specific rating category can be computed by using a specific default equity threshold per rating band. Using the linkages of assets returns to macroeconomic factors as well as the equity threshold levels, one is able to obtain an individual default probability for each firm. Ultimately, this methodology "…provides an empirical implementation of the Merton model using only two pieces of publicly available information for each firm, namely market returns and credit ratings, in a multi-country setting" (PSTW (2006)).

From a practical point of view, the appeal of this method lies in the fact that it is based on publicly available information, steering away from the noisy and asymmetric accounting data and/or non-publicly available information usually used in commercially-available credit portfolio models. The return of individual firms is driven by the domestic economy as well as the impact of the international business cycle. Using the firm's credit rating with this dynamic return process the portfolio loss distribution can be obtained by means of a much richer and practically applicable method, in turn allowing for provision of more detailed information through scenario analysis.

From an academic point of view the method is appealing in that it bridges the gap between economic, finance, credit and econometric literature and combines the three disciplines, thus providing a theoretically sound credit portfolio model.

At the heart of the credit portfolio model lies the macroeconometric "risk driver model". Although macroeconometrics has been a central part of economic literature since the seminal work by Klein (1947), only recently has the application of econometrics found its way into the credit risk framework. In order to link the firm-specific return distributions to macroeconomic business cycles, a South African specific vector error-correcting model (VECM) will be estimated (based on the methods proposed by Johansen (1988)), the latter being similar to the global vector error-correcting model proposed by PSTW (2006)). However, many practitioners usually explore whether or not the standard residual-based macroeconometric cointegration framework (proposed by Engle and Granger (1987)) could be applied successfully to a credit portfolio model. Although this method is much simpler from a theoretical point of view, the loss in theoretical accuracy is in some instances more than offset by the increase in application and practicality. Such a simplified method would also allow for much simpler and more accessible scenario analysis and for linkage to other similar macroeconometric models usually used by Central Banks, Departments of Finance and other international organisations such as the United Nations, International Monetary Fund and World Bank.

The firm-specific return process will be modelled in the context of the multi-factor models of equity returns which are based on the basic Capital Asset Pricing Model principles pioneered by Ross (1976) through his Arbitrage Pricing Theory (APT). The attractiveness of this approach lies in the fact that firm heterogeneity is preserved, while it is usually lost due to simplification when using commercially-available and other credit portfolio models (for example CreditRisk+ and CreditPortfolioView). APT theory models the return process of a firm as a function of key macroeconomic variables. As such, each firm will be influenced by a set of selected macroeconomic variables based on its own sensitivity to each factor. While this model approach lends itself to regression analysis and would be the standard practice to follow, searching for the correct specification for each firm in a credit portfolio can become a daunting task (an average portfolio consists of over 500 exposures. Assuming a reasonable five factor model per exposure leaves as many as 3000 parameters to be estimated). However, due to the stationary characteristics of firm returns, a panel estimation approach can be applied to the set of exposures which in turn allows for individual coefficient estimation and some degree of firm heterogeneity to be recognised while also easing computational capacity constraints.

The methodology can be summarised as follows. The macroeconometric risk driver model will specify and represent the macroeconomic environment in which the credit portfolio operates. Using Monte Carlo simulations, various possible simulation paths of the economy will be forecast over a forecast period. These macro factors will be fed into the firm specific return models in order to produce the value-generating process of each firm. Using the return dynamics and the estimated equity default thresholds from PSTW (2006), the probability of default can be obtained using the structure-based credit default model described above. Finally a conditional loss distribution for the credit portfolio can be obtained and used to estimate various credit-related parameters such as economic capital, in turn allowing for various scenario analyses to be performed.

1.6 DATA

A credit portfolio model will be build for a sample of firms spanning the range of the rating spectrum. Economic data will be obtained from the usual databases such as the International Monetary Fund (IMF), the South African Reserve Bank (SARB) and Statistics South Africa (Stats SA). Firm-specific return data will be obtained from the McGregor database in addition to well-known data vendors such as Bloomberg and I-Net Bridge. Due to the fact that the estimates of the equity default threshold of PSTW (2006) were obtained through an exhaustive data analysis of the Standard and Poor's (S&P) rating databases, these estimates will be used in the current study but will be evaluated to confirm their applicability and relevance in the current setting. In order to assign default thresholds to individual companies, the ratings as assigned to a fictitious portfolio of corporate loans are obtained from internally derived rating models as derived by FirstRand Bank. These ratings therefore constitute the only non-publicly available data necessary for using the current methodology. Since most banks are required under Basel II to have some form of internally derived rating for all their counterparties, this requirement is therefore not deemed as being restrictive when applying said methodology.

1.7 OUTLINE OF THE STUDY

Credit portfolios are ultimately exposed to macroeconomic cycles, even though idiosyncratic risk within a portfolio can be diversified away to some extent in a large corporate loan portfolio. In order to perform credit portfolio scenario analysis the portfolio manager must be able to link firm-specific dynamics with macroeconomic factors through statistical models.

In chapter 2 we provide details of the PSTW (2006) methodology which gives a complete framework for linking macroeconomic dynamics and corporate credit risk. Within the framework the main elements include a structural macroeconomic risk driver engine, a default model which governs the default states within the macroeconomic environment and finally a translation function which transforms macroeconomic conditions into firm credit risk. Chapter 3 provides a South African specific VECM model which includes both domestic and global macroeconomic risk factors believed to drive credit risk in the South African economy. Together with the Merton-type default model as provided in chapter 4 it will be shown that conditional loss credit portfolio modelling is possible in the South African context. Our credit portfolio model provides stochastic simulation results and allows for correlation between macroeconomic factors, the correlation of firms with these macro factors as well as the correlation of firms amongst themselves. The PSTW (2006) methodology will be extended by providing an individual multifactor model for each exposure in the study portfolio, thus supporting the argument that the enhancement allows for more diversification to be recognised in the portfolio than what is assumed in normal ASRF-type models.

In chapter 4 the credit portfolio model will also be tested on a fictitious South African corporate loan portfolio through provision of scenario analysis. This study's *a priori* expectation is confirmed in that the asymmetric behaviour of credit risk, i.e. negative economic shocks translates into proportionally much higher increases in portfolio risk than a subsequent decrease in risk from a similar positive economic shock.

This macroeconometric-based credit portfolio model and methodology provides a theoretically consistent and direct method of estimating credit risk while also performing scenario analysis.

CHAPTER 2

A THEORETICAL FRAMEWORK FOR ESTIMATING A CONDITIONAL PORTFOLIO LOSS DISTRIBUTION: THE PESARAN, SCHUERMANN, TREUTLER AND WEINER (2006) METHODOLOGY

2.1 INTRODUCTION

At the heart of the structural default models lies the Merton (1974) approach to default risk. As argued in chapter 1, a firm is expected to default if the underlying asset value (where assets value is equal to the sum of equity and debt) falls below a specific threshold level. The threshold level is determined by the underlying callable liabilities and will essentially change over time with changes in liability, equity and asset value. The fundamental insight of Merton (1974) is that debt can be regarded as a put option on the value of the assets of the firm i.e. debt holders sell a put option on the underlying firm's assets to equity holders. If the value of the firm falls below this threshold level or strike price, the equity holders would effectively put the company to the debt holders. Typically, default is defined by banks and rating agencies as a non-payment of any coupon or interest payment or any principal due, although stricter definitions such as delisting activities are sometimes also applied for internal use by banks.

As such, Merton (1974) was able to apply debt valuation to the option-valuation field for which increasingly sophisticated valuation techniques were becoming available. The option-value view of debt has since been applied in both credit and interest rate risk management techniques and is a fundamental part of financial risk management and portfolio management techniques and models (see for example Van Deventer, Imai and Mesler (2004)).

Similar to the option valuation field, quantification of default risk will therefore require modelling of three aspects:

- The evolution of the underlying firm's value;

- The default threshold of the specific firm; and
- The degree to which firm value is correlated with other companies; and the macroeconomic environment, also known as the asset-value correlation.

Although equity value data is easily available for listed companies, the underlying asset value of companies is not readily available. In fact, most Merton-type models such as the Moody's KMV Portfolio Manager™ uses a propriety database in which firm asset value is estimated from balance sheet and equity data. From this database, asset value correlation and volatility are estimated while the default thresholds are taken to be some function of the short-term and long-term debt in each period. In this framework, a distance to default variable for each company is estimated as the number of standard deviations the underlying firm value is, from the default threshold. Default probability is then inferred through historical observations of the distance to default and actual default experiences.

Clearly, such a framework is heavily dependent on the propriety data underlying both the asset value evolution as well as the distance to default and default probability estimation. The methodology proposed by PSTW (2006) has made a significant advance in credit risk modelling in that it avoids the usage of proprietary balance sheet and distance to default data, instead focussing on credit ratings which are more freely available. By linking this adjusted structural default model to a structural global econometric (GVAR) model, credit risk analysis and portfolio management can be done through the use of a conditional loss distribution estimation and simulation process.

This chapter aims to provide a comprehensive discussion of the PSTW (2006) approach and argues that the methodology provides a foundation through which credit risk managers in South African financial markets can evaluate and analyse their portfolios not only from a domestic macroeconomic perspective, but by linking into the existing GVAR model will also be able to use a global perspective in portfolio analysis.

2.1 AN ADJUSTED MERTON-BASED MODEL OF DEFAULT

Consider a firm *j* in a country *i* having a total asset value of $V_{ji,t}$ at time *t* while the underlying debt obligation is designated by $D_{ji,t}$. Using the Merton (1974) approach default would occur at maturity date of debt, *t*+*H* if the firm's assets are less than the value of the debt, i.e. default occurs if $V_{ji,t+H} < D_{ji,t+H}$. Clearly, this default definition is similar to a European put option which is only exercisable at maturity. The first passage model proposed by Black and Cox (1976) allows default to occur the first time $V_{ji,t} < D_{ji,t}$ over the time period *t* to maturity date *t*+*H*. The default probability is therefore determined by the probability distribution of asset values at the terminal date, *t*+*H* or over the period *t* to *t*+*H* in the Merton and first passage models respectively. Although the method proposed by PSTW (2006) can be adapted to suit both approaches it is applied here to the Merton European put option specification.

Using the accounting definition equation, the value of a firm should equal the value of debt and equity, i.e.

$$V_{ji,t} = D_{ji,t} + E_{ji,t}\text{, } with\ D_{ji,t} > 0\,. \qquad (1)$$

Dividing both sides by the value of debt, equation 1 can alternatively be presented as,

$$\frac{V_{ji,t}}{D_{ji,t}} = 1 + \frac{E_{ji,t}}{D_{ji,t}}\ . \qquad (2)$$

Therefore, default will take place at time *t*+*H* if

$$V_{ji,t+H} \leq D_{ji,t+H}\,,$$

or from (2) if

$$\frac{E_{ji,t+H}}{D_{ji,t+H}} \leq 0. \qquad (3)$$

From equation 3 it is clear that default will only occur if the equity value of a firm is negative. This condition is rather restrictive and not necessarily true in practice. Often shareholders work pro-actively and put the firm up for receivership before the equity value of the firm hits zero. Several studies have also shown that equity owners receive some compensation even though debt holders have not been paid in full (Eberhart and Weiss (1998) and Longhofer (1997)) and data suggests that equity values stay positive even for insolvent firms (Betker (1995), Franks and Torous (1991) and LoPucki (1991)). From a bank perspective, various loan conditions allow banks to force firms into default if equity values fall below a specific non-negative threshold (Garbade (2001)). As argued by PSTW (2006), the value of equity does not only take into account firm asset value but also includes an option that firms may recover before creditors take control of assets. On the other hand, borrowers often work out refinancing arrangements if, for example, one or two coupon payments have been missed, avoiding the firm going into bankruptcy. As such PSTW (2006) assumes that default takes place if

$$0 < E_{ji,t+H} < C_{ji,t+H}. \tag{4}$$

In equation 4, $C_{ji,t+H}$ represents some positive default threshold which can vary over time and differ between firms depending on firm-specific characteristics such as sector or industry classification, leverage, profitability, firm size or age and qualitative factors such as management style. Clearly, accounting-based factors such as leverage are measurable and obtainable through data vendors. However, although new accounting standards are increasingly trying to improve the quality of data provided through financial statement disclosures, such data is still noisy and includes information asymmetries between firm management and investors (Wittenberg-Moerman (2006)).

In order to estimate economic capital through an economic capital model, one implicitly encounters the latter-mentioned measurement and information asymmetry problems as with a default model. PSTW (2006) attempts to alleviate these challenges by using credit ratings for firms (*R*). *R* may take the values usually depicted by either

Moody's (Aaa, Ba, Baa,…, Caa) or S&P's (AAA, AA, BBB,…,CCC) rating notations. The use of ratings facilitates the estimation of default thresholds in order to obtain the default probabilities of each firm. Most rating agencies perform a rigorous process of interviews with firm officials and in-depth analysis of financial statements and observable market data to assign a particular rating to a firm. Moreover, rating agencies are explicit in their commitment to assigning consistent ratings between firms and also over time so that comparisons can be performed over longer periods. As such it is reasonable to assume that the information contained in the ratings outcome, *R*, contains estimates of current balance sheet and equity return data, historic return data and non-publicly available information. This is an attempt to bridge the information asymmetry gap and provide information on all the firms in the past which have been given a similar rating.

Consider then a particular *R*-rated firm at time *t* and assume that arriving at their rating, the credit rating agency uses the standard geometric random walk model of equity values as assumed in fundamental financial pricing models such as CAPM, then:

$$\ln\left(E_{R,t+1}\right) = \ln\left(E_{Rt}\right) + \mu_R + \sigma_R \eta_{R,t+1}, \qquad \eta_{R,t+1} \sim IID(0,1) \tag{5}$$

with a non-zero drift μ_R and an idiosyncratic Gaussian innovation, σ_R, with a zero mean and a fixed volatility. Further, also assume that depending on data at a specific point in time *t*, a firm's rating does not change over a fixed horizon, *t+H*. It is indeed a well known fact that rating agencies are trying to provide ratings that represent counterparty credit quality that is "through the cycle" in nature. As such they would aim to provide ratings that represents and are able to withstand anticipated ups and downs of business cycles not only a snapshot of the present situation. One can therefore assume that the rating agencies are taking a longer term view on their ratings in order to provide investors with more stable ratings over time. Therefore equation 5 becomes:

$$\ln\left(E_{R,t+H}\right) = \ln\left(E_{Rt}\right) + H\mu_R + \sigma_R \sum_{s=1}^{H} \eta_{R,t+s}.$$

Using the information in equation 4, default will therefore occur if:

$$\ln(E_{R,t+H}) = \ln(E_{Rt}) + H\mu_R + \sigma_R \sum_{s=1}^{H} \eta_{R,t+s} < \ln(C_{R,t+H}). \tag{6}$$

Or using the log equity threshold, default occurs if the H^{th}-period return falls below the log-threshold equity ratio:

$$\ln\left(\frac{E_{R,t+H}}{E_{Rt}}\right) < \ln\left(\frac{C_{R,t+H}}{E_{Rt}}\right). \tag{7}$$

Equation 7 illustrates that over the horizon *H,* the relative decline in firm value must be big enough to result in default. As such the default becomes independent of firm size in the default process, but is an important determinant of the initial rating assigned by the rating agencies to a specific firm. Naturally a small firm will need to hold a larger equity cushion relative to a big firm if it is likely to withstand a given shock.

As usual, if firm equity values follow equation 5, $\ln(E_{R,t+H}/E_{Rt})$ can be approximated by cumulative returns so that (7) becomes

$$Hu_R + \sigma_R \sum_{s=1}^{H} \eta_{R,t+s} < \ln\left(\frac{C_{R,t+H}}{E_{Rt}}\right).$$

Therefore the default probability of an *R*-rated firm at the terminal date *t*+*H* is given by

$$\pi_R(t,H) = \Phi\left(\frac{\ln(C_{R,t+h} / E_{Rt}) - H\mu_R}{\sigma_R \sqrt{H}}\right), \tag{8}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Now, denote the H-period forward log threshold equity ratio to be $\lambda_R(t,H)= \ln(C_{R,t+H}/E_{Rt})$ so that

$$\lambda_R(t,H) = H\mu_R + Q_R(t,H)\sigma_R\sqrt{H}$$

where $Q_R(t,H) = \Phi^{-1}[\pi_R(t,H)]$ is the quantile associated with the default probability $\pi_R(t,H)$.

Essentially an estimate of $\lambda_R(t,H)$ can be obtained using past observations of equity returns, $r_{Rt}=ln(E_{R,t+H}/E_{Rt})$, and the empirical default frequencies, $\hat{\pi}_R(t,H)$, of R-rated firms over a given time period say t=1,2,…,T. Not surprisingly, a source of heterogeneity between firms will result from the varying bankruptcy laws, differences in financial market sophistication, and regulations such as exchange controls etc. which are displayed by countries across the globe. The use of rating agency data allows one to overcome these heterogeneities since in assessing each firm a specific rating is only assigned after these factors are taken into account. Specific ratings thus reflect underlying default risk after adjustment for the latter factors has been taken into account. Using empirical estimates for the mean return, $\hat{\mu}_R$ and standard deviation, $\hat{\sigma}_R$ for R rated firms over the sample period we have:

$$\hat{\lambda}_R(t,H) = H\hat{\mu}_R + \hat{Q}_R(t,H)\hat{\sigma}_R\sqrt{H} \tag{9}$$

where

$$\hat{Q}_R(t,H) = \Phi^{-1}[\hat{\pi}_R(t,H)]. \tag{10}$$

Admittedly the estimate of $\hat{\pi}_R(t,H)$ may not be a reliable estimate of $\pi_R(t,H)$ since defaults in higher rating categories are not very common and estimates will be based on very few defaults over any particular horizon $(t,t+H)$. Therefore to make estimates more robust, an average estimate of $\lambda_R(t,H)$ can be obtained from a reasonably long time period (10 to 20 years on a rolling basis). Therefore, based on a sample period of t=1,2,…,T,

$$\hat{\lambda}_R(t,H) = H\hat{\mu}_R + \hat{Q}_R(t,H)\hat{\sigma}_R\sqrt{H} \tag{11}$$

would be estimated with $\hat{Q}_R(H)$ given by

$$\hat{Q}_R(H) = T^{-1}\sum_{t=1}^{T}\left\{\Phi^{-1}\left[\hat{\pi}_R(t,H)\right]\right\}, \tag{12}$$

for example assuming a one year horizon used by rating agencies when assessing a firm, H=4 quarters.

The PSTW (2006) framework allows one to obtain estimates of the default-equity threshold ratios by credit rating. Also, if sufficient data is available one can estimate different default frequencies for specific countries or regions and even firms over particular rating categories. However, as already stated default is a rare event and in the absence of multiple defaults and sufficient regional default experiences, individual default frequency estimates are impossible to obtain. As such the following reasonable identification condition is made:

$$\left(\frac{C_{jiR,t+H}}{E_{jiRt}}\right) = \left(\frac{C_{R,t+H}}{E_{Rt}}\right), \qquad \text{for all } j \tag{13}$$

where E_{jiRt} and $C_{jiR,t+H}$ are the equity and default threshold values of firm j in region i, with credit rating R, at time t. Essentially condition 13 states that at any given time, any two firms that have received a similar rating will have the same default equity-thresholds ratios. As such the condition allows different threshold levels, allowing for heterogeneity between firm equity growth-paths, but assumes that the same ratio applies.

Data permitting, other factors can also be used to solve the identification problem. For example, it can be assumed that all firms with the same credit rating have the same distance to default (DD) ratio as opposed to equity threshold level where $DD = \left[\lambda_R(t,H) - H\mu_R\right]/\hat{\sigma}_R\sqrt{H}$. Moreover, one can group firms into homogenous sub-groups by using other criteria such as rating category within industry or sector.

However, these different criteria will be dependent on data availability as well as the fact that a reasonable sample of data will be required within each sub-group to allow reliable estimations to be obtained.

2.3 FIRM-SPECIFIC DEFAULTS

Using this adapted Merton model of default it is now possible to specify firm-specific default conditions and a default probability model. Following the standard multifactor model specification, assume that the return of firm *j* in region *i* over the time period *t* to *t*+1, i.e. $r_{ji,t+1}$ can be decomposed into two portions, conditional on information available at time *t*, given the information set Ω_t , i.e.

$$r_{ji,t+1} = \ln\left(\frac{E_{ji,t+1}}{E_{ji,t}}\right) = \mu_{ji,t} + \varepsilon_{ji,t+1} \tag{14}$$

with $\mu_{ji,t}$ the conditional mean and $\varepsilon_{ji,t+1}$ the innovation component of the return process. Since the conditional mean of the return process can be forecasted using macroeconomic factors (or risk drivers) this creates the channel through which one can simulate the impact that economic shocks will have on firm returns. As usual the individual innovation components will be assumed to take on the normal Merton model form

$$\varepsilon_{ji,t+1} \big| \Omega_t \sim N\left(0, \omega^2_{\varepsilon,ji}\right). \tag{15}$$

The assumption that the conditional variance of returns is time invariant has been heavily disputed throughout the literature in the case of high frequency data. As such techniques such as GARCH modelling which allows for time-dependent variances should be employed when dealing with such cases. Although the use of such techniques will slightly alter the current framework the general form is still preserved. In this application quarterly data will be employed and time-invariant return variances will be assumed to hold true.

From equation 7 it follows that a firm will be in default if

$$\ln(E_{R,t+H} / E_{Rt}) < \ln(C_{R,t+H} / E_{Rt}) \quad \text{or} \quad \ln(r_{ji,t+1}) < \lambda_{ji}(t,1).$$

Using an indicator function it is possible to separate the default and non-default states such that:

$$\begin{aligned} &I\left(r_{ji,t+1} < \lambda_{ji}(t,1)\right) = 1 \quad \text{if} \quad r_{ji,t+1} < \lambda_{ji}(t,1) \Rightarrow \text{Default} \\ &I\left(r_{ji,t+1} < \lambda_{ji}(t,1)\right) = 0 \quad \text{if} \quad r_{ji,t+1} \geq \lambda_{ji}(t,1) \Rightarrow \text{No Default.} \end{aligned} \tag{16}$$

The default probability of firm j over a one quarter ahead (H=1) forecast period follows from the framework above and is given by:

$$\pi_{ji}(t,1) = \Phi\left(\frac{\lambda_{ji}(t,1) - \mu_{ji,t}}{\omega_{\varepsilon,ji}}\right). \tag{17}$$

Using the firm-specific return regressions $\mu_{ji,t}$ and $\omega_{\varepsilon ji}$ can be estimated while $\lambda_{ji}(t,1)$ will be estimated using the assumptions made in the identification condition. As such for an R- rated firm $\lambda_{ji}(t,1)$ will be estimated as $\hat{\lambda}_R(t,1)$. Using these parameter estimates the default condition for an R-rated firm can therefore be written as:

$$\begin{aligned} &I\left(r_{ji,t+1} < \hat{\lambda}_R(t,1)\right) = 1 \quad \text{if} \quad r_{ji,t+1} < \hat{\lambda}_R(t,1) \Rightarrow \text{Default} \\ &I\left(r_{ji,t+1} < \hat{\lambda}_R(t,1)\right) = 0 \quad \text{if} \quad r_{ji,t+1} \geq \hat{\lambda}_R(t,1) \Rightarrow \text{No Default.} \end{aligned} \tag{18}$$

Although the default threshold ratios of R-rated firms are assumed to be the same, the default probabilities will differ conditional on the firm-specific return characteristics. More formally, by analysing equation 17 firm j's default probability (and also its distance to default) will be driven by:

- The firm's credit rating at the beginning of the forecast period. Lower credit ratings will by "closer" to the default thresholds;
- The volatility of equity returns, $\omega_{\varepsilon ji}$. The more volatile the more likely a firm is to cross the threshold at any time; and
- The unconditional equity return, $\mu_{ji,t}$. The higher the expected return, the "further" the firm is from default.

A macroeconometric risk driver model (the GVAR model in the case of PSTW (2006)) provides a conditional loss distribution through the ability to link the changes in macroeconomic variables (in region *i* and globally) to firm-specific credit risk. The linkages in the macroeconomy are established through the interaction of the changes in macro-variables and firm-specific (conditional) means $u_{ji,t}$.

2.4 CONDITIONAL CREDIT RISK MODELLING

2.4.1 THE MACROECONOMIC RISK DRIVER MODEL

2.4.1.1 Introduction

The macroeconometric risk driver model (GVAR) used in PSTW (2006) comprises a total of 25 countries which are grouped into 11 regions and account for 80 per cent of world production. However, as stated by PSTW (2006) a cointegration framework can become computationally demanding and therefore seven key economies are modelled, namely the U.S., U.K., Japan, China, Germany, France and Italy alone while all other countries are modelled as part of regional groups, i.e. Western Europe, South East Asia, Latin America and the Middle East. Clearly, in the case of South Africa the GVAR model lacks applicability, since it does not include an African region. However, the approach is general enough so that country-specific cointegration models can be linked into the global and already established GVAR model. Therefore, the use of cointegration is applied in such a fashion that heterogeneity that exists across regions and countries is acknowledged.

Although explained in detail below the basic intuition of the GVAR model is that specific vector error-correcting models (VECM), which relate macroeconomic variables such as gross domestic product GDP, inflation, interest rates, money supply, exchange rates and equity prices to foreign variables through the linkages of international trade patterns, are estimated for each region or country. Within the GVAR framework there are three channels through which economies and factors in each economy interact:

- Contemporaneous dependence of domestic and foreign variables;
- Dependence of country-specific variables on common global effects such as oil prices; and
- Weak cross-sectional dependence of idiosyncratic shocks.

In this study a country-specific macroeconometric risk driver engine which can feed into the GVAR model and framework proposed by PSW (2004) will be constructed. This will allow conditional loss estimation of a South African-specific credit portfolio but also opens the door for credit portfolio modelling on a global scale, because such a model can easily be linked into the GVAR model. In order to estimate and provide such a South African specific model it is necessary to analyse the construction of the GVAR model proposed by PSW (2004).

2.4.1.2 The global error-correcting macroeconometric model

Applications of cointegrating systems have been limited to single-country models covering only some sub-sample of key macroeconomic variables or sector-specific variables such as labour-market dynamics. The limitations of applying cointegrating systems and models on a global basis are not due to theoretical constraints but rather due to the computational burden and the data intensity required by such a system. Specifically in an unrestricted vector autoregressive (VAR) model covering N regions the unknown parameters that need to be estimated increase with the number of regions included in such a system. Specifically in a VAR system there will be $p(kN\text{-}1)$

unknown parameters (excluding intercepts and exogenous variables) to be estimated for each equation where p is the lag order and k is the number of endogenous variables for each region. For example, in a world composed of 6 regions, p=2 and k=5, there will be at least 58 parameters to be estimated. Clearly such a system would require a sufficient number of time-series observations to allow enough degrees of freedom to obtain efficient and consistent parameter estimates.

Not surprisingly, these constraints have forced global forecasting models into using single-equation cointegration techniques (see for example Lawrence Klein's Project Link adopted by the United Nations). These models are, however, difficult to use in risk management since they do not adequately account for the financial interlinkages that exist in the global economy.

The macroeconometric (GVAR) model proposed by PSW (2004) aims to provide a flexible framework for use in a variety of applications. Individual vector error-correcting models are estimated for each region individually where domestic variables are related to corresponding foreign variables through the international trade pattern of the individual country. Individual country models are then combined to generate forecasts and impulse response functions for all variables in the world economy simultaneously through the GVAR model.

2.4.1.3 Individual-country VECMs

Assume that there are N+1 countries in the global economy (i.e. i = 0, 1, 2,…,N) Following PSW (2004) let the reference country be denoted by 0 and assume that it is the U.S. Also let each country's domestic variables be related to global economic variables measured as country-specific weighted averages of foreign variables plus deterministic variables and weakly exogenous global factors such as oil prices. Confining the specification to a first order dynamic specification (for exposition purposes), the $k_i \times 1$ country specific factors, $\mathbf{x}_{it}$ are related to $\mathbf{x}_{it}^*$, a $k_i^* \times 1$ vector of foreign variables specific to country i such that

$$\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \mathbf{\Phi}_i \mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{i0}\mathbf{x}^*_{it} + \mathbf{\Lambda}_{i1}\mathbf{x}^*_{i,t-1} + \boldsymbol{\varepsilon}_{it}, \qquad t = 1, 2, ..., T, \quad i = 1, 2, ..., N \tag{19}$$

where $\mathbf{\Phi}_i$ is a $k_i \times k_i$ matrix of lagged coefficients, $\mathbf{\Lambda}_{i0}$ and $\mathbf{\Lambda}_{i1}$ are $k_i \times k^*_i$ matrices of coefficients associated with the foreign-specific variables, and $\boldsymbol{\varepsilon}_{it}$ is a $k_i \times 1$ vector of idiosyncratic country-specific shocks which are serially uncorrelated with mean $\mathbf{0}$ and a non-singular covariance matrix, $\mathbf{\Sigma}_{ii} = (\sigma_{ii,ls})$, where $\sigma_{ii,ls} = \text{cov}(\varepsilon_{i,lt}, \varepsilon_{ist})$, i.e.

$$\boldsymbol{\varepsilon}_{it} \sim iid(\mathbf{0}, \mathbf{\Sigma}_{ii}). \tag{20}$$

As stated previously, the assumption of non-serially correlated residuals is not overly simplistic when using quarterly data observations and can be relaxed when estimating the model using higher frequency data. $\boldsymbol{\varepsilon}_{it}$ is also allowed to be weakly correlated with shocks in other regions through the link between country-specific, $\mathbf{x}_{it}$ and foreign variables, $\mathbf{x}^*_{it}$ as explained below.

Following PSW (2004), for illustrative purposes, the set of domestic variables $\mathbf{x}_{it}$ includes six variables i.e. $k_i = 6$ and $\mathbf{x}_{it} = (y_{it}, p_{it,} q_{it}, e_{it}, \rho_{it}, m_{it})'$ where:

$$\begin{aligned} y_{it} &= \ln\left(\frac{GDP_{it}}{CPI_{it}}\right), & p_{it} &= \ln(CPI_{it}), \\ q_{it} &= \ln\left(\frac{EQ_{it}}{CPI_{it}}\right), & m_{it} &= \ln\left(\frac{M_{it}}{CPI_{it}}\right), \\ e_{it} &= \ln(E_{it}), & \rho_{it} &= 0.25\ln\left(1+\frac{R_{it}}{100}\right), \end{aligned} \tag{21}$$

and GDP_{it} = nominal gross domestic product, CPI_{it} = consumer price index, M_{it} = nominal money supply in domestic currency, EQ_{it} = nominal equity price index, E_{it} = exchange rate of country i at time t with respect to the U.S. dollar, and R_{it} = nominal rate of interest per annum in percent.

$\mathbf{x}_{it}$ can include any set of domestic variables and does not have to be constrained to the above specification. Also, other adjustments of variables may be necessary in certain countries, such as is the case when real effective exchange rates are used instead of nominal exchange rates. Also data limitations will in certain cases be present, such as in the case of emerging markets, where there may not be information on equity prices over the full time period. These factors also dictate the variables that can be included in $\mathbf{x}_{it}$.

The set of foreign variables $\mathbf{x}_{it}^*$ is also not constrained to a specific set of variables but once again for illustrative purposes PSW (2004) will be followed, thus confining k_i^* to 6. $\mathbf{x}_{it}^*$ is constructed as a set of indices of weighted averages of country or regional weights, i.e.:

$$\begin{aligned} y_{it}^* &= \sum_{j=0}^{N} w_{ij}^y y_{it}, & p_{it}^* &= \sum_{j=0}^{N} w_{ij}^p p_{it}, \\ q_{it}^* &= \sum_{j=0}^{N} w_{ij}^q q_{it}, & e_{it}^* &= \sum_{j=0}^{N} w_{ij}^e e_{it}, \\ \rho_{it}^* &= \sum_{j=0}^{N} w_{ij}^\rho \rho_{it}, & m_{it}^* &= \sum_{j=0}^{N} w_{ij}^m m_{it}. \end{aligned} \tag{22}$$

The weights $w_{ij}^y, w_{ij}^p, w_{ij}^q, w_{ij}^e, w_{ij}^\rho$ and w_{ij}^m for $i, j = 0,1,\ldots,N$ *and* $w_{ii}^m = 0$, could be based on trade shares i.e. the share of country *j* in the total trade of country *i* measured in U.S. dollars in the case of y_{it}^*, p_{it}^*, e_{it}^* and m_{it}^*, and on capital flows in the case of equity prices, q_{it}^* and interest rates, ρ_{it}^*. Calculation of the weights can be done on a dynamic basis but may result in the introduction of randomness into the analysis. As such the weights can be kept constant based on an average of trade flows over a 3-year period. The data sample over which the trade weights are calculated should preferably be representative of the current economic environment if the aim is to use the model to forecast and simulate forward-looking estimates of variables.

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As discussed and illustrated in PSW (2004), the exchange rate variable e_{it}^* is not the familiar "effective exchange rate" encountered in the literature. Only in the case of the base country will the two concepts coincide. Also in the case of the base country, e_{0t}^* will be determined by the models of the rest of the world via equation 19, for i = 1,2,…,N. Therefore, e_{0t}^* should be treated as an exogenous variable for the base country. In the case of countries who peg their currencies to that of another country multicollinearity may arise when both e_{it} and e_{it}^* are included in the model since they are bound to be highly correlated. In such cases e_{it} should be sufficient to capture both domestic and international exchange rate impacts.

In general, the GVAR model proposed by PSW (2004) allows for interaction amongst countries and economies through three channels:

- Contemporaneous dependence of $\mathbf{x}_{it}$ on $\mathbf{x}_{it}^*$ and on its own past lagged variables;
- Dependence of country-specific variables on common global effects such as oil prices; and
- Weak cross-section dependence of the idiosyncratic shocks in country j and those of country i, measured by the cross-country covariances, $\mathbf{\Sigma}_{ij}$, :

$$\sum\nolimits_{ij} = \mathrm{cov}\left(\varepsilon_{it}, \varepsilon_{jt}\right) = E\left(\varepsilon_{it}\varepsilon_{jt}^{'}\right), \qquad \text{for } i \neq j, \qquad (24)$$

where ε_{it} is defined by (19). A typical element of $\mathbf{\Sigma}_{ij}$, will be denoted by $\sigma_{ij,ls} = \mathrm{cov}\left(\varepsilon_{it}, \varepsilon_{jt}\right)$ which is the covariance of the l^{th} variable in country i with the s^{th} variable in country j.

The specification above provides a complete system of N+1 country-specific variables which can be estimated simultaneously if data permits. However, this is hardly ever possible and as a result PSW (2004) proposes that the individual country models should be estimated separately assuming that the foreign variables are weakly exogenous to the system. This exogeneity assumption should in practice hold for small open economies where the impact of global market leaders and/or regions is usually exogenously given. Certainly such a weak exogeneity assumption seems reasonable to a small player in the global economy such as South Africa.

2.4.1.4 Solving the GVAR model

To construct the GVAR model from the individual country models, let $z_{it}^{'}$ be a $\left(k_i + k_i^*\right)\times 1$ vector such that:

$$\mathbf{z}_{it} = \begin{pmatrix} \mathbf{x}_{it} \\ \mathbf{x}_{it}^* \end{pmatrix}, \tag{25}$$

therefore equation 19 can be defined as:

$$\mathbf{A}_i \mathbf{z}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1t} t + \mathbf{B}_i \mathbf{z}_{i,t-1} + \boldsymbol{\varepsilon}_{it} \tag{26}$$

where

$\mathbf{A}_i = \left(\mathbf{I}_{ki}, -\boldsymbol{\Lambda}_{i0}\right)$ and $\mathbf{B}_i = \left(\boldsymbol{\Phi}_i, \boldsymbol{\Lambda}_{i1}\right)$ are $k_i \times \left(k_i + k_i^*\right)$ and $\mathbf{A}_i$ has full row rank,

i.e. rank($\mathbf{A}_i$)= k_i . (27)

Also, collect all the country-specific variables together in a $k \times 1$ global vector $\mathbf{x}_t = \left(\mathbf{x}_{0t}^{'}, \mathbf{x}_{1t}^{'}, ..., \mathbf{x}_{Nt}^{'}\right)^{'}$ where $k = \sum_{i=0}^{N} k_i$ is equal to the total number of endogenous variables in the system. Therefore, the country-specific variables can all be written in terms of $\mathbf{x}_t$:

$$\mathbf{z}_{it} = \left(\mathbf{W}_i \mathbf{x}_t\right), \qquad i = 0,1,2,....N \tag{28}$$

where $\mathbf{W}_i$ is a $(k_i + k_i^*) \times k$ matrix of fixed constants defined in terms of the country-specific weights as defined and calculated above. PSW (2004) therefore defines $\mathbf{W}_i$ as the "link" matrix which allows the individual-country models to be written in terms of the global variable vector $\mathbf{x}_t$.

Using the results presented in equations 26 and 28, we have:

$$\mathbf{A}_i \mathbf{W}_i \mathbf{x}_t = \mathbf{a}_{i0} + \mathbf{a}_{i1t} t + \mathbf{B}_i \mathbf{W}_i \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_{it}$$

where $\mathbf{A}_i \mathbf{W}_i$ and $\mathbf{B}_i \mathbf{W}_i$ are both $(k_i \times k)$ matrices. Stacking these equations will yield:

$$\mathbf{G}\mathbf{x}_t = \mathbf{a}_0 + \mathbf{a}_{1t} t - \mathbf{H}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t \tag{29}$$

where

$$\mathbf{a}_0 = \begin{pmatrix} \mathbf{a}_{00} \\ \mathbf{a}_{10} \\ \cdots\cdots \\ \mathbf{a}_{N0} \end{pmatrix}, \qquad \mathbf{a}_1 = \begin{pmatrix} \mathbf{a}_{01} \\ \mathbf{a}_{11} \\ \cdots\cdots \\ \mathbf{a}_{N1} \end{pmatrix}, \qquad \boldsymbol{\varepsilon}_t = \begin{pmatrix} \boldsymbol{\varepsilon}_{0t} \\ \boldsymbol{\varepsilon}_{1t} \\ \cdots\cdots \\ \boldsymbol{\varepsilon}_{Nt} \end{pmatrix} \tag{30}$$

and

$$\mathbf{G}_0 = \begin{pmatrix} \mathbf{A}_0 \mathbf{W}_0 \\ \mathbf{A}_1 \mathbf{W}_1 \\ \cdots\cdots\cdots \\ \mathbf{A}_N \mathbf{W}_N \end{pmatrix}, \qquad \mathbf{H}_0 = \begin{pmatrix} \mathbf{B}_0 \mathbf{W}_0 \\ \mathbf{B}_1 \mathbf{W}_1 \\ \cdots\cdots\cdots \\ \mathbf{B}_N \mathbf{W}_N \end{pmatrix} \tag{31}$$

$\mathbf{G}$ will be a $(k \times k)$ dimensional matrix and in general will be of full rank and therefore also non-singular. As such the GVAR model on all of the variables can be written and solved forward to obtain future values of $\mathbf{x}_t$ as discussed below[1], i.e.:

[1] PSW (2004) provides an illustration of this solution technique in section 3.

$$\mathbf{x}_t = \mathbf{G}^{-1}\mathbf{a}_0 + \mathbf{G}^{-1}\mathbf{a}_{1t}t + \mathbf{G}^{-1}\mathbf{H}\mathbf{x}_{t-1} + \mathbf{G}^{-1}\boldsymbol{\varepsilon}_t$$

since the country-specific weights satisfy the adding-up restrictions $\sum_{i=1}^{N} w_i = 1$, the link matrixes must be of full row rank allowing the link matrix $\mathbf{G}$ to be non-singular as well.

2.4.1.5 Short-run dynamics of the global model

The error-correcting representation of equation 19 is given by:

$$\Delta\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1t}t - (\mathbf{I}_{ki} - \boldsymbol{\Phi}_i)\mathbf{x}_{i,t-1} + (\boldsymbol{\Lambda}_{i0} + \boldsymbol{\Lambda}_{i1})\mathbf{x}^*_{i,t-1} + \boldsymbol{\Lambda}_{i0}\Delta\mathbf{x}^*_{it} + \boldsymbol{\varepsilon}_{it} \qquad i = 1, 2, ..., N \tag{32}$$

and, recalling the $(k_i + k_i^*)\times 1$ vector:

$$z_{it} = \begin{pmatrix} x_{it} \\ x^*_{it} \end{pmatrix},$$

and the $k_i \times (k_i + k_i^*)$ matrices:

$$A_i = (\mathbf{I}_{ki} - \boldsymbol{\Lambda}_{i0}) \qquad \text{and } B_i = (\boldsymbol{\Phi}_i, \boldsymbol{\Lambda}_{i1})$$

Then:

$$\Delta\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1t}t - (\mathbf{A}_i - \mathbf{B}_i)\mathbf{z}_{i,t-1} + \boldsymbol{\Lambda}_{i0}\mathbf{x}^*_{i,t-1} + \boldsymbol{\varepsilon}_{it}, \tag{33}$$

where $z_{it} = (x'_{it}, x^{*'}_{it})'$. The error-correcting properties of the model for each country i are summarised in the $k_i \times (k_i + k_i^*)$ matrix:

$$\boldsymbol{\Pi}_i = \mathbf{A}_i - \mathbf{B}_i. \tag{34}$$

As usual, the rank(r) of $\mathbf{\Pi}_i$, with $r_i \leq k_i$ will specify the number of cointegrating or "long- run" relationships between the country-specific and foreign variables. As such we have:

$$\mathbf{A}_i - \mathbf{B}_i = \alpha_i \beta_i^{'}, \tag{35}$$

where α_i is a $k_i \times r_i$ loading matrix of full column rank and β_i is the $\left(k_i + k_i^*\right) \times r_i$ matrix of cointegrating vectors, also of full column rank. In the case where $\mathbf{\Pi}_i$ is rank deficient it is important to retain the same deterministic properties of $\mathbf{x}_{it}$ by restricting the rank of $\mathbf{\Pi}_i$. PSW (2004) draws on the results presented by Pesaran, Shin and Smith (2000) who show that this can be achieved by restricting the coefficients so that:

$$\mathbf{a}_{it} = \left(\mathbf{A}_i - \mathbf{B}_i\right)\boldsymbol{\kappa}_i \tag{36}$$

where $\boldsymbol{\kappa}_i$ is a $\left(k_i + k_i^*\right) \times 1$ vector of fixed constants. As such this specification imposes $k_i - r_i$ restrictions on the trend coefficients.

Although beyond the scope of this study, it is also important to note that PSW (2004) (sections 3 and 4) show that the global model and its error-correcting specification follow from equation 29 and can be represented by

$$\mathbf{G}\Delta\mathbf{x}_t = \mathbf{a}_0 + \mathbf{a}_1 t - \left(\mathbf{G}_i - \mathbf{H}_i\right)\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t. \tag{37}$$

They also prove that it is possible to combine all individual-country models using the trade-share weighting factors as "link" matrices in such a way that it is possible to uniquely solve for all endogenous variables within the GVAR model. Moreover, they show that the error-correcting properties of the individual models are carried over to the global model and prove that the number of cointegrating relationships in the global model cannot exceed the sum of the individual country models' cointegrating relationships. Once again the choice of the "link" matrix is important as this will determine the number of cointegrating relationships in the global model.

2.4.1.6 Forecasting and dynamic properties of the GVAR model

For risk-management and policy-analysis purposes it is important to be able to give a forward-looking estimate of possible future events. As such it is important that the GVAR model is stable enough to allow consistent forecasts of macroeconomic conditions. Moreover, it is often desirable that the model be general enough to allow for external policy variables or other common global variables such as oil prices to be externally determined from the system but be allowed to impact other country- and region-specific variables within the GVAR model. This adjustment is given by PSW (2004) through the VARX* model and can be represented as:

$$\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \mathbf{\Phi}_i \mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{i0}\mathbf{x}_{it}^* + \mathbf{\Lambda}_{i1}\mathbf{x}_{i,t-1}^* + \boldsymbol{\psi}_{i0}\mathbf{d}_t + \boldsymbol{\psi}_{i1}\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{it}, \qquad t = 1, 2, \ldots, T, \quad i = 1, 2, \ldots, N \tag{38}$$

where $\mathbf{d}_t$ is an $(s \times 1)$ vector of common global variables assumed to be weakly exogenous to the global economy. It is also important to note that the distinction between foreign variables $\mathbf{x}_t^*$ and exogenous variables $\mathbf{d}_t$ is only relevant for analysis of the dynamic properties of the model and they do not influence the estimation of the country-specific models. This property is especially desirable in this study since the South African specific model simulation and forecasts will be done separately from those of the global GVAR model already estimated by PSW (2004). Therefore, in simulating forward-looking estimates of the domestic macroeconomic variables, the other regional variables will also be assumed to be exogenous and thus similar to exogenous global variables.

The country-specific global model including the exogenous variables will now be given by:

$$\mathbf{G}\mathbf{x}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{H}\mathbf{x}_{t-1} + \boldsymbol{\psi}_0\mathbf{d}_t + \boldsymbol{\psi}_1\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{a}_0, \mathbf{a}_1, \mathbf{G}, \mathbf{H}$ and $\boldsymbol{\varepsilon}_{it}$ are defined above and:

$$\boldsymbol{\psi}_0 = \begin{pmatrix} \boldsymbol{\psi}_{00} \\ \boldsymbol{\psi}_{10} \\ .... \\ \boldsymbol{\psi}_{N0} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\psi}_1 = \begin{pmatrix} \boldsymbol{\psi}_{01} \\ \boldsymbol{\psi}_{11} \\ .... \\ \boldsymbol{\psi}_{N1} \end{pmatrix}. \tag{39}$$

It is reasonable to assume that $\mathbf{G}$ is non-singular, thus the reduced form global model will be given by:

$$\mathbf{x}_t = \mathbf{b}_0 + \mathbf{b}_1 t + \mathbf{F}\mathbf{x}_{t-1} + \boldsymbol{\gamma}_0 \mathbf{d}_t + \boldsymbol{\gamma}_1 \mathbf{d}_{t-1} + \boldsymbol{\mu}_t, \quad t = 1, 2, ..., T, T+1, ..., T+n \tag{40}$$

where

$$\mathbf{b}_i = \mathbf{G}^{-1}\mathbf{a}_i, \quad i = 0,1; \qquad \mathbf{F} = \mathbf{G}^{-1}\mathbf{H}, \\ \boldsymbol{\gamma}_0 = \mathbf{G}^{-1}\boldsymbol{\psi}_0 \qquad \boldsymbol{\gamma}_1 = \mathbf{G}^{-1}\boldsymbol{\psi}_1 \qquad \text{and} \qquad \boldsymbol{\mu}_t = \mathbf{G}^{-1}\boldsymbol{\varepsilon}_t. \tag{41}$$

Now, suppose that the global economy is observed over the period $t = 1, 2, ..., T$ but that the aim is to forecast $\mathbf{x}_t$ over the forecast period *n*, i.e. $t = T+1, ..., T+n$. In order to simplify the analysis it is assumed that the exogenous variables $\mathbf{d}_t$ are given for all periods over the forecast horizon. As explained below the exogenous variables will only impact the simulated loss distribution since this is a non-linear function of macroeconomic variables, but the impulse response functions are not influenced by the underlying data-generating processes of the exogenous variables since they are linearly correlated.

So, using (40), the difference equation is solved forward over the forecast period *n* and the solution of all macroeconomic variables in the system is obtained, i.e.:

$$\mathbf{x}_{T+n} = \mathbf{F}^n \mathbf{x}_T + \sum_{\tau=0}^{n-1} \mathbf{F}^\tau \left[\mathbf{b}_0 + \mathbf{b}_1 (T+n-\tau)\right] + \sum_{\tau=0}^{n-1} \mathbf{F}^\tau \left[\boldsymbol{\gamma}_0 \mathbf{d}_{T+n-\tau} + \boldsymbol{\gamma}_1 \mathbf{d}_{T+n-\tau-1}\right] + \\ + \sum_{\tau=0}^{n-1} \mathbf{F}^\tau \boldsymbol{\mu}_{T+n-\tau} \tag{42}$$

Equation 42 states that the future realisation of the domestic macro variables is a function of four components. The first component, $\mathbf{F}^n \mathbf{x}_T$, is the effect of the initial state of the variables themselves, i.e. $\mathbf{x}_T$. The second component is the effect of the deterministic trend within the underlying VAR model, while the third component is the impact of the exogenous variables. The last component (the random errors) represents the stochastic undetermined component of the forecast of $\mathbf{x}_{T+N}$.

Using the expectation operator the point forecast of $\mathbf{x}_T$ can be determined and as such will be conditional on the initial values of $\mathbf{x}_T$ and the global exogenous variables, i.e.:

$$\begin{aligned} \mathbf{x}^*_{T+n} &= \mathrm{E}\left(x_{T+N} \middle| x_T, \cup_{\tau=1}^{n} d_{t+\tau}\right) \\ &= \mathbf{F}^n \mathbf{x}_T + \sum_{\tau=0}^{n-1} \mathbf{F}^{\tau}\left[\mathbf{b_0} + \mathbf{b_1}(T+n-\tau)\right] + \sum_{\tau=0}^{n-1} \mathbf{F}^{\tau}\left[\boldsymbol{\gamma}_\mathbf{0} \mathbf{d}_{T+n-\tau} + \boldsymbol{\gamma}_\mathbf{1} \mathbf{d}_{T+n-\tau-1}\right] \end{aligned} \tag{43}$$

Equation 42 also enables one to compute the probability distribution function of $\mathbf{x}_{T+n}$. This is an important component needed for computing the loss distribution of a given portfolio. Therefore, assuming that the random errors, $\boldsymbol{\varepsilon}_\mathbf{t}$ or innovations are normally distributed, it follows from 42 that:

$$\mathbf{x}^*_{T+n} \left| x_T, \cup_{\tau=1}^{n} d_{t+\tau} \sim N\left(x^*_{T+N}, \mathbf{\Omega}_n\right)\right., \tag{44}$$

where x^*_{T+n} is given by (43) and

$$\mathbf{\Omega}_n = \sum_{\tau=0}^{n-1} \mathbf{F}^{\tau} \mathbf{G}^{-1} \mathbf{\Sigma} \mathbf{G}^{'-1} \mathbf{F}^{'\tau}, \tag{45}$$

where $\mathbf{\Sigma}$ is the $k \times k$ variance-covariance matrix of the shocks $\boldsymbol{\varepsilon}_\mathbf{t}$ such that $\mathbf{\Sigma}_{ij}$, as given in equation 24, is the (i, j) block of $\mathbf{\Sigma}$.

Clearly the dynamic properties of the global model are highly influenced by the eigenvalues of $\mathbf{F}$. In fact a very desirable outcome would be if the eigenvalues of $\mathbf{F}$ lie within the unit circle. This can be achieved by restricting the trend coefficient $\mathbf{b}_1$ in each country-specific model in such a way that the linear trend in $\mathbf{x}_T$ is carried over to the error-correction model. Practical ways to apply such constraints are discussed below. If the eigenvalues are indeed within the unit circle, $\mathbf{x}_{T+n}$ will have a stable distribution and thus satisfy the following properties:

- The dependence of $\mathbf{x}_{T+n}$ on the initial values of $\mathbf{x}_T$ will decrease and disappear for long forecast horizons i.e. sufficient values of n.
- The forecast covariance matrix $\mathbf{\Omega}_n$ will converge to a finite value as the forecast horizon increases, i.e. $n \to \infty$.
- The trending properties of the underlying country-specific VAR models will be transposed onto the point forecasts of x^*_{T+n}.

2.4.1.7 Impulse response and scenario analysis

Policy and scenario analysis is fundamental to any dynamic modelling framework. Traditionally this is accomplished through the methodology proposed by Sims (1980). The orthogonalised-impulse-response (OIR) analysis computes the impact of a set of orthogonal impulses, ξ_t instead of the original shocks $\boldsymbol{\varepsilon}_\mathbf{t}$. The set of independent shocks, ξ_t are transformed into correlated shocks $\boldsymbol{\varepsilon}_\mathbf{t}$ through the $k \times k$ lower triangle Cholesky factorization of the variance-covariance matrix of $\boldsymbol{\varepsilon}_\mathbf{t}, \mathbf{\Sigma}$ such that $\xi_t = \mathbf{P}^{-1}\boldsymbol{\varepsilon}_t$. Therefore:

$$\mathbf{PP}' = \mathbf{\Sigma} \quad \text{and} \quad \mathrm{E}\left(\xi_t \xi_t^{'}\right) = \mathbf{I}. \tag{46}$$

The $k \times 1$ vector of the OIR function of a unit shock (one standard deviation) of the j^{th} equation on $\mathbf{x}_{t+n}$ is then given by:

$$\boldsymbol{\psi}_j^0(n) = \mathbf{F}^n \mathbf{G}^{-1} \mathbf{P} \mathbf{s}_j, \qquad n = 0,1,2,..... \tag{47}$$

where s_j is a $k \times 1$ selection vector with unity as its j^{th} element (the particular shock under consideration) and zeros elsewhere. Although informative, the OIR suffers from one fundamental drawback i.e. the response of the variables is conditional on the ordering of factors within the specific-region VAR systems due to the non-uniqueness of the Cholesky factorization matrix, $\mathbf{P}$. In small systems, such as a regional model for South Africa the ordering can be inferred through theoretical restrictions. However in larger systems such as the GVAR model this approach will be too onerous. As such PSW (2004) proposes an alternative method which is invariant to the ordering of the variables in the VAR models. The generalized-impulse response function (GIRF) has been advanced by Koop, Pesaran, and Potter (1996), Pesaran and Shin (1998), and Pesaran and Smith (1998).

By using equation 42 directly, and by shocking only one element (say the j^{th} element in $\boldsymbol{\varepsilon}_t$, corresponding to the l^{th} variable in the i^{th} country) the effects of other shocks can be integrated out using either assumed or historical distributions of the errors. Therefore under GIRF:

$$\mathbf{GI}_{x:\varepsilon_{il}}\left(n, \sqrt{\sigma_{ii,ll}}, I_{t-1}\right) = \mathrm{E}\left(\mathbf{x}_{t+n} \middle| \varepsilon_{ilt} = \sqrt{\sigma_{ii,ll}}, I_{t-1}\right) - \mathrm{E}\left(\mathbf{x}_{t+n} \middle| I_{t-1}\right), \tag{48}$$

and $I_t = (\mathbf{x}_t, \mathbf{x}_{t-1},...)$ is the information set at time $t-1$ and $\mathbf{d}_t$ is still assumed to be exogenously determined. Once again assuming that $\boldsymbol{\varepsilon}_t$ has a multivariate normal distribution it follows from equation 42 that:

$$\boldsymbol{\psi}_j^g(n) = \frac{1}{\sqrt{\sigma_{ii,ll}}} \mathbf{F}^n \mathbf{G}^{-1} \boldsymbol{\Sigma} \mathbf{s}_j. \qquad n = 0,1,2,..... \tag{49}$$

Equation 49 measures a one standard deviation shock to the j^{th} equation (corresponding to the l^{th} variable in the i^{th} country) at time t on expected values of $\mathbf{x}$

at time *t*+*n*. It also follows that $\boldsymbol{\psi}_j^g(n)$ will be identical to $\boldsymbol{\psi}_j^0(n)$ when $\boldsymbol{\Sigma}$ is diagonal (i.e. there is no cross correlation between the errors) or when the analysis shocks the first element of $\boldsymbol{\varepsilon}_\mathbf{t}$.

PSW (2004) also derived a GIRF function for a unit shock to exogenous variables, say the i^{th} exogenous variable, d_{it}. Suppose the exogenous variables $\mathbf{d}_t$ are assumed to be generated by an autoregressive system and in particularly an AR(1) process, i.e.:

$$\mathbf{d}_t = \boldsymbol{\mu}_t + \boldsymbol{\Phi}_d \mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{dt}, \qquad \boldsymbol{\varepsilon}_{dt} \sim iid(\mathbf{0}, \boldsymbol{\Sigma}_d) \tag{50}$$

where $\boldsymbol{\mu}_t$ is a $s \times 1$ vector of constants, $\boldsymbol{\Phi}_d$ is a $s \times s$ matrix of lagged coefficients, $\boldsymbol{\varepsilon}_{dt}$ is a $s \times 1$ vector of idiosyncratic shock variables, and $\boldsymbol{\Sigma}_d$ is the covariance matrix of the shocks which are allowed to be singular. Similar to the above, the effects of a one unit shock to the i^{th} exogenous variable on the vector of endogenous variables *n* periods ahead can be defined by:

$$\mathbf{GI}_{x:d_i}(n, \sigma_{d,ii}, I_{t-1}) = \mathrm{E}\left(\mathbf{x}_{t+n} \middle| d_{it} = \sqrt{\sigma_{d,ii}}, I_{t-1}\right) - \mathrm{E}\left(\mathbf{x}_{t+n} \middle| I_{t-1}\right), \tag{51}$$

where $\sigma_{d,ii}$ is the i^{th} diagonal element of $\boldsymbol{\Sigma}_d$. Now, using equation 40, PSW (2004) shows that:

$$\begin{aligned}\mathbf{GI}_{x:d_i}(n, \sigma_{d,ii}, I_{t-1}) = {} & \mathbf{FGI}_{x:d_i}(n-1, \sigma_{d,ii}, I_{t-1}) + \boldsymbol{\gamma}_0 \mathbf{GI}_{d:d_i}(n, \sigma_{d,ii}, I_{t-1}) \\ & + \boldsymbol{\gamma}_1 \mathbf{GI}_{d:d_i}(n-1, \sigma_{d,ii}, I_{t-1}), \quad \text{for } n = 0,1,2,....\end{aligned} \tag{52}$$

where

$$\mathbf{GI}_{d:d_i}(n, \sigma_{d,ii}, I_{t-1}) = \mathrm{E}\left(\mathbf{d}_{t+n} \middle| d_{it} = \sqrt{\sigma_{d,ii}}, I_{t-1}\right) - \mathrm{E}\left(\mathbf{d}_{t+n} \middle| I_{t-1}\right). \tag{53}$$

Now, for $n < 1$, $\mathbf{GI}_{x:d_i}(n-1, \sigma_{d,ii}, l_{t-1}) = \mathbf{GI}_{d:d_i}(n-1, \sigma_{d,ii}, l_{t-1}) = 0$ and furthermore

$$\mathbf{GI}_{x:d_i}(0, \sigma_{d,ii}, l_{t-1}) = \boldsymbol{\gamma}_0 \mathbf{GI}_{x:d_i}(n-1, \sigma_{d,ii}, l_{t-1}),$$

and

$$\mathbf{GI}_{d:d_i}\left(0,\sigma_{d,ii},l_{t-1}\right)=\frac{1}{\sqrt{\sigma_{d,ii}}}\mathbf{\Sigma}_d\mathbf{e}_i,$$

where $\mathbf{e}_i$ is once again a $s\times 1$ selection vector with the i^{th} element being unity and zero for all other elements. Also:

$$\mathbf{GI}_{d:d_i}\left(n,\sigma_{d,ii},I_{t-1}\right)=\mathbf{\Phi}_d\mathbf{GI}_{d:d_i}\left(n-1,\sigma_{d,ii},l_{t-1}\right). \qquad \text{for } n=1,2,....$$

Therefore:

$$\mathbf{GI}_{d:d_i}\left(n,\sigma_{d,ii},I_{t-1}\right)=\frac{1}{\sqrt{\sigma_{d,ii}}}\Phi_d^n\mathbf{\Sigma}_d\mathbf{e}_i. \qquad \text{for } n=0,1,2,....$$

Substituting this result into equation 52, we have:

$$\mathbf{GI}_{x:d_i}\left(n,\sigma_{d,ii},l_{t-1}\right)=\mathbf{FGI}_{x:d_i}\left(n-1,\sigma_{d,ii},l_{t-1}\right)+\frac{1}{\sqrt{\sigma_{d,ii}}}\left(\boldsymbol{\gamma}_0\mathbf{\Phi}_d+\boldsymbol{\gamma}_1\right)\mathbf{\Phi}_d^{n-1}\mathbf{\Sigma}_d\mathbf{e}_i. \qquad (54)$$
$$\text{for } n=1,2,...$$

where:

$$\mathbf{GI}_{d:d_i}\left(0,\sigma_{d,ii},l_{t-1}\right)=\frac{1}{\sqrt{\sigma_{d,ii}}}\boldsymbol{\gamma}_0\mathbf{\Sigma}_d\mathbf{e}_i. \qquad (55)$$

As an example, if $\mathbf{d}$ is a scalar such as oil prices, $\frac{1}{\sqrt{\sigma_{d,ii}}}\boldsymbol{\gamma}_0\mathbf{\Sigma}_d\mathbf{e}=\sqrt{\sigma_{d,ii}}$.

2.4.2 USING INDIVIDUAL-COUNTRY MODELS TO CONSTRUCT THE GVAR MODEL

As argued above the GVAR model is theoretically very appealing, but estimation of the model as a single system would not be feasible for any moderately high values of N. In fact, PSW (2004) argues that the number of parameters that need to be estimated is often more than the number of data observations available. As such it would seem as if the model is practically unusable. However, PSW (2004) proves that the model can indeed be estimated due to the fact that the individual-country weightings w_{ij}, $i,j = 0,1,\ldots,N$, are computed independently based on trade and/or capital flow data. Moreover they also argue that the estimation of the country-specific models on a name by name basis rather than simultaneously is feasible for large values of *N,* if the following three conditions hold:

- The global model in equation 40 must be dynamically stable i.e. the eigenvalues of the matrix ***F*** should lie on or within the unit circle so that the trending properties of the underlying VAR model is maintained and is not explosive.
- The individual weights used to construct the foreign-specific variables, w_{ij} should be sufficiently small so that, $\sum_{j=0}^{N} w_{ij}^2 \to 0, \quad \text{as } N \to \infty$ for all i;
- The cross correlation of the idiosyncratic shocks should be sufficiently small so that $\frac{\sum_{j=0}^{N} \sigma_{ij,ls}^2}{N} \to 0 \quad \text{as } N \to \infty,$ for all i, l and s, where $\sigma_{ij,ls}^2 = \text{cov}(\varepsilon_{ilt}, \varepsilon_{jst})$ is the covariance of the l^{th} variable in country i with the s^{th} variable in country j.

Essentially these conditions provide a formal definition of a small open economy and in the context of this study will suffice, since South Africa can certainly be regarded as a small open economy in the global market. Although the first two conditions can easily be satisfied, the third condition can become fairly difficult to prove theoretically, especially in the case of a bigger global market country such as the U.S.

Under weak exogeneity conditions the parameters of the country-specific models can be estimated using the reduced-rank procedure directly on equation 38. Using the reduced-rank approach pioneered and developed by Johansen (1989, 1992 and 1995) allowances can be made for the possibility that the levels of the macroeconomic variables might be cointegrated in the long run. Although the methods developed by Johansen assume that all variables are endogenously determined and of order I(1), Pesaran, Shin and Smith (2000) have modified the method to allow for weakly exogenous variables to be included in a reduced-rank estimation procedure.

Therefore to estimate country-specific models, subject to reduced-rank restrictions, the error-correcting model should be re-written as:

$$\Delta \mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1} t + \mathbf{\Pi}_i \mathbf{v}_{i,t-1} + \mathbf{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \boldsymbol{\psi}_{i0} \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}_{it}, \tag{56}$$

where

$$\mathbf{\Pi}_i = (\mathbf{A}_i - \mathbf{B}_i, \boldsymbol{\psi}_{i0} - \boldsymbol{\psi}_{i1}) \tag{57}$$

and

$$\mathbf{v}_{i,t-1} = \begin{pmatrix} \mathbf{z}_{i,t-1} \\ \mathbf{d}_{t-1} \end{pmatrix}. \tag{58}$$

As before, to avoid introducing a quadratic trend into the levels of the variables it is necessary to impose the following restriction in the case where $\mathbf{\Pi}_i$ is rank deficient i.e. $\mathbf{a}_{i1} = \mathbf{\Pi}_i \boldsymbol{\kappa}_i$ where the dimensions of $\boldsymbol{\kappa}_i$ are now $(k_i + k_i^* + s) \times 1$. Therefore equation 56 becomes:

$$\Delta \mathbf{x}_{it} = \mathbf{c}_{i0} + \mathbf{\Pi}_i [\mathbf{v}_{i,t-1} - \boldsymbol{\kappa}_i (t-1)] + \mathbf{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \boldsymbol{\psi}_{i0} \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}_{it}, \tag{59}$$

where

$$\mathbf{c}_{i0} = \mathbf{a}_{i0} + \mathbf{\Pi}_i \boldsymbol{\kappa}_i. \tag{60}$$

The information regarding the long-run cointegration relationships between the levels of the variables is contained in the $(k_i + k_i^* + s) \times 1$ matrix $\mathbf{\Pi}_i$. If there is no cointegration amongst the variables $\mathbf{\Pi}_i = 0$, then equation 59 reduces to the normal first difference model:

$$\Delta \mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \boldsymbol{\psi}_{i0} \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}. \tag{61}$$

In general, one would expect $\mathbf{\Pi}_i \neq 0$, since there should be important interlinkages between the economic variables (both between domestic variables themselves and with foreign variables). However $\mathbf{\Pi}_i$ would probably be rank deficient so that

$rank(\mathbf{\Pi}_i) = r_i < k_i$ and therefore the error-correcting model in equation 59 needs to be estimated using reduced rank restrictions, i.e.:

$$H_{r_i} : \quad rank(\mathbf{\Pi}_i) = r_i < k_i \tag{62}$$

and

$$\mathbf{\Pi}_i = \boldsymbol{\alpha}_i \boldsymbol{\beta}_i^{'}, \tag{63}$$

where $\boldsymbol{\alpha}_i$ is a $k_i \times r_i$ matrix of rank r_i and $\boldsymbol{\beta}_i^{'}$ is a $(k_i + k_i^* + s) \times r_i$ matrix of rank r_i. So given a specific choice of $\boldsymbol{\beta}_i$, and using (63) and (59), we have:

$$\Delta \mathbf{x}_{it} = \mathbf{c}_{i0} + \boldsymbol{\alpha}_i \boldsymbol{\eta}_{it-1} + \mathbf{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \boldsymbol{\psi}_{i0} \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}_{it}, \tag{64}$$

where $\boldsymbol{\eta}_{it} = \boldsymbol{\beta}_i^{'} \mathbf{v}_{it} - (\boldsymbol{\beta}_i^{'} \boldsymbol{\kappa}_i) t = \boldsymbol{\beta}_i^{'} \mathbf{v}_{it} + \boldsymbol{\delta}_i \mathbf{t}$ is a $r_i \times 1$ vector of cointegrating relationships.

Following the normal procedures, $\boldsymbol{\beta}_i$ can be determined through a two-step process. First the rank of $\mathbf{\Pi}_i$ is determined through the rank or maximum eigenvalue statistics. Then $\boldsymbol{\beta}_i$ is estimated by imposing theoretical exact or overriding restrictions on the elements of $\boldsymbol{\beta}_i$. As such, a total of r_i^2 restrictions are necessary to identify and estimate $\boldsymbol{\beta}_i$.

For simulation and impulse-response purposes it is also necessary to estimate the covariance matrix of $\boldsymbol{\varepsilon}_t$. Using the reduced-rank estimates of $\boldsymbol{\varepsilon}_{it}$, i.e. $\hat{\boldsymbol{\varepsilon}}_{it}$, the following is obtained:

$$\hat{\text{cov}}(\boldsymbol{\varepsilon}_{it}, \boldsymbol{\varepsilon}_{jt}) = T^{-1} \sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_{it}, \hat{\boldsymbol{\varepsilon}}_{jt},$$

$$\hat{\text{cov}}(\boldsymbol{\varepsilon}_t) = \begin{pmatrix} \hat{\text{cov}}(\boldsymbol{\varepsilon}_{0t}, \boldsymbol{\varepsilon}_{0t}) & \hat{\text{cov}}(\boldsymbol{\varepsilon}_{0t}, \boldsymbol{\varepsilon}_{1t}) & \ldots & \hat{\text{cov}}(\boldsymbol{\varepsilon}_{0T}, \boldsymbol{\varepsilon}_{Nt}) \\ \hat{\text{cov}}(\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{0t}) & \ldots\ldots\ldots\ldots\ldots & \ldots & \hat{\text{cov}}(\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{Nt}) \\ \ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots & \ldots & \ldots\ldots\ldots\ldots\ldots \\ \hat{\text{cov}}(\boldsymbol{\varepsilon}_{Nt}, \boldsymbol{\varepsilon}_{jt}) & \ldots\ldots\ldots\ldots & \ldots & \hat{\text{cov}}(\boldsymbol{\varepsilon}_{Nt}, \boldsymbol{\varepsilon}_{Nt}) \end{pmatrix}$$

(65)

and

$$\hat{\boldsymbol{\varepsilon}}_{it} = \mathbf{x}_{it} + \hat{\mathbf{a}}_{i0} + \hat{\mathbf{a}}_{i1}\mathbf{t} - \hat{\boldsymbol{\Phi}}_i \mathbf{x}_{i,t-1} - \hat{\boldsymbol{\Lambda}}_{i0}\mathbf{x}^*_{it} - \hat{\boldsymbol{\Lambda}}_{i0}\mathbf{x}^*_{it-1} - \hat{\boldsymbol{\psi}}_{i0}\mathbf{d}_t - \hat{\boldsymbol{\psi}}_{i1}\mathbf{d}_{t-1}$$

with $\hat{\mathbf{a}}_{i0}, \hat{\mathbf{a}}_{i1}, \hat{\boldsymbol{\Phi}}_i, \hat{\boldsymbol{\Lambda}}_{i0}, \hat{\boldsymbol{\Lambda}}_{i0}, \hat{\boldsymbol{\psi}}_{i0}$, and $\hat{\boldsymbol{\psi}}_{i1}$ being the reduced-rank country-specific estimates from above.

2.5 FIRM-SPECIFIC RETURN DYNAMICS

The unique properties of the GVAR model allow one to model firm-specific return behaviour not only as a function of one global risk factor but as any of a set of global and foreign variables. Using the arbitrage pricing theory (APT) it is possible to include more variables in the return specification without loss of theoretical underpinning. As such a firm's change in value, $r_{ji,t+1}$ is assumed to be a function of a correlated systematic component, say the k_i region-specific, $\mathbf{x}_{i,t+1}$ and k_i^* foreign, $\mathbf{x}^*_{i,t+1}$ macroeconomic variables, a set of exogenous global factors $\mathbf{d}_t$ and a firm-specific idiosyncratic shock, $\eta_{ji,t+1}$:

$$r_{ji,t+1} = \alpha_{ju} + \beta'_{ji}\Delta\mathbf{x}_{i,t+1} + \beta'_{ji}\Delta\mathbf{x}^*_{i,t+1} + \gamma'_{ji}\Delta\mathbf{d}_{t+1} + \eta_{ji,t+1}, \tag{66}$$

where $\eta_{ji,t+1}$ is normally distributed and has a mean of zero and a constant time-invariant variance $\omega^2_{\eta,ji}$, i.e. $\eta_{ji,t+1} \sim IIN(0, \omega^2_{\eta,ji})$.

Recalling that:

$$z_{it} = \begin{pmatrix} x_{it} \\ x_{it}^{*} \end{pmatrix} = \mathbf{W}_i \mathbf{x}_{t+1},$$

and then using the properties contained in the weighting matrix, $\mathbf{W}_i$, the specification can be rewritten as:

$$r_{ji,t+1} = \alpha_{ju} + \mathbf{B}^{'}_{ji} \mathbf{W}_i \Delta \mathbf{x}_{t+1} + \gamma^{'}_{ji} \Delta \mathbf{d}_{t+1} + \eta_{ji,t+1}, \tag{67}$$

where $\mathbf{B}^{'}_{ji} = \left(\beta^{'}_{ji}, \beta^{*'}_{ji}\right)^{'}$. The GVAR model should capture all systematic risk while firm-specific idiosyncratic risk should be uncorrelated.

It is also possible to re-write the return equation to take into account the dependence between the macroeconomic variables (determined within the VECM) and other exogenous macroeconomic variables (determined outside of the VECM). This is particularly relevant when scenario analysis is done based on certain assumptions with respect to the exogenous variables e.g. higher oil prices. As such equation 65 can be written as:

$$r_{ji,t+1} = \alpha_{ju} + \Gamma^{'}_{ji}(\boldsymbol{\mu} + \boldsymbol{\delta}) + \Gamma^{'}_{ji}(\mathbf{I} - \boldsymbol{\Phi})(\mathbf{y}_t - \gamma t) + \Gamma^{'}_{ji} D \nu_{t+1} + \eta_{ji,t+1}. \tag{68}$$

This specification divides the return dynamics into a predictable component, $\Gamma^{'}_{ji}(\mathbf{I} - \boldsymbol{\Phi})(\mathbf{y}_t - \gamma t)$, and an unpredictable component $\Gamma^{'}_{ji} D \nu_{t+1} + \eta_{ji,t+1}$ which itself is a function of the macroeconomically unpredictable components $\Gamma^{'}_{ji} D \nu_{t+1}$ where D is a $(k+s)\times(k+s)$ matrix of fixed coefficients out of the GVAR model and $\nu_{t+1} = \left(\varepsilon^{'}_{t+1}, \varepsilon_{d,t+1}\right)^{'}$ is a matrix with both macroeconomic and other exogenous variable innovations. The last two terms of equation 68 are comprised of a firm-specific fixed effect component, α_{ju} and the drift terms originating from the macroeconomic models, $\Gamma^{'}_{ji}(\boldsymbol{\mu} + \boldsymbol{\delta})$.

The extent to which this specification is able to predict returns will depend on the factor loading, Γ'_{ji}. Although the model allows for an operational procedure for relating excess returns to macroeconomic factors, it is not meant to represent perfect predictability. As such, if actual returns deviate from those predicted by the model, it would possibly be an indication of time-varying risk premia rather then market inefficiencies.

2.6 LOSS GIVEN DEFAULT AND EXPECTED LOSS ESTIMATION

Based on the log-equity default threshold ratio, $\hat{\lambda}_R(t,H)$ as defined in section 3 above, and the value-change process of firm j which is driven by the outcomes of the GVAR model, it is possible to define the expected loss to firm j at time T.

Given the information set available to the bank at time, T, say Ω_T recall the default conditions set out in equation 18:

$$
\begin{aligned}
&I\left(r_{ji,t+1} < \hat{\lambda}_R(t,1)\right) = 1 \quad \text{if} \quad r_{ji,t+1} < \hat{\lambda}_R(t,1) \Rightarrow \text{Default} \\
&I\left(r_{ji,t+1} < \hat{\lambda}_R(t,1)\right) = 0 \quad \text{if} \quad r_{ji,t+1} \geq \hat{\lambda}_R(t,1) \Rightarrow \text{No Default.}
\end{aligned}
$$

Now, define the expected loss that the bank faces at time T (but actually occurring at time ($T+1$), i.e. $\mathrm{E}\left(L_{ji,T+1}\right) = \mathrm{E}\left(L_{ji,T+1} \middle| \Omega_T\right)$ as:

$$
\begin{aligned}
\mathrm{E}\left(L_{ji,T+1}\right) = \mathrm{E}\left(L_{ji,T+1} \middle| \Omega_T\right) = \Pr\left(r_{ji,t+1} < \hat{\lambda}_R(t,1) \middle| \Omega_T\right) \times \mathrm{E}_T\left(\chi_{ji,T+1}\right) \times \mathrm{E}_T\left(S_{ji,T+1}\right) \\
+ \left[1 - \Pr\left(r_{ji,t+1} < \hat{\lambda}_R(t,1) \middle| \Omega_T\right)\right] \times \tilde{L},
\end{aligned}
\tag{69}
$$

where

$\chi_{ji,T+1}$ is the exposure at default assuming no recoveries (usually assumed to be the face value of the loan and referred to as the exposure at default (EAD)) and it is known at time T.

$S_{ji,T+1}$ is the percentage of exposure that is not recoverable in the event of default, $(1 - S_{ji,T+1})$ is also know as the loss severity or loss given default (LGD).

$\tilde{L}$ is the loss in event of no default (usually assumed to be zero).

$S_{ji,T+1}$ will typically not be known at time T and takes the form of a random variable assumed to be between 0 and 1. In commercial and other applications $S_{ji,T+1}$ is usually drawn from a beta distribution where the smoothness parameters are set to match a predefined central tendency obtained from empirical observations. As such $S_{ji,T+1}$ is assumed to be uncorrelated or independent of the default probability. The PSTW (2006) methodology also uses this application to estimate $S_{ji,T+1}$ and will also be used in our own application. However, several studies have shown that there is in fact a correlation between default probabilities and loss severity. Intuitively this is due to the fact that macroeconomic conditions usually result in default-generating conditions across a wide range of firms, very similar to the emerging-market contagion problems within financial markets. As such banks will not only face higher default rates but also lower recovery amounts since aggregate borrower quality will simultaneously deteriorate. We will therefore also investigate the possibility that loss severity is driven by the same macroeconomic conditions as realized out of the GVAR model.

More formally, one can allow $LGD_{ji,T+1} = (1 - S_{ji,T+1})$ to be generated by the set of country and foreign macroeconomic variables through the logit transformation function as proposed by Schonbucher (2003) and Dullmann and Trapp (2004):

$$Y_{ji,T+1} = \log\left(\frac{LGD_{ji,T+1}}{1 - LGD_{ji,T+1}}\right) = \alpha_{ji} + \mathbf{H}_{ji}\mathbf{z}_{ji,t+1} + \zeta_{it}, \tag{70}$$

where $\mathbf{z}_{ji,t+1}$ is defined above and represents the systematic risk component and ζ_{it} the idiosyncratic component of $LGD_{ji,T+1}$ which is also assumed to be normally distributed with zero mean and a finite time-invariant variance.

Now, using the firm return dynamics in equation 67 and in equation 69 and setting $\tilde{L} = 0$ we obtain the following specification:

$$\mathrm{E}\left(L_{ji,T+1}\right) = \pi_{ji,T+1|T} \times \mathrm{E}_T\left(\chi_{ji,T+1}\right) \times \mathrm{E}_T\left(S_{ji,T+1}\right) \tag{71}$$

where

$$\pi_{ji,T+1|T} = \Pr\left(\alpha_{ji} + \Gamma'_{ji}\Delta y_{T+1} + \eta_{ji,T+1} < \hat{\lambda}_R(T,1)\middle|\Omega_T\right).$$

Equation 71 can be interpreted as the conditional default probability as at time T for the time period $T+1$. More importantly from a practitioner's point of view, the modelling framework presented by PSTW (2006) allows a direct and explicit derivation of $\pi_{ji,T+1|T}$ using the returns as characterized by equation 68. One is then able to prove that:

$$\pi_{ji,T+1|T} = \Pr\left(\xi_{ji,T+1} < \hat{\lambda}_R(T,1) - \mu_{ji,T+1|T}\middle|\Omega_T\right), \tag{72}$$

where

$$\xi_{ji,T+1} = \Gamma'_{ji} D v_{t+1} + \eta_{ji,t+1}, \tag{73}$$

and

$$\mu_{ji,T+1|T} = \alpha_{ji} + \Gamma'_{ji}\left[\boldsymbol{\mu} + (T+1)\boldsymbol{\delta}\right] - \Gamma'_{ji}\left(\mathbf{I} - \boldsymbol{\Phi}\right)\mathbf{y}_T \tag{74}$$

Finally, under the assumption that all the shock variables are normally distributed and that the parameter estimates are given, the probability of default over the horizon T to $T+1$ formed at time T is given by:

$$\pi_{ji,T+1|T} = \Phi\left(\frac{\hat{\lambda}_R(T,1) - \mu_{ji,T+1|T}}{\sqrt{Var\left(\xi_{ji,T+1}\middle|\Omega_T\right)}}\right), \tag{75}$$

where

$$Var\left(\xi_{ji,T+1}\middle|\Omega_T\right) \equiv \omega^2_{\xi,ji} = \omega^2_{\eta,ji} + \Gamma'_{ji}\mathbf{B}\Gamma_{ji} \tag{76}$$

$$\mathbf{B} = \mathbf{D}\boldsymbol{\Sigma}_{\mathbf{v}}\mathbf{D}'$$

$\mathbf{\Sigma}_{\mathbf{v}}$ = covariance matrix of the systematic innovations, and

$\omega^2_{\eta,ji}$ = the variance of the idiosyncratic firm-specific innovations.

The expected loss of a bank credit portfolio can finally be computed by aggregating the expected losses over the different loans in the portfolio, i.e.:

$$\mathrm{E}_T\left(L_{T+1}\right)=\sum_{i=0}^{N}\sum_{j=0}^{nc_i}\pi_{ji,T+1|T}\times\mathrm{E}_T\left(\chi_{ji,T+1}\right)\times\mathrm{E}_T\left(S_{ji,T+1}\right), \tag{77}$$

where nc_i is the number of loans in the portfolio in region or country i.

2.7 SIMULATING THE LOSS DISTRIBUTIONS

Given the fact that the GVAR model will provide the macroeconomic parameter estimates, and that estimates for the individual return processes can be obtained for equation 67 as well as the equity threshold level as in equation 9, the entire loss distribution can be simulated for a time period T to $T+1$.

Rewriting the firm-specific return equation 68 as:

$$r_{ji,t+1}=\mu_{ji,T+1|T}+\Gamma_{ji}^{'}\mathbf{D}\mathbf{v}_{t+1}+\eta_{ji,t+1}, \tag{78}$$

where the $\mu_{ji,T+1|T}$ is given by equation 74 above and the innovations $\mathbf{v}_{t+1}$ and $\eta_{ji,t+1}$ are again independently distributed normally with zero mean and covariance matrices Σ_ν and $\omega^2_{\eta,ji}$. As such, $\Gamma_{ji}^{'}\mathbf{D}\mathbf{v}_{t+1}$ will also be distributed normally, $N\left(0,\Gamma_{ji}^{'}\mathbf{D}\Sigma_\nu\mathbf{D}^{'}\Gamma_{ji}\right)$ and the firm returns can be estimated as:

$$r_{ji,t+1}^{(r)}=\mu_{ji,T+1|T}+\xi_{ji,t+1}^{(r)}, \tag{79}$$

where $r_{ji,t+1}^{(r)}$ is the r^{th} replication or simulation of the firm-specific return and $\xi_{ji,t+1}^{(r)}$ the r^{th} simulation of the "composite innovation" given as,

$$\xi^{(r)}_{ji,t+1} = \left(\Gamma^{'}_{ji}\mathbf{D}\Sigma_{v}\mathbf{D}^{'}\Gamma_{ji}\right)^{1/2} Z^{(r)}_{0} + \omega_{\eta,ji} Z^{(r)}_{ji}, \tag{80}$$

where $Z^{(r)}_{0}$ and $Z^{(r)}_{ji}$ are independent draws from a standard normal distribution.

Using this information at time $T+1$, together with the loan face value, $FV_{ji,T}$, and estimates of the loss severity (either from the beta distribution or the LGD logit transformation model), the loss can be simulated as:

$$\left(L^{(r)}_{T+1}\right) = \sum_{i=0}^{N}\sum_{j=0}^{nc_i} I\left(r^{(r)}_{ji,T+1} < \hat{\lambda}_{R}(T,1)\right) \times FV_{ji,T} \times \left(S^{(r)}_{ji,T+1}\right), \tag{81}$$

and finally the simulated expected loss is given by:

$$\tilde{L}_{R,T+1} = \frac{1}{R}\sum_{r=1}^{R} L^{(r)}_{T+1}. \tag{82}$$

For high enough values of R i.e. as $R \to \infty$, $\tilde{L}_{R,T+1} \xrightarrow{p} \mathrm{E}_{T}\left(L_{T+1}\right)$. The simulated loss distribution is given by ordering the simulated values of $L^{(r)}_{T+1}$. As such any percentile value of the loss distribution e.g. a capital point of the 99.9th percentile can be obtained from the simulated loss distribution.

2.8 CONDITIONAL DEFAULT AND CONDITIONAL EXPECTED LOSS

Following the impulse response analysis of the GVAR specification in section 2.4.1.7 it is natural that the impact of changes in macroeconomic variables will have an influence on the loss distribution of the portfolio. These scenario analyses and stress testing play a fundamental part in risk and credit portfolio management. Similar to the GIRF, the impact of changes to the loss distribution due to macroeconomic shocks can be analysed by evaluating the change in the loss distribution relative to a base scenario. However, an analysis such as the one in section 2.4.1.7 which only relies on the impact of macroeconomic factors on the loss distribution would underestimate the impact on the loss distribution since it only assumes shocks which are generated out of the macroeconomy.

In fact, PSTW (2006) proposes that firm-specific elements should be included in the analysis through the assumption that firm-specific and macroeconomic innovations are both normally distributed, and further that:

$$r_{ji,T+1} = \mu_{ji,T+1|T} + \Gamma_{ji}^{'}\mathbf{D}\mathbf{v}_{T+1} + \eta_{ji,T+1}.$$

Recalling that $\mu_{ji,T+1|T} = \alpha_{ji} + \Gamma_{ji}^{'}[\boldsymbol{\mu} + (T+1)\boldsymbol{\delta}] - \Gamma_{ji}^{'}(\mathbf{I} - \boldsymbol{\Phi})\mathbf{y}_T$ it can be shown that if a shock is anticipated, the return dynamics change to:

$$r_{ji,T+1}\left|\Omega_T, \varepsilon_{iT+1,l} = \sqrt{\sigma_{ii,ll}} \sim N\left(\mu_{ji,T+1|T} + \Gamma_{ji}^{'}\psi_{il}(\Delta\mathbf{y},1), \omega_{\xi,ji,il}^2\right),\right.$$

where $\varepsilon_{iT+1,l} = s_{il}^{'}\upsilon_{T+1}$, and $\psi_{il}(\Delta\mathbf{y},1)$ are defined in a similar way to equation 49. Now

$$\omega_{\xi,ji,il}^2 = \omega_{\eta,ji}^2 + \Gamma_{ji}^{'}\mathbf{B}_{il}\Gamma_{ji} \qquad (83)$$

where Γ_{ji} are the factor loadings and $\mathbf{B}_{il} = \mathbf{D}\left[\boldsymbol{\Sigma}_{\nu} - \boldsymbol{\Sigma}_{\nu}\mathbf{s}_{il}\left(\mathbf{s}_{il}^{'}\boldsymbol{\Sigma}_{\nu}\mathbf{s}_{il}\right)^{-1}\mathbf{s}_{il}^{'}\boldsymbol{\Sigma}_{\nu}\right]\mathbf{D}^{'}$. However, if the shock is unanticipated (probably more relevant in credit risk analysis) PSTW (2006) show that the return dynamics change to:

$$r_{ji,T+1}\left|\Omega_T, \varepsilon_{iT+1,l} = \sqrt{\sigma_{ii,ll}} \sim N\left(\mu_{ji,T+1|T} + \Gamma_{ji}^{'}\psi_{il}(\Delta\mathbf{y},1), \omega_{\xi,ji,il}^2\right)\right.$$

where $\omega_{\xi,ji,il}^2$ will now take the form as in equation 73 above. Therefore, to allow for macroeconomic as well as idiosyncratic shocks to effect the loss distribution, the simulation of the loss distribution needs to be carried out using draws such that:

$$r_{ji,T+1}^{il,(r)} = \mu_{ji,T+1|T} + \Gamma_{ji}^{'}\psi_{il}(\Delta\mathbf{y},1) + \xi_{ji,t+1}^{(r)} \qquad (84)$$

and $\xi^{(r)}_{ji,t+1}$ is defined by equation 80 above. Equation 84 shows that relative to the baseline, the mean return is augmented by $\Gamma'_{ji}\psi_{il}(\Delta\mathbf{y},1)$ and as previously, default will occur if the r^{th} simulation falls below the equity threshold level $\hat{\lambda}_R(T,1)$ defined by equation 9. Using these results in equation 81 the scenario-specific loss distribution can be obtained and compared to the baseline scenario.

Once again, it is also possible to estimate the scenario-augmented default probabilities, $\pi^{il}_{ji,T+1|T}$ and compare them to the baseline default probability, $\pi_{ji,T+1|T}$ as in equation 75. As such the augmented default probabilities under unanticipated shocks to a macroeconomic variable $x_{i,T+1,l}$, can be estimated as

$$\pi_{ji,T+1|T} = \Phi\left(\frac{\hat{\lambda}_R(T,1) - \mu_{ji,T+1|T}}{\omega_{\xi,ji}}\right),$$

$$\pi^{il}_{ji,T+1|T} = \Phi\left(\frac{\hat{\lambda}_R(T,1) - \mu_{ji,T+1|T} - \Gamma'_{ji}\psi_{il}(\Delta\mathbf{y},1)}{\omega_{\xi,ji}}\right). \tag{85}$$

2.9 MULTI-PERIOD LOSS DISTRIBUTIONS

Due to the fact that the PSTW (2006) methodology is based on ratings assigned by agencies, it is natural that the forecast horizon of the loss distribution should also be constrained by the horizon used to assign ratings. Usually this is defined as a one-year period but in general should not be shorter since rating agencies aim to present a longer time period rating for counterparties.

The following illustration is for a two-period forecast horizon (i.e. two quarters in our analysis). Recalling that the Merton model as applied here only assumes that default occurs at the terminal date and therefore at time T, firm j will default if:

$$r_{ji,T+1} + r_{ji,T+2} < 2\hat{\mu}_R + \hat{\sigma}_R \sqrt{2}\hat{Q}_R(2)$$

where the quantile estimate $\hat{Q}_R(2)$ is given by the equation 12 and by extending this to the period H will result in:

$$R_{ij,T+H} = \sum_{\tau=1}^{H} r_{ji,T+\tau} < H\hat{\mu}_R + \hat{\sigma}_R \sqrt{H}\hat{Q}_R(H), \tag{86}$$

where future returns are generated from the simulation outcomes of the GVAR model.

2.9.1 BASELINE MULTI-PERIOD LOSS DISTRIBUTION

The loss distribution of firm j in region i over the forecast horizon T to $T+H$ is now given by:

$$\begin{aligned} L_{ji}(T+1, T+H) = L_{ji,T+1} &+ \varphi I\left(R_{ij,T+H} \geq \hat{\lambda}_R(T,1)\right) \times L_{ji,T+2} + \ldots \\ &+ \varphi^{H-1} \prod_{k=1}^{H-1} I\left(R_{ij,T+k} \geq \hat{\lambda}_R(T,k)\right) \times L_{ji,T+K}, \end{aligned} \tag{87}$$

where φ is the multi-period discount factor, (PSTW (2006) proposes that $\varphi = \frac{1}{(1+\rho)}$ with ρ being set equal to the average real rate of interest), $R_{ij,T+k}$ is as before in equation 86 and:

$$L_{ji,T+K} = I\left(R_{ij,T+k} < \hat{\lambda}_R(T,k)\right) \times \chi_{ji,T+k} \times S_{ji,T+k} \qquad \text{for } k = 1,2,..,H.$$

As such, the multi-period loss distribution is based on a conditional survival probability basis i.e. losses are computed in period $T + \tau + 1$ only if the firm has survived the previous period, i.e. $T + \tau$.

Using the simulation results for the individual firm return dynamics as presented through the simulation of the GVAR model, the empirical distribution of $L_{ji}(T+1,T+H)$ can be constructed from $L_{ji}^{(r)}(T+1,T+H)$ where:

$$L_{ji}^{(r)}(T+1,T+H) = L_{ji,T+1}^{(r)} + \sum_{t=2}^{H} \varphi^{t-1} \prod_{k=1}^{t-1} I\left(R_{ij,T+k} \geq \hat{\lambda}_R(T,k)\right) \times L_{ji,T+t}^{(r)} ,$$

and

$$L_{ji,T+k}^{(r)} = I\left(\sum_{\tau=1}^{k} R_{ji,T+\tau} < \hat{\lambda}_R(T,k)\right) \times \chi_{ji,T+k}^{(r)} \times S_{ji,T+k}^{(r)} \quad \text{for } k = 1,2,\ldots,H.$$

Finally, as before, by aggregating over all firms in all countries one obtains the time T conditional H-period ahead simulated loss distribution of the total credit portfolio, i.e.

$$L^{(r)}(T+1,T+H) = \sum_{i=0}^{N} \sum_{j=1}^{nc_i} L_{ji}^{(r)}(T+1,T+H) \quad \text{for } r = 1,2,\ldots,H.$$

2.9.2 CONDITIONAL MULTI-PERIOD LOSS DISTRIBUTION

Using the impulse response techniques as defined above, a one standard error shock to factor l in country i on the multi-period loss distribution will once again flow through the individual return equations:

$$r_{ji,T+k}^{il,(r)} = \mu_{ji,T+k|T} + \Gamma_{ji}^{'} \psi_{il}(\Delta \mathbf{y}, k) + \xi_{ji,t+k}^{(r)} \quad \text{for } k = 1,2,\ldots,H. \tag{88}$$

where $\psi_{il}(\Delta \mathbf{y}, k)$ is now given by:

$$\begin{aligned} \psi_{il}(\Delta \mathbf{y}, k) &= \frac{1}{\sqrt{\sigma_{i,ll}}} D\Sigma_{\upsilon} s_{il} \quad \textit{for } k = 1 \\ &= \frac{1}{\sqrt{\sigma_{i,ll}}} \left(\Phi^{n-1} - \Phi^{n-2}\right) D\Sigma_{\upsilon} s_{il} \quad \text{for } k = 2,3,\ldots,H. \end{aligned}$$

and

$$\xi_{ji,t+k}^{(r)} = \left(\Gamma_{ji}^{'}\mathbf{B}\Gamma_{ji}\right)^{1/2} Z_0^{(r)} + \sum_{\tau=1}^{k-1}\left(\Gamma_{ji}^{'} H_\tau \mathbf{B} H_\tau^{'} \Gamma_{ji}\right)^{1/2} Z_\tau^{(r)} + \omega_{\eta,ji} Z_{jik}^{(r)}, \tag{89}$$

where $\mathbf{B} = D\Sigma_\upsilon D^{'}$, while $Z_\tau^{(r)}$ and $Z_{jik}^{(r)}$ are independent draws from the standard normal distribution, $N(0,1)$, for all τ, j, i and k. Clearly, for $k = 1$ equation 89 reduces to equation 80 above. The return simulations in the baseline case will then be given by:

$$r_{ji,T+k}^{il,(r)} = \mu_{ji,T+k|T} + \xi_{ji,t+k}^{(r)} \quad \text{for } k = 1,2,\ldots,H.$$

2.10 CONCLUSION

In this chapter we have provided a detailed specification of the PSW (2004) and PSTW (2006) credit risk modelling methodology. The methodology is unique in that it specifically links macroeconomic credit risk factors to firm-specific return dynamics to provide correlated credit risk analytics. It also steers clear from proprietary data requirements usually employed in commercially available credit portfolio models. Due to its strong theoretical underpinning the methodology can be regarded as being significantly robust and should enhance the ability of credit risk portfolio managers to perform scenario and stress testing based on macroeconomic factors.

CHAPTER 3

AN EMPIRICAL APPLICATION OF A SOUTH AFRICAN GLOBAL ERROR-CORRECTING MACROECONOMETRIC MODEL

3.1 INTRODUCTION

Driven by intense competition for market share, banks across the globe have increasingly allowed credit portfolios to become less diversified (across all dimensions – country, industry, sector and size) and they have been willing to accept lesser quality assets on their books. As a result, even well-capitalised banks could come under severe solvency pressure when global economic conditions turn. The banking industry have realised the need for more sophisticated loan origination, and credit and capital management practices. To this end, the reforms introduced by the Bank of International Settlement through the New Basel Accord (Basel II) aims to include exposure-specific credit risk characteristics within the regulatory capital requirement framework. The new regulatory capital framework still does not allow diversification and concentration risk to be fully recognised within credit portfolios because it does not account for systematic and idiosyncratic risk in a multi-factor framework.

The core principle for addressing practical questions in credit portfolio management lies in the ability to link the cyclical or systematic components of firm credit risk with the firm's own idiosyncratic credit risk as well as the systematic credit risk component of every other exposure in the portfolio. Simple structural credit portfolio management approaches have opted to represent the general economy or systematic risk by a single risk factor. The systematic component of all exposures, the process generating asset values and therefore default thresholds are homogeneous across all firms. Indeed, this Asymptotic Single Risk Factor (ASRF) model has been the foundation for Basel II. While the ASRF framework is appealing due to its analytical closed-form properties for regulatory and generally universal application in large portfolios, the single risk factor characteristic is also its major drawback. Essentially it does not allow for enough flexibility in answering real life questions. Commercially available credit portfolio models make an effort to address this by introducing more systematic factors in the asset value generating process, but from a practitioner's point of view, these models are often a "black-box" allowing little economic meaning or inference to be attributed to systematic factors.

The methodology proposed by Pesaran, Schuermann, and Weiner (PSW) (2004) and supplemented by Pesaran, Schuermann, Treutler and Weiner (PSTW) (2006) has made a significant advance in credit risk modelling in that it avoids the use of proprietary balance sheet and distance-to-default data, focussing on credit ratings which are more freely available. By linking an adjusted structural default model to a structural global econometric (GVAR) model, credit risk analysis and portfolio management can be done through the use of a conditional-loss distribution estimation and simulation process. The GVAR model used in PSW (2004) comprises a total of 25 countries which are grouped into 11 regions and accounts for 80 per cent of world production. In the case of South Africa the GVAR model lacks applicability since it does not include an African component.

In this chapter we construct a country-specific macroeconometric risk driver engine which is compatible with and could feed into the GVAR model and framework of PSW (2004) using vector error-correcting (VECM) techniques. This will allow conditional loss estimation of a South African-specific credit portfolio but also opens the door for credit portfolio modelling on a global scale as such a model can easily be linked into the GVAR model. We extend the set of domestic factors beyond those used in PSW (2004) in such a way that the risk driver model is applicable for both retail and corporate credit risk. As such, the model can be applied to a total bank balance sheet, incorporating the correlation and diversification between both retail and corporate credit exposures.

3.2 DATA SOURCES AND GLOBAL VARIABLE TIME SERIES CONSTRUCTION

3.2.1 THE TRADE WEIGHTS

As discussed in chapter 3, section 4, the impact that the global economy has on individual country macroeconomic dynamics can be captured by constructing country-specific global time series of economic variables. PSW (2004) proposes that this time series should be constructed through weighting the major foreign role players' individual time series by utilising foreign-trade weighting.

With global integration and an ever-increasing free-trade environment, South Africa has seen an increase the number of its major trading partners over the last decade. As such the South African Reserve Bank has increased the number of trading partners considered in constructing the real effective exchange rate from four to fourteen, in order to more accurately reflect current foreign trade relations. According to Walters and De Beer (1999) South Africa's major trading partners are comprised of 14 countries which account for over 85 per cent of South Africa's trade in manufacturing goods. The 14 major trade partners include the Euro area, the United States of America, the United Kingdom, Japan, Switzerland, the People's Republic of China: Mainland, the People's Republic of China: Hong Kong, Korea, Zimbabwe, Canada, Australia, Sweden, Singapore and Israel. Walters and De Beer (1999) presents a full discussion on the Information Notice System (INS) of the International Monetary Fund's methodology used to calculate the new trade weights for South Africa and also apply the methodology retrospectively in order to provide historical information on the trade weights. Table 3.1 illustrates the weights as presented by Walters and De Beer (1999) and is used to construct the global (or "starred") macroeconomic variables.

Clearly, due to the implosion of its economy in recent years, the inclusion of Zimbabwe would skew the global variables over the near term. Zimbabwe is excluded from the sample and its trade share is apportioned pro-rata to the remaining foreign trade partners.

Table 3.1 South African trade weights

	Weights before Jan 1999 (%)	Weights after Jan 1999 (%)
Euro Area:		
Germany	16.91	
Italy	5.07	
France	4.98	
Netherlands	3.90	
Belgium-Luxembourg	3.54	
Spain	1.34	
Ireland	0.86	
Austria	0.83	
Finland	0.81	
Portugal	0.34	
Euro Area	38.58	35.70
USA	14.44	15.15
UK	14.09	14.91
Japan	9.90	10.26
Switzerland	4.99	5.28
China: Mainland	2.91	3.11
China: Hong-Kong	2.59	2.62
Korea	2.50	2.57
Zimbabwe	2.27	2.27
Canada	1.87	1.93
Australia	1.59	1.62
Sweden	1.58	1.79
Singapore	1.55	1.62
Israel	1.14	1.17

Source: Walters and De Beer (1999)

3.2.2 GLOBAL MACROECONOMIC VARIABLES AND DATA SOURCES

Since this study proposes adding an African component to the PSW (2004) GVAR model, it is necessary to use the same underlying data in construction so that it is possible to link this model to the outcomes generates by the PSW (2004) model in a dynamic global simulation and forecasting model. Therefore, in order to construct the global macroeconomic data we reference the same data sources as used in PSW (2004) to construct its global variables. The primary variables, data sources and data adjustments used in this study are therefore similar to those used and explained in PSW (2004) and are summarised in table 3.2.

Table 3.2 Global data series and data sources

Variables	Data source	Short name
Output (GDP)	IMF's International Financial Statistics (IFS) GDP (2000) series	y^*
General price indices	IMF's International Financial Statistics (IFS) Consumer Price Index (2000) series	p^*
Equity price indices	Bloomberg's	q^*
Exchange rates	IMF's International Financial Statistics (IFS) rf. series	e^*
Interest rates	IMF's International Financial Statistics (IFS) series 60B (the money rate)	ρ^*
Money supply	The sum of the IMF's International Financial Statistics (IFS) series 34 (money) and series 35 (quasi-money)	m^*

The construction of time-series data for the Euro area for the time period before unification in 1998 was done based on the trade weights as provided by Walters and De Beer (1999). Clearly this would have led to some significant data gaps as well as unavailable time series for some countries, particularly for earlier time periods. Moreover, data series for the Euro Area after 1998 may not always add up or link perfectly with the weighting system applied before 1998. Values for missing observations were approximated by first interpolating the quarterly data from the annual data where available. In cases where annual data were not available, quarterly data were generated by back casting, using the average of the earliest available quarterly growth rates for that series. Thereafter, the time series for the Euro Area before 1998 were adjusted by scaling the data to match the data obtained from the IFS data base for the time period after 1998.

Therefore, the 10-country individual time series for the Euro area before 1998 was combined into a single Euro Area series for each variable and by applying the country weights in table 3.1, this series was augmented to link to the Euro time series for the time period after 1998.

In a similar fashion the other foreign variables were constructed from the 14 individual economic time series by weighting them with their individual weights as obtained from table 3.1. Similarly to the Euro area data series other data series, such as, for example, China: Mainland and China: Hong Kong, where missing data were encountered the time series were constructed by interpolation of the annual time series or through approximation of the series by using growth rates[2].

As such, 6 time series have been constructed from the 14 individual-country series and these represent the dynamics of the global economic variables which are assumed to impact and shape macroeconomic variables in South Africa's domestic economy. As formally specified in equation 22 section 2.4.1.3, the composite indices can be represented as:

$$y_{it}^{*} = \sum_{j=0}^{N} w_{ij}^{y} y_{it}, \qquad p_{it}^{*} = \sum_{j=0}^{N} w_{ij}^{p} p_{it},$$

$$q_{it}^{*} = \sum_{j=0}^{N} w_{ij}^{q} q_{it}, \qquad e_{it}^{*} = \sum_{j=0}^{N} w_{ij}^{e} e_{it},$$

$$\rho_{it}^{*} = \sum_{j=0}^{N} w_{ij}^{\rho} \rho_{it}, \qquad m_{it}^{*} = \sum_{j=0}^{N} w_{ij}^{m} m_{it}$$

with y_{it}, p_{it}, q_{it}, e_{it}, ρ_{it}, m_{it}, as defined above in table 3.2 and weights $w_{ij}^{y}, w_{ij}^{p}, w_{ij}^{q}, w_{ij}^{e}, w_{ij}^{\rho}$, and w_{ij}^{m} presented in table 3.1.

The composite world time series for interest rates, exchange rates, inflation, output, equity prices and money supply are illustrated in figure 3.1. . Although illustrated in nominal terms variables, analysis estimation and empirical results are done after converting data into real terms in line with PSW (2004). According to the constructed time series, world interest rates have structurally decreased from significantly high levels of up to 21.3 per cent in the mid 1980s to levels of below 5 per cent experienced over the last 5 years. Average nominal interest rates over the last 26 years have been 7.36 per cent with real interest rates at 3.94 per cent. The lowest point of interest rates occurred in 2004 with a figure as low as 1.92 per cent, and after this period, rates increased to a less accommodating level of 3.71 per cent at the end of 2006.

[2] Details on the construction and approximation of data for missing observations are available from the author on request.

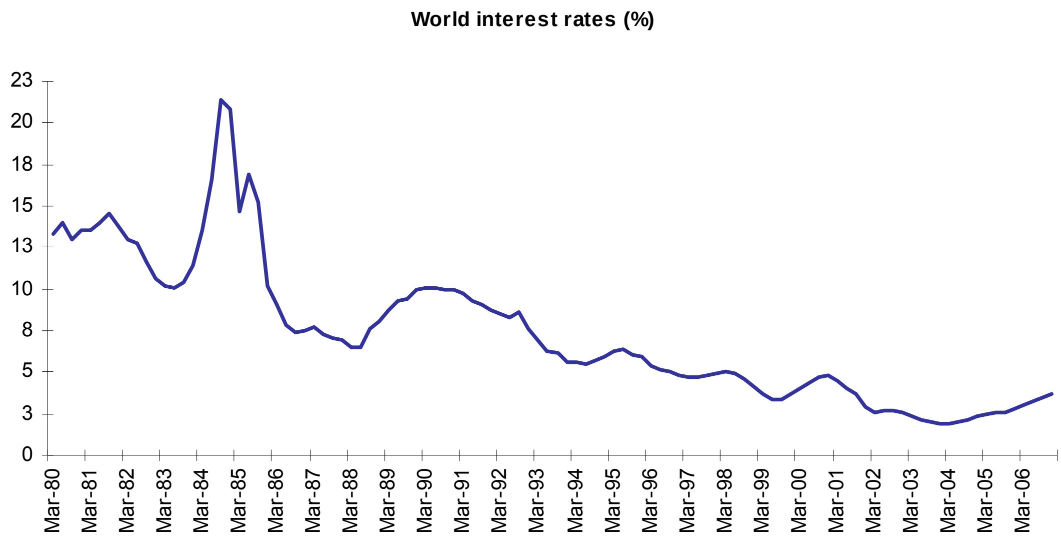
Analysis of the trend in world exchange rates is quite challenging since these rates actually represent the value of the currencies of the group of countries that represent South Africa's major trading partners and not necessarily those of the world. However, it is clear from the exchange rate series that the emerging market crisis in 1998 has had a major impact on the value of the composite world index. When analysing the individual data series for an indication of the major contributors to the huge depreciation in the exchange rates, it is not surprising that sharp decreases in the currency can be attributed to Asian economies. While the Asian crisis in 1998 was short-lived it is clear that the composite exchange rate have not recovered to the same levels as before this crisis period.

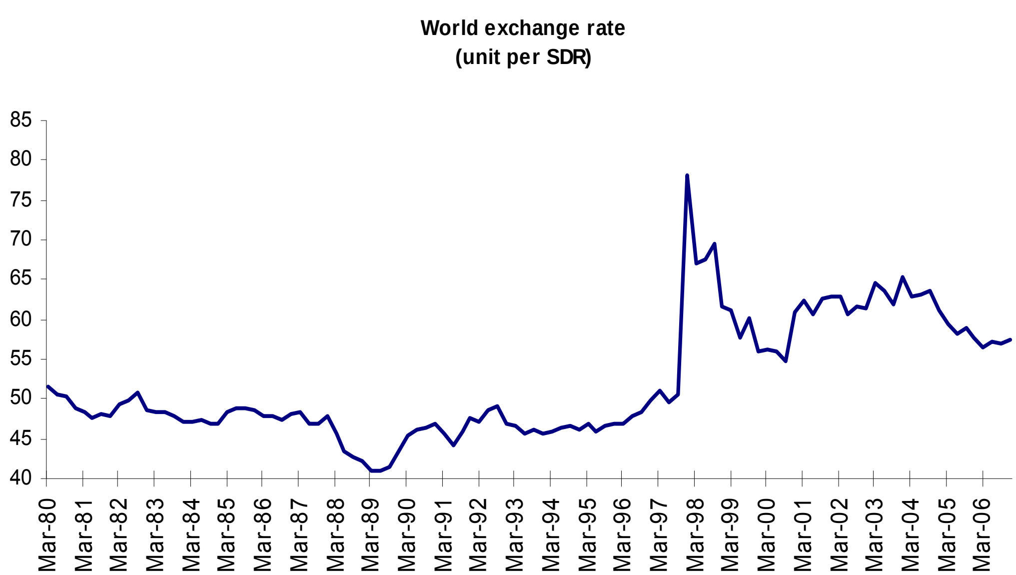
As one would expect, world inflation shows results which reflect the movements in interest rates. Starting at significantly high levels of 12.47 per cent (quarter on quarter annualised (q.a.)) in the beginning of 1980, inflation has subsequently decreased to around 2.5 per cent q.a. over the last 5 years while average inflation over the period was 3.36 per cent (q.a.). Although structurally significantly lower towards the latter part of the estimation period, it is clear that world inflation displays definite seasonal effects. Economic growth over the last decade is estimated at 2.83 per cent by the composite GDP index. Similar to global inflation, global growth rates display significant seasonality with a recorded maximum of 9.47 per cent q.a. in December 1988. Money supply growth has been fairly stable over the estimation period with an average of 12.64 per cent q.a. in the 1980s, decreasing to 8.5 per cent during the 1990s and again decreasing to 6.94 per cent in the last six years.

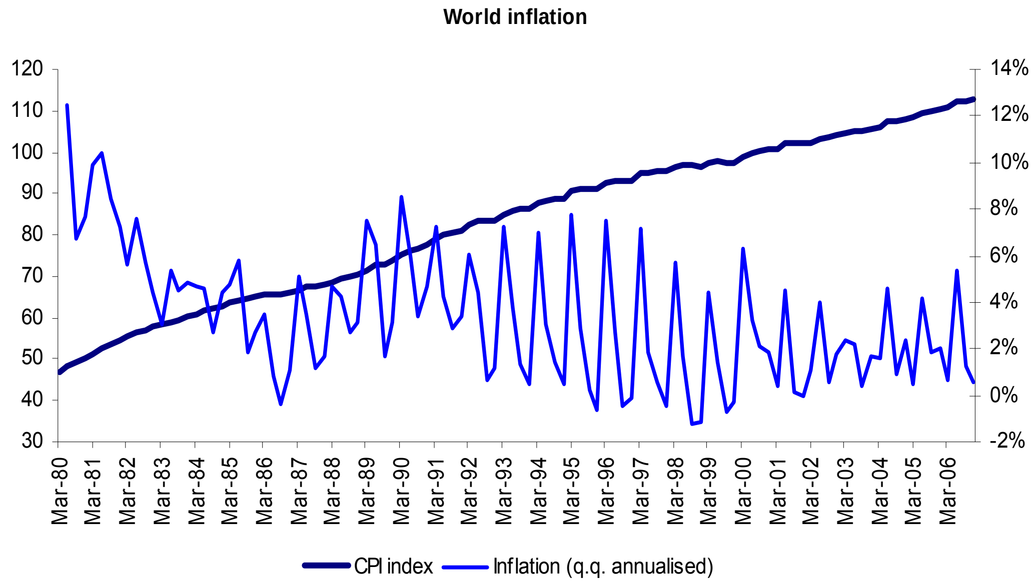
The world equity price index tells a rather compelling story. Building on long-term momentum the equity index showed significant increases until the 1998 period when the Asian crisis had a correcting impact on global equity markets. Although growth rates were still positive, on average the 2001 emerging-market and financial-market crisis' impacts are evident from the significant losses suffered during the period from 1999 to 2003. The strong global growth experienced since 2003 has lead to a significant increase in the index. Average equity return over the 26-year period has been 24.71 per cent q.a.

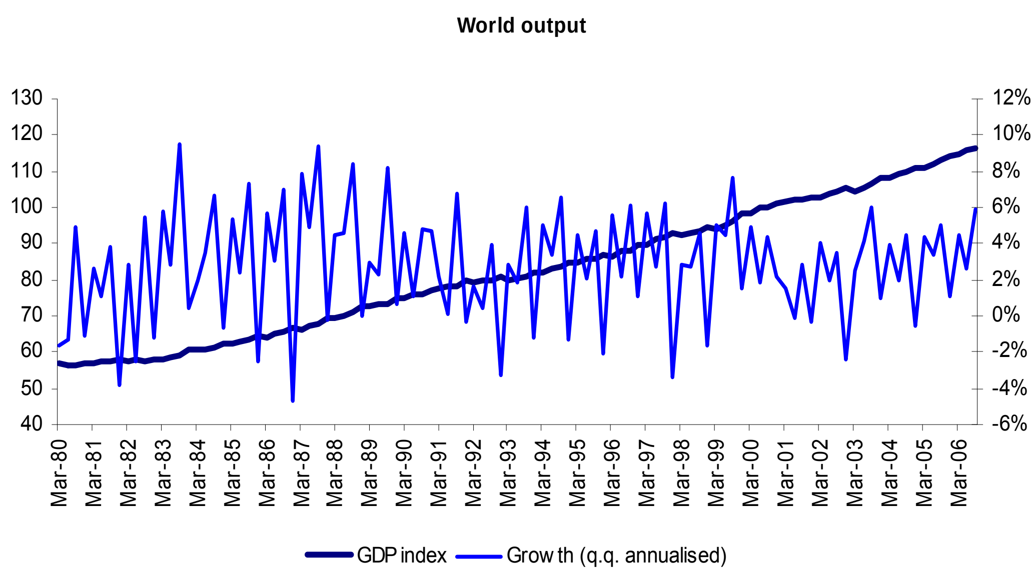
After analysing the data and considering the seasonality in the world output, inflation and equity time series the data were adjusted in line with PSW (2004) by first constructing the real time series and then applying seasonality adjustments.

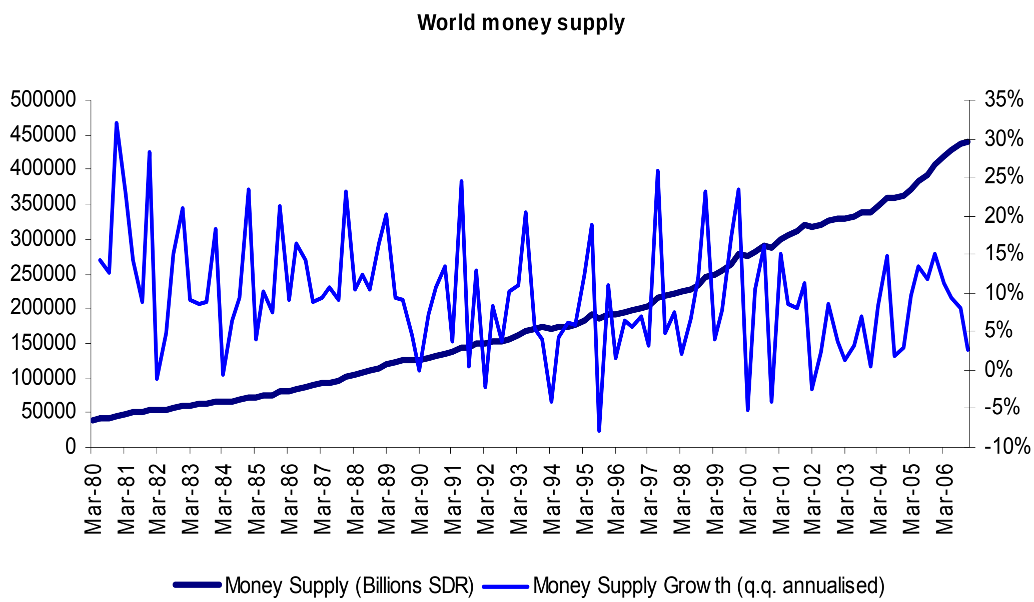
Figure 3.1 Composite global macroeconomic variables

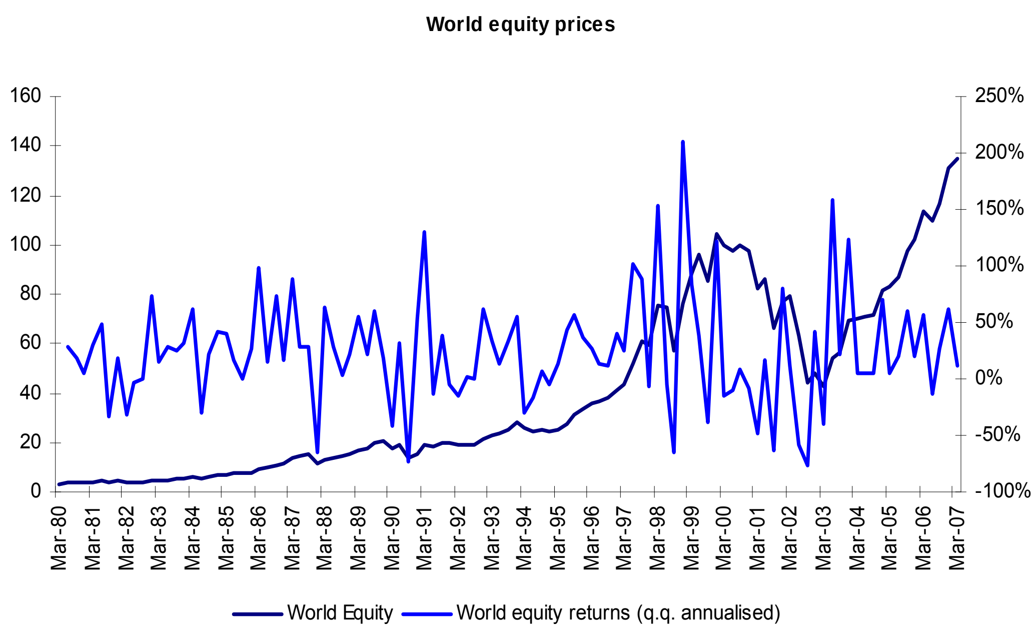





World inflation
120
110
100
90
80
70
60
50
40
30
14%
12%
10%
8%
6%
4%
2%
0%
-2%
Mar-80
Mar-81
Mar-82
Mar-83
Mar-84
Mar-85
Mar-86
Mar-87
Mar-88
Mar-89
Mar-90
Mar-91
Mar-92
Mar-93
Mar-94
Mar-95
Mar-96
Mar-97
Mar-98
Mar-99
Mar-00
Mar-01
Mar-02
Mar-03
Mar-04
Mar-05
Mar-06
CPI index
Inflation (q.q. annualised)


World output
130
120
110
100
90
80
70
60
50
40
12%
10%
8%
6%
4%
2%
0%
-2%
-4%
-6%
Mar-80
Mar-81
Mar-82
Mar-83
Mar-84
Mar-85
Mar-86
Mar-87
Mar-88
Mar-89
Mar-90
Mar-91
Mar-92
Mar-93
Mar-94
Mar-95
Mar-96
Mar-97
Mar-98
Mar-99
Mar-00
Mar-01
Mar-02
Mar-03
Mar-04
Mar-05
Mar-06
GDP index
Growth (q.q. annualised)


World money supply
500000
450000
400000
350000
300000
250000
200000
150000
100000
50000
0
35%
30%
25%
20%
15%
10%
5%
0%
-5%
-10%
Mar-80 Mar-81 Mar-82 Mar-83 Mar-84 Mar-85 Mar-86 Mar-87 Mar-88 Mar-89 Mar-90 Mar-91 Mar-92 Mar-93 Mar-94 Mar-95 Mar-96 Mar-97 Mar-98 Mar-99 Mar-00 Mar-01 Mar-02 Mar-03 Mar-04 Mar-05 Mar-06
Money Supply (Billions SDR)
Money Supply Grow th (q.q. annualised)


World equity prices
160
140
120
100
80
60
40
20
0
250%
200%
150%
100%
50%
0%
-50%
-100%
Mar-80 Mar-81 Mar-82 Mar-83 Mar-84 Mar-85 Mar-86 Mar-87 Mar-88 Mar-89 Mar-90 Mar-91 Mar-92 Mar-93 Mar-94 Mar-95 Mar-96 Mar-97 Mar-98 Mar-99 Mar-00 Mar-01 Mar-02 Mar-03 Mar-04 Mar-05 Mar-06 Mar-07
World Equity
World equity returns (q.q. annualised)

3.2.3 DOMESTIC AND EXOGENOUS MACROECONOMIC VARIABLES AND DATA SOURCES

In this study, the selection of domestic economic variables deviates slightly from the variables selected and applied in PSW (2004). Due to the fact that this study does not aim to provide region and country-specific models for all trading partners, but only to supply a South African-specific element to the GVAR model, it is possible to construct a domestic VECM which includes more variables. The selection of data series is extended beyond those capturing the domestic macroeconomic environment, to include additional variables which are deemed to be important to credit markets in South Africa. Other than output, general price levels, interest rates and money supply representing the global macroeconomic environment, two additional domestic variables are included for the South African domestic model, adding to the domestic counterparts of global variables.

Household debt-to-income ratios have increased significantly over the last two years of the sample due to structurally lower interest rates. Although debt repayment amounts have been fairly stable, the absolute volume underlying credit extension could have a significant adverse effect on the economy if interest rates were to increase to historically observed levels. As such, the household debt-to-income ratio is included in the set of domestic variables as it includes significant information with respect to the risk drivers of the domestic credit market.

A second significant development in the domestic market in recent times has been the increase experienced in the value of the property market. Similar to the global arena, property prices have seen phenomenal growth rates over the last three years of the sample under discussion and have lead to significant wealth creation. In fact, many of the increases in debt-to-income ratios are underpinned by increases in property which are supplied for collateral. Therefore, although the South African economy's balance sheet has seen an increase in liabilities due to higher debt, on a net wealth basis balance sheet quality has improved due to the increase in property values. A house price index variable is therefore included in the set of domestic variables in order to capture the impact and solvency risk posed to the macroeconomy by the development of the property market.

The final set of domestic variables included in the domestic VECM and the relevant data sources are summarised in table 3.3. Again it follows from equation 21, section 2.4.1.3, that the formal variable description is given as:

$$y_t = \ln\left(\frac{GDP_t}{CPI_t}\right), \qquad p_t = \ln\left(CPI_t\right), \qquad d_t = \ln\left(Household\ \ debt/\ Income\right)$$

$$q_t = \ln\left(\frac{EQ_t}{CPI_t}\right), \qquad m_t = \ln\left(\frac{M_t}{CPI_t}\right), \qquad h_t = \ln\left(\frac{HPI_t}{CPI_t}\right)$$

$$e_t = \ln\left(E_t\right), \qquad \rho_t = 0.25\ln\left(1+\frac{R_t}{100}\right) \qquad o_t = \ln\left(Oilp\$\right).$$

where GDP_t = nominal gross domestic product, CPI_t = consumer price index, M_t = nominal money supply in domestic currency, EQ_t = nominal equity price index, E_t = real effective exchange rate, R_t = nominal rate of interest per annum in percent. $Household\ debt/Income_t$ = debt-to-income ratio of households, HPI_t = house price index depicting the general increase in property values, and *Oilp$* = Brent crude oil price in U.S. dollar terms.

Table 3.3 Domestic and exogenous data series and data sources

Variables	Data source	Short name
Output (GDP)	South African Reserve Bank Quarterly Bulletin	*y*
General price indices	Statistics South Africa	*p*
Equity price indices	Johannesburg Stock Exchange	*q*
Exchange rates	IMF's International Financial Statistics (IFS) rf series	*e*
Interest rates	South African Reserve Bank Quarterly Bulletin	*ρ*
Money supply	South African Reserve Bank Quarterly Bulletin	*m*
Household debt-to-income ratio	South African Reserve Bank Quarterly Bulletin	*d*
House price index	ABSA bank	*h*
Oil prices	Inet bridge	*o*

Missing data were again approximated by interpolating the quarterly data from the annual data where available. If annual data were not available the quarterly data were generated by backcasting using the average of the earliest available quarterly growth rates for that series. Figure 3.2 contains graphical representations of domestic and exogenous data series. Once again we present data in nominal terms in the graphs.

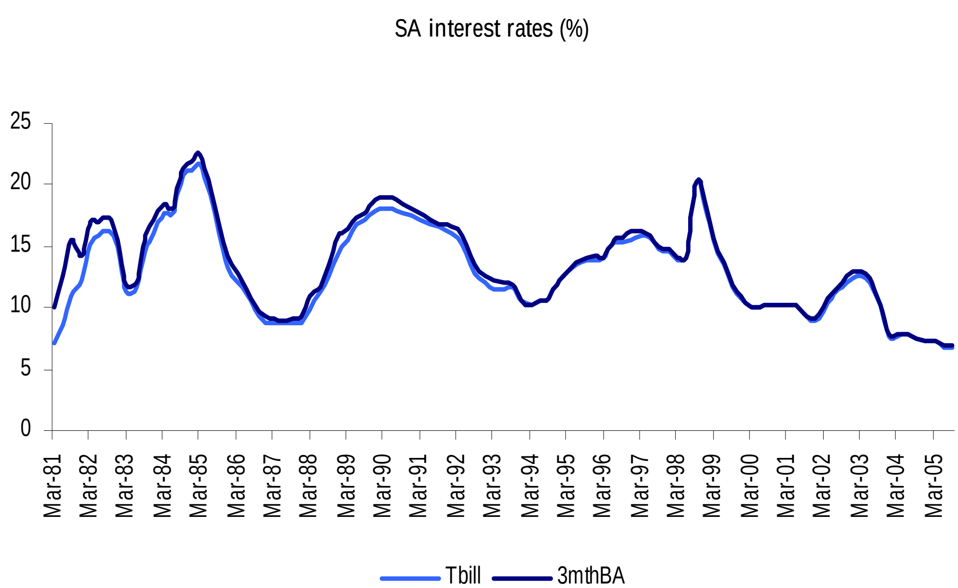
The official policy rate in South Africa is the repurchase rate (*Repo*) of the South African Reserve Bank. However, this policy instrument was only introduced in 1998 rendering the use of the time series of observations insufficient for inclusion in this study. We illustrate both the 3 month Banker's Acceptance (BA) and 91 day Treasury bill (Tbill) rates and propose to use the Tbill rate as a proxy for domestic short-term or policy rates. Interest rates in South Africa have shown trends similar to international rates over the last 25 years. Starting from a high 10-year average of 13.1 per cent and 14.37 per cent in the 1980s and 1990s, interest rates have structurally decreased to record an average of 9.06 per cent over the last 7 years (excluding the 2001 currency crisis effects, the average for the last 3 years has been 7.26 per cent). With inflationary pressure building in the South African economy since mid-2006 resulting, in a 250bps increase in interest rates over the course of one year, it is expected that rates should be stable with an upward drift over the short to medium term.

The constructed world exchange rate has been transformed into a Rand per world unit rate. This series is analogous to the reciprocal of the real effective exchange rate series more generally known in the literature. For illustrative purposes the Rand/US dollar exchange rate is shown for the estimation period as well. As indicated by the graph the two series display similar trends, since the U.S. is a major trade partner for South Africa. In general it is clear that the Rand has been depreciating against the currencies of its major trade partners over the last quarter of a century. More recently, the Rand recovered from the sharp fall in value in 2001-2002 and has continued its long-run trend.

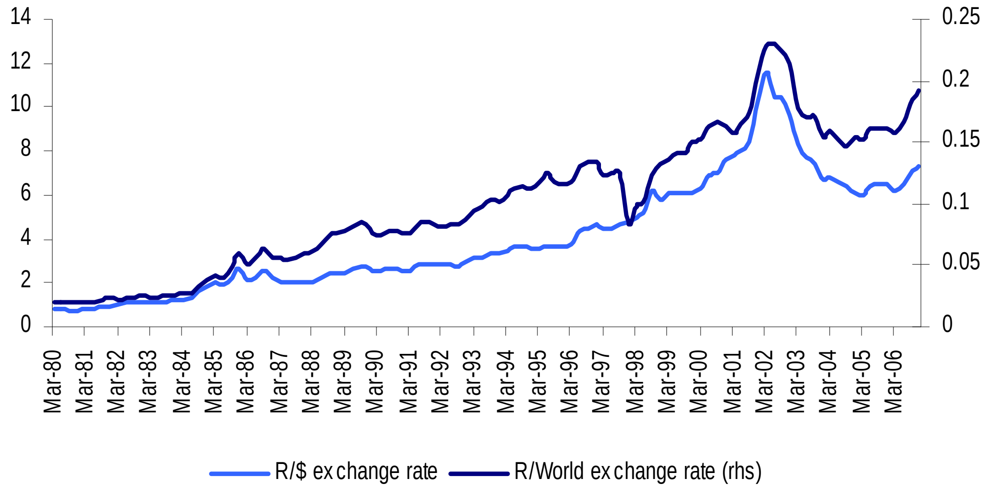
Domestic inflation has structurally decreased from an average annualized rate of 14.81 per cent and 9.48 per cent for the 1980s and 1990s, to 5.33 per cent over the last 6 years of the sample. These decreases are not as pertinent as those shown by world inflation rates. Money supply and nominal output growth averaged 15.69 per cent and 13.81 per cent but did not show any structural changes over the sample period. However, the drop in inflation implies that real money supply and growth have increased significantly since 1980. In fact, while real money supply and output averaged around 3.5 per cent before 2000, real growth increased to close to 6 per cent while real money supply increased to 10.86 per cent the last 6 years of the sample.

Analysing the equity market index trend, it is clear that the 1998 and 2001 emerging-market and currency-crisis periods had very negative consequences for the equity market but that the market has recovered strongly over the last five years of the sample period. This strong growth in the equity market is somewhat overshadowed by the 20 per cent average growth rate experienced in South African house prices since 2000. Although this excessive increase in house prices can partly be explained by the fact that house prices were generally deflated during the 1990s (the average annualized growth rate was only 8.87 per cent relative to the 13.95 per cent average growth rate for the previous ten years) it is clear that this increase has lead to significant wealth creation in the economy.

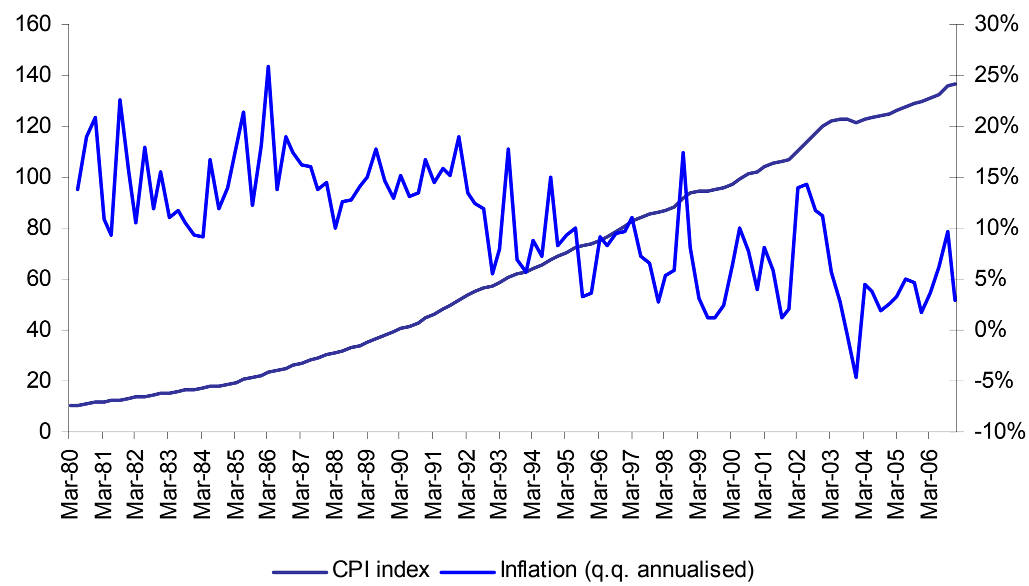
Figure 3.2 South African domestic and exogenous macroeconomic variables



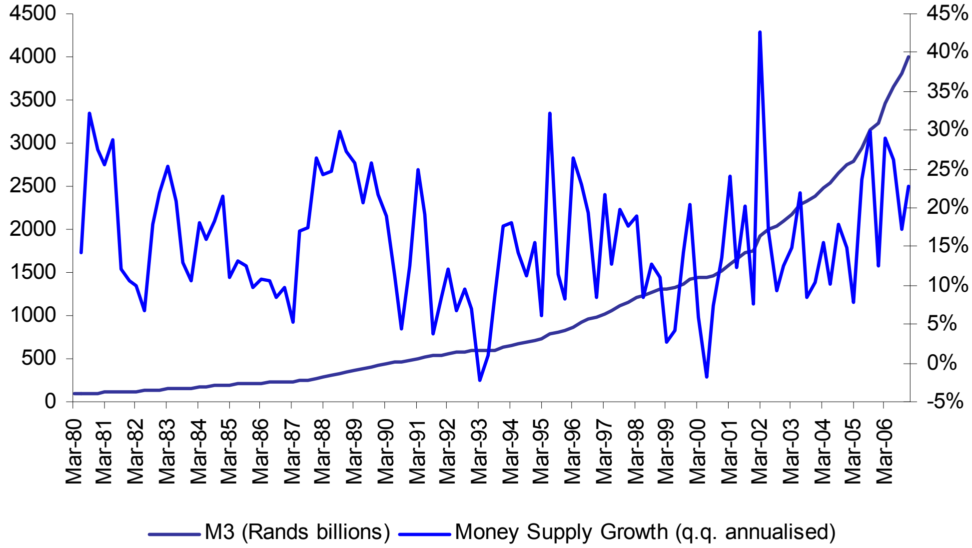
Exchange rates



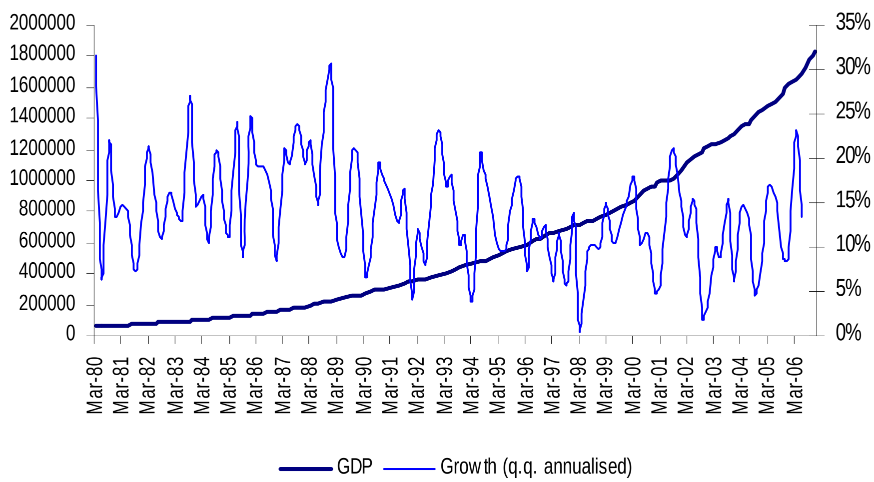
SA inflation



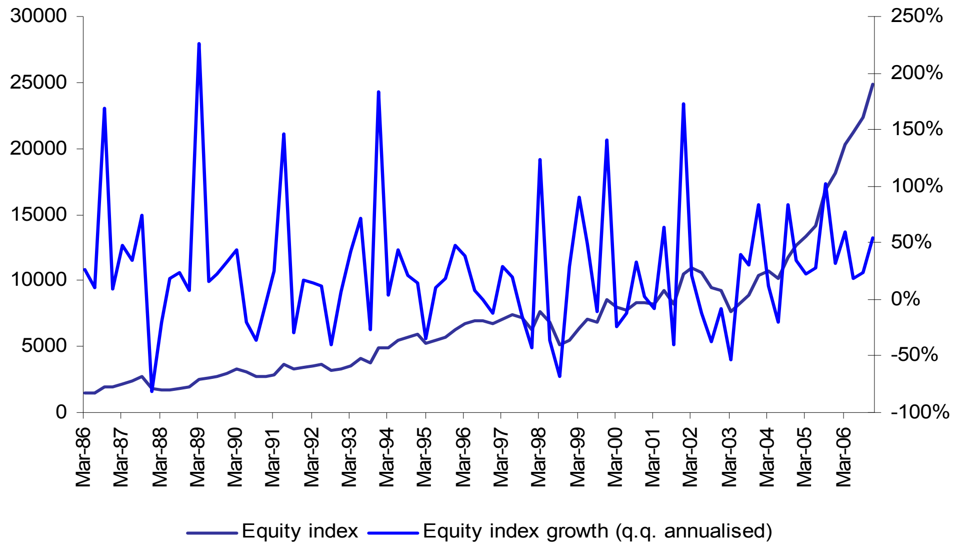
SA money supply



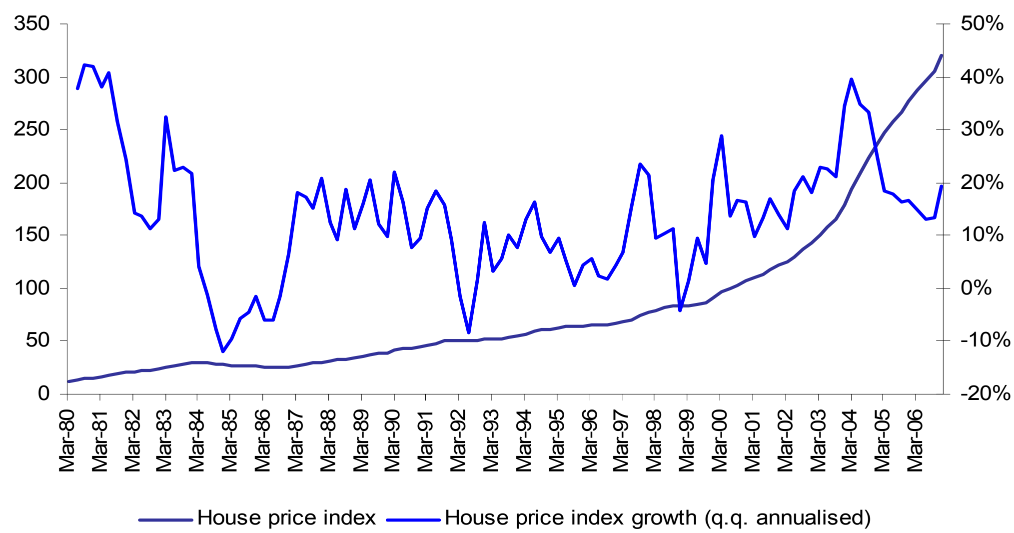
SA Output

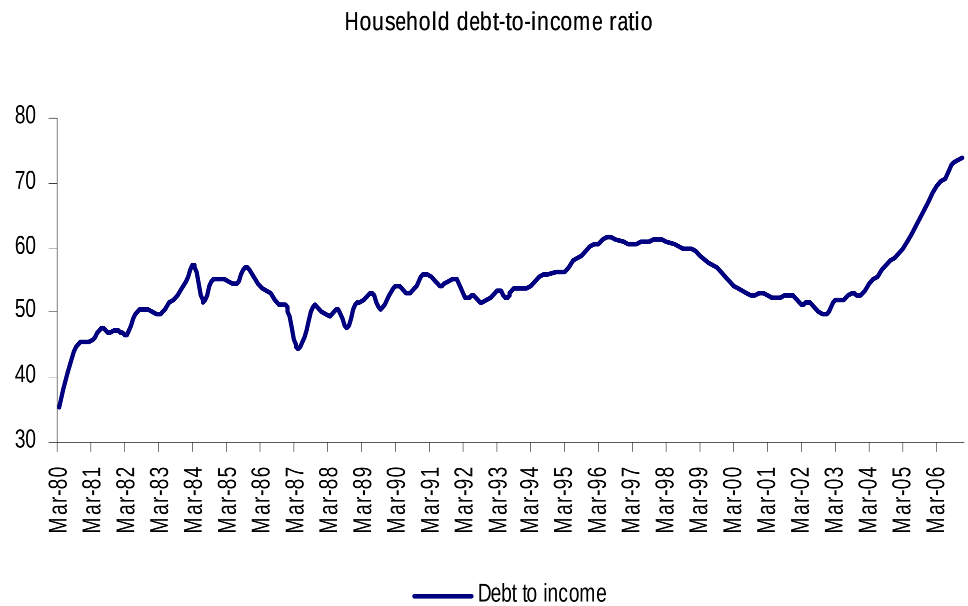


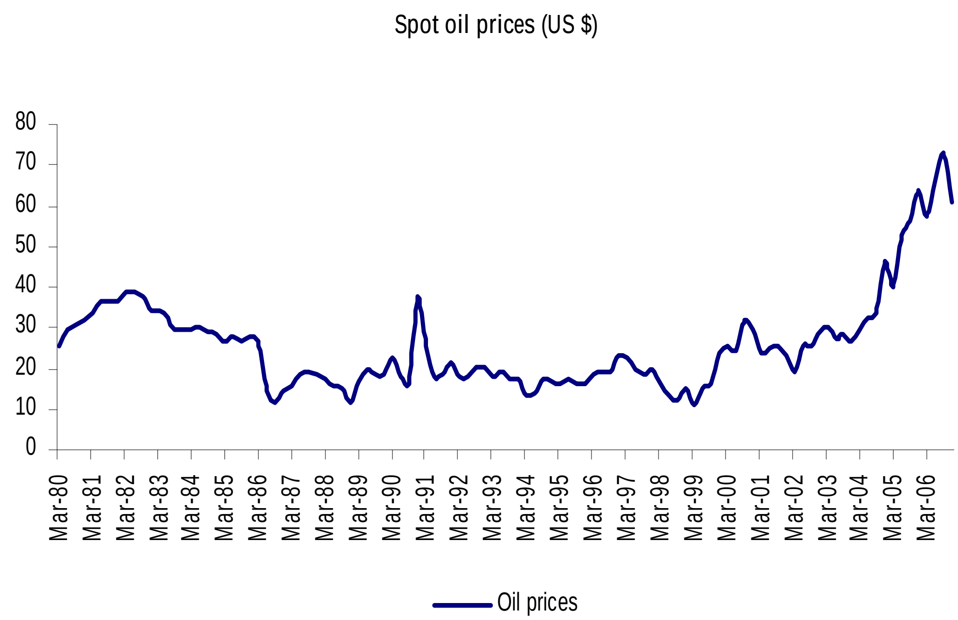
SA equity index



SA house prices







As argued above, these wealth and property value increases in addition to structurally lower interest rates, have allowed households to borrow more and increase their leveraged positions to over 70 per cent of income. While these figures are still well below the levels experienced in developed countries (around 100 per cent), increases in debt levels experienced in the last 3 years are definitely a structural shift in the domestic economy and could lead to volatility and instability if interest rates were suddenly to increase at any future time.

The last time series which will be employed in the South African GVAR model is the exogenously determined oil price variable. Analyzing the trend in oil prices, it is clear that the political tensions in the Middle-East over the last 4 years have had a significant impact on oil prices. While the average price of Brent-crude oil was just above US$23 until 2005, oil prices increased to over US$70 per barrel in Q3 of 2006. While the domestic economy was effectively sheltered from this massive increase due to the recovery of the Rand relative to the U.S. dollar, oil prices now pose significant inflationary and subsequent higher interest rate risk if current oil price increases persist.

3.3 INTEGRATION PROPERTIES OF THE TIME SERIES

As argued in section 2.4.2 the reduced-form cointegration methodology proposed by Johansen (1989, 1992 and 1995) assumes that the underlying data series within the system of equations are integrated of order one i.e. I(1). Although it is generally assumed that macroeconomic variables display I(1) behaviour, the integration properties of each series are formally assessed in each case. Recognizing the fact that the Augmented Dickey-Fuller (ADF) test for unit roots may suffer power problems in small samples we also apply the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) unit root tests. The KPSS test differs from other unit root tests in that the underlying data series are assumed to be (trend-) stationary under the null as opposed to non-stationary. By applying both the ADF and KPSS unit root tests we reduce the risk of spurious inference due to low power properties displayed by the underlying unit root test.

The unit root tests are applied to the full sample of data available, i.e. from 1980Q1 to 2006Q4 for all time series. Series were tested for unit roots, based on the transformation as set out in section 4 of chapter 2. We start testing with a maximum lag length based on the underlying AR process assuming an AR process of order 5 and allow the final order of the ADF test statistic to be calculated based on the Akaike information criterion (AIC) in the case of the ADF test, while the bandwidth of the KPSS test statistic is calculated based on the Newey-West criteria. We test for stationarity of variables in levels and first difference terms. For variables in level form, the ADF test statistic is evaluated assuming stationarity where there is no intercept (none), an intercept only and an intercept and trend in the underlying data generating process. The KPSS test is evaluated with an intercept and intercept plus trend in the underlying data series. For variables in first differences no trend stationarity specification is included, i.e. ADF excluding and including an intercept and KPSS including an intercept are tested.

The following domestic and exogenous variables are tested for containing unit roots in their underlying data generating processes and results are reported in table 3.4: real output (y), the price index (p), real money supply (m), real equity prices (q), interest rates (ρ), the exchange rate (e), the real housing price index (h) and the household debt ratio (d). The oil price index (o) serves as exogenous variable and is also subjected to unit root testing.

In general the results from the unit root test are in line with intuition and the results found in the literature. Although the KPSS test suggests that some of the domestic variables might also be I(0) as opposed to I(1), the ADF test indicates unambiguously that all domestic variables are all I(1).

Table 3.4 Unit root test statistics: domestic and exogenous variables

Variables	Unit root test equation specification	ADF lag length	ADF test statistic H0: Non-stationary	KPSS band width	KPSS test statistic H0: Stationary
y	none	4	7.40		
	intercept	4	6.23	9	1.08*
	intercept and trend	4	2.26	8	0.31*
Δy	none	3	-7.44*		
	intercept	3	-9.21*	7	1.0
p	none	0	1.06		
	intercept	0	-0.49	8	0.91*
	intercept and trend	0	-1.93	7	0.14
Δp	none	0	-10.68*		
	intercept	4	-6.59*	2	0.2
m	none	1	3.10		
	intercept	1	2.66	9	1.05*
	intercept and trend	1	0.61	8	0.30*
Δm	none	0	-5.86*		
	intercept	0	-6.60***	7	0.62
q	none	0	1.06		
	intercept	0	-0.49	8	0.91*
	intercept and trend	0	-1.93	7	0.15*
Δq	none	0	-10.61***		
	intercept	0	-10.68***	2	0.2
ρ	none	1	-0.40		
	intercept	4	-3.14	8	0.38
	intercept and trend	1	-3.70	8	0.17*
Δρ	none	0	-5.87*		
	intercept	0	-5.84*	4	0.14
e	none	0	-3.87*		
	intercept	1	-1.73	9	1.09*
	intercept and trend	0	1.70	8	0.25*
Δe	none	0	-9.37*		
	intercept	0	-9.95*	2	0.21
h	none	1	0.43		
	intercept	1	-0.91	9	0.27
	intercept and trend	4	-0.21	9	0.26
Δh	none	0	-3.33*		
	intercept	0	-3.34*	8	0.44
d	none	1	1.97		
	intercept	1	-2.93	8	0.71
	intercept and trend	0	-3.32	8	0.07
Δd	none	0	-10.63*		
	intercept	0	-10.55*	6	0.15
o*	none	0	0.25		
	intercept	0	-1.70	8	0.31
	intercept and trend	0	-1.87	8	0.27*
Δo*	none	1	-9.41*		
	intercept	1	-9.39*	3	0.19

*/**/*** indicates a rejection of the null hypothesis at a 10%/5%/1% level of significance.

The results from the global variables are similar and suggest that real output (y^*), the price index (p^*), real money supply (m^*), real equity prices (q^*) and interest rates (ρ^*) are I(1) if evaluated by either the ADF or KPSS test statistics. We will also be guided by the literature and empirical results presented by for example PSW(2004) on their global variable integration tests in order to assume that variables are weakly I(1). Results are contained in table 3.5.

Table 3.5 Unit root test statistics: global variables

Variables	Unit root test equation specification	ADF lag length	ADF test statistic H0: Non-stationary	KPSS band width	KPSS test statistic H0: Stationary
y*	none intercept intercept and trend	4 4 4	2.23 -7.32*** -8.25***	8 8	0.35*** 0.25***
Δy*	none intercept	3 3	-6.01**** -6.91****	9	0.59
p*	none intercept intercept and trend	5 5 5	2.00 -1.66 -1.26	9 8	1.17* 0.31*
Δp*	none intercept	4 4	-2.49** -3.06**	9	0.79
m*	none intercept intercept and trend	5 5 4	4.04 -0.60 -2.13	9 8	1.18* 0.15*
Δm*	none intercept	3 4	-2.23** -4.70***	5	0.13
q*	none intercept intercept and trend	0 0 0	1.70 -0.32 -4.20***	9 8	0.0.9 1.03
Δq*	none intercept	0 0	-3.67*** -3.89***	2	0.15
ρ*	none intercept intercept and trend	4 4 3	-1.48 -1.81 -5.39***	8 7	1.15* 0.06
Δρ*	none intercept	5 5	-5.75*** -5.89***	4	0.04

*/**/*** indicates a rejection of the null hypothesis at a 10%/5%/1% level of significance.

The unit root results above demonstrate that the VECM approach is appropriate within the South African context. After careful analysis of possible theoretical relationships and multicollinearity that may exist between the variables we select the set of domestic macroeconomic factors to be; real output (y), the price index (p), real equity prices (q), exchange rate (e), interest rates (ρ), real house price index (h) and the household debt ratio (d). Due to the fact that South Africa is a small global role player it is reasonable to assume that the South African economy can be regarded as a small open economy. Theoretically, this implies that domestic economic variables should in the long run not have a significant impact on global macroeconomic events. As such, the theoretical assumption is imposed that the set of global variables are world output ($y*$), real world equity prices ($q*$) and world interest rates ($\rho*$).

Although these variables are estimated within the GVAR model in order to provide estimates of the global factors for simulation purposes, these variables will be included within the VECM system and modelled accordingly. While it is not the aim of this paper to obtain structural estimates for the global economic factors, the estimation results and forecasts from the VECM model will be evaluated for reasonability to ensure that these do not unduly bias the domestic variables' forecasts.

VECM and cointegration estimation techniques have become very popular and well known in the empirical econometric literature. Various asymptotic tests and procedures have been developed to test the cointegrating rank of a system as well as identify the system of equations in order to provide a unique solution. Asymptotically these tests are very appealing and most certainly correct, "…but for the sample sizes available in most practical situations it is argued that the interaction of dynamic identification and long-run identification can have enormous effects on the size and power of the testing procedures conventionally used" and that "…the small sample properties of now familiar cointegration tests can be very poor in many practical situations". Moreover, "…in a common realistic modelling situation of a limited data set and the theory requirements of a fairly rich model, the techniques proposed in the existing literature are almost impossible to implement successfully." (Greenslade, Hall and Henry (2000)). Many researchers have thus been discouraged by the statistical results from their modelling efforts and abandoned their efforts due to some statistical properties that have not been satisfied completely or could not be married with economic theory. In our application the final VECM and specification is arrived at and accepted based on economic theory, statistical correctness, in and out of sample estimation results, forecast simulation outcomes as well as keeping the specification as parsimonious as possible.

3.4 COINTEGRATING RANK PROPERTIES OF THE SYSTEM

The first step in estimating the South African specific VECM component to the GVAR model is to test the cointegrating rank of the system. As a result of the short time span of data series, the order of the lag structure is assumed to be 1 and the cointegration rank tests are performed accordingly. The data is assumed to have a linear deterministic trend while including an intercept in the cointegration equation as well as the test VAR. The trace and maximum eigenvalue test statistics and critical values are provided in table 3.6.

Table 3.6 Cointegration rank test statistics

Unrestricted Cointegration Rank Test (Trace)				
Hypothesized		Trace	0.05	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None *	0.668898	357.9437	239.2354	0
At most 1 *	0.568988	269.5175	197.3709	0
At most 2 *	0.448352	202.1879	159.5297	0
At most 3 *	0.433694	154.6003	125.6154	0.0003
At most 4 *	0.319116	109.1107	95.75366	0.0044
At most 5 *	0.299598	78.36163	69.81889	0.0089
At most 6 *	0.215491	49.87358	47.85613	0.0319
At most 7 *	0.191815	30.45785	29.79707	0.0419
At most 8	0.147213	13.42068	15.49471	0.1003
Trace test indicates 8 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)				
Hypothesized		Max-Eigen	0.05	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None *	0.668898	88.42621	64.50472	0.0001
At most 1 *	0.568988	67.32956	58.43354	0.0054
At most 2	0.448352	47.58759	52.36261	0.1422
Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

The trace test indicates that there are 8 cointegrating relationships between the variables included in our VAR model while the maximum eigenvalue test indicates that there are only 2 cointegrating relationships. Although it is generally accepted in the literature that the trace test has better power properties in small samples and is often preferred to the maximum eigenvalue test as a result it also suffers from size distortions is some cases. Moreover it is highly unlikely that there could be 8 structural theoretical relationships within the system of 10 variables and would most certainly make it practically impossible to identify the system. As such this study bases its inference on the maximum eigenvalue statistic which indicates 2 cointegrating equations in the system.

3.5 IMPOSING IDENTIFICATION RESTRICTIONS ON THE VECM SYSTEM

Due to the theoretical exogeneity assumptions placed on the global variables it is necessary to identify two long-run cointegrating equations from a set of seven endogenous domestic variables by imposing just-identifying or over-identifying restrictions on the system. Due to the complexity of multivariate cointegration techniques and the 13 different VECM models comprising the GVAR model, it is practical for PSW (2004) to derive their country-specific VECM models based on an exact-identification restriction. Although all exact-identified systems produce a just-identified system and similar forecast and impulse response estimates, these results may not always conform to economic theory. While this study does not aim to provide an unchallengeable structural relationship to the South African VECM model, but rather a theoretically consistent simulation model to be used in a South African credit portfolio model, it is necessary to ensure that the restrictions placed on the system are in line with economic rationale. The study therefore aims to identify the system as well as to ensure that the coefficient estimates are consistent with expectations. The set of possible normalization restrictions can be based on the following set of domestic variables: real output (y), the price index (p), real equity prices (q), exchange rate (e), interest rates (ρ), real house price index (h) and the household debt ratio (d). The study proposes the identification of equations for the exchange rate (e), and real equity prices (q).

For the real equity price (q) equation the principles behind the dividend discount model used in equity stock analysis are used to derive the main components of the specification. The dividend discount model estimates the present value of a share as the value of future cash flows discounted by an appropriate interest rate. As such, real world output (y) is included as a cash flow variable with interest rates as the representative discount rate. Equity prices should increase with increased future cash flows and decrease if discount rates increase. Due to the massive increase in house price asset values in recent times, many investors have substituted equities for property in order to diversify portfolios and also tap into the massive property price boom experienced in global markets. Especially in South Africa, property is the primary substitution asset for equities and therefore should be included in the specification. Increases in property values should therefore result in a substitution away from equities and into property, resulting in equity price deflation. Financial liquidity is a significant driving force behind my asset price bubbles and especially price volatility in equity markets. Many previous equity market meltdowns have resulted from liquidity draining quickly from the economic system. As a result we include the household debt to income ratio as a proxy for liquidity (due to its long correlation with money supply) and expect that an increase in liquidity should lead to increases in equity prices. The domestic equity price specification is therefore:

$$q = f\left(y^*, \rho, h, d\right) \tag{90}$$

The second relationship that will be identified within the VECM specification is the exchange rate. Following the theoretical discussions from Abel and Bernanke (2001), the theoretical constraints, as applicable to a small open economy, are imposed. According to Abel and Bernanke (2001) one can expect that the exchange rate of a small open economy (real or nominal) could be influenced by domestic and global currency demand and supply factors. An increase in foreign income or liquidity would increase demand for domestic goods and currency resulting in a strengthening in the value of the domestic currency. Higher real rates of return on domestic assets e.g. interest rates, equity returns and property values would also increase the demand for domestic currency as more investors would be looking to invest in the domestic market. On the other hand, increases in domestic income and inflation rates would result in an increase in the demand for foreign goods and a loss in purchasing power, which would lead to a deterioration in the domestic currency value. The following exchange rate equation is thus proposed:

$$e = f(\rho, q, y, h) \tag{91}$$

The final theoretical specification provides a total of 12 over-identification restrictions on the system of 2 long-run cointegrating vectors i.e. 2 normalization and 10 theoretical exclusion restrictions.

3.6 ESTIMATION RESULTS

The VECM results for the theoretical specification outlined above are presented in table 3.7. The estimation results of the cointegration equations are of particular interest as they would govern the long-run relationship of the model simulations in the portfolio model and should therefore provide theoretically consistent estimates of the interaction between the variables. According to the Likelihood-ratio test, the Chi-square statistic of 15.71 (probability equal to 0.15) indicates that the theoretical constraints placed on the system are valid and binding and identify all cointegrating vectors. While the R-squared values from the dynamic estimation output indicate that a significant portion of the variation in the dynamics of the variables is explained by the system, the discussion would focus on the long-run estimation results.

The first cointegrating relationship capturing domestic equity price movements illustrates the high dependence of South African financial markets on global economic conditions and prosperity. While the substitution between equity and fixed assets is still significant, domestic interest rates are clearly very significant discounting factors. As expected the positive correlation between domestic liquidity (as proxied by the household income to debt ratio) and equity movements illustrates that equity prices can be inflated by high liquidity levels.

The second cointegrating equation displays the theoretical specification for the exchange rate and shows that the estimation results conform to economic theory. The coefficients for domestic income growth and inflation show the expected depreciation in the currency due to higher import volumes and a loss in purchasing power while increases in asset values would lead to increased demand for Rand denominated assets and a strengthening of the currency.

Table 3.7 VECM estimation output

Cointegrating relationships

Vector Error Correction Estimates		
Standard errors in ( ) & t-statistics in [ ]		
Restrictions identify all cointegrating vectors		
LR test for binding restrictions (rank = 2): Chi-square(10): 15.714 Probability: 0.152		
Cointegrating Eq:	CointEq1	CointEq2
ρ* (-1)	0.000	0.000
q* (-1)	0.000	0.000
y* (-1)	3.020 -1.537 [1.964]	0.000
e(-1)	0.000	-1
ρ(-1)	-8.807 -1.246 [-7.067]	-6.465 -1.850 [-3.494]
q(-1)	-1	-0.513 -0.261 [-1.968]
y (-1)	0.000	5.633 0.385 [14.641]
d (-1)	4.073 0.400 [10.178]	0.000
h(-1)	-0.685 -0.194 [-3.537]	-2.460 0.212 [-11.598]
p(-1)	0.000	0.000
c	-13.961	-59.672

3.7 RESIDUAL DIAGNOSTICS AND VECM STABILITY TEST

Due to the fact that the VECM model will be used to simulate “economic states” in the credit portfolio model, it is important that the model exhibits stability in order to avoid generating unrealistic economic realizations. As such, various diagnostic tests have been performed and are presented below to assess the appropriateness of the VECM specification and also to give comfort to the identification restrictions imposed in the system.

As presented in table 3.8, overall residual unit root tests assuming a common unit root process as well as individual unit root processes, indicate that the null hypothesis of a unit root is rejected, while only the residual for the world output equation indicates that there is still significant information which is not captured by the model specification.

Finally, autocorrelation tests indicate that there are no systematic patterns in the errors up to lag 4. As such, the results indicate that the model does not suffer significant miss-specification and does not possess serial correlated error terms as a result.

Table 3.8 Residual unit root and serial correlation tests

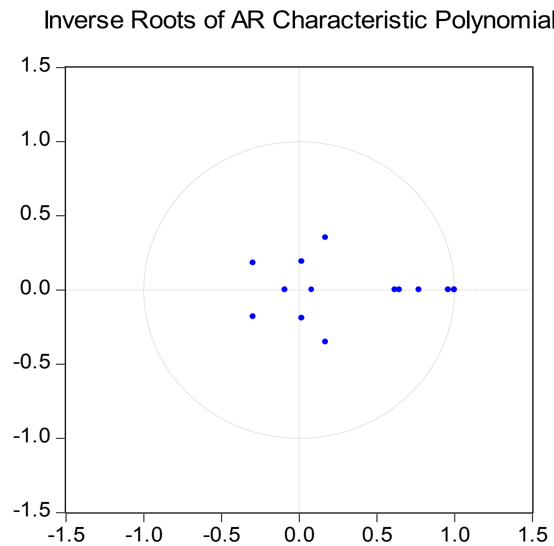
Group unit root test: Summary				
Newey-West bandwidth selection using Bartlett kernel				
Method	**Statistic**	**Prob.****	**Cross-sections**	**Obs**
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-28.289	0.0000	10	870
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-28.940	0.0000	10	870
ADF - Fisher Chi-square	307.414	0.0000	10	870
PP - Fisher Chi-square	307.201	0.0000	10	870
** Probabilities for Fisher tests are computed using an asymptotic Chi-square distribution. All other tests assume asymptotic normality.				

VEC Residual Portmanteau Tests for Autocorrelations					
Lags	**Q-Stat**	**Prob.**	**Adj Q-Stat**	**Prob.**	**df**
1	28.378	NA*	28.704	NA*	NA*
2	105.567	0.332	109.689	0.282	100
3	219.007	0.170	226.132	0.107	200
4	315.497	0.258	326.217	0.143	300
*The test is valid only for lags larger than the VAR lag order.					
df is degrees of freedom for (approximate) chi-square distribution					

The next test for appropriateness is to assess the number of roots created in the AR characteristic polynomial. As outlined by PSW (2004), this is a significant condition which should be met for the GVAR model to be estimated as individual country-specific models. In the VECM context, for a VECM with r cointegrating relations, $k-r$ roots should be equal to unity. In the current system this implies that there should be 8 (10–2) unit roots. The results of the stability conditions check are summarized in table 3.9 and show that the VEC specification imposes the expected 8 unit roots and should be significantly stable when used in simulations and forecasts within the portfolio model. Most of the roots are also close to the origin which indicates that the system should be stable. These results give further support to the theoretical.

Table 3.9 Stability conditions: AR characteristic polynomial roots

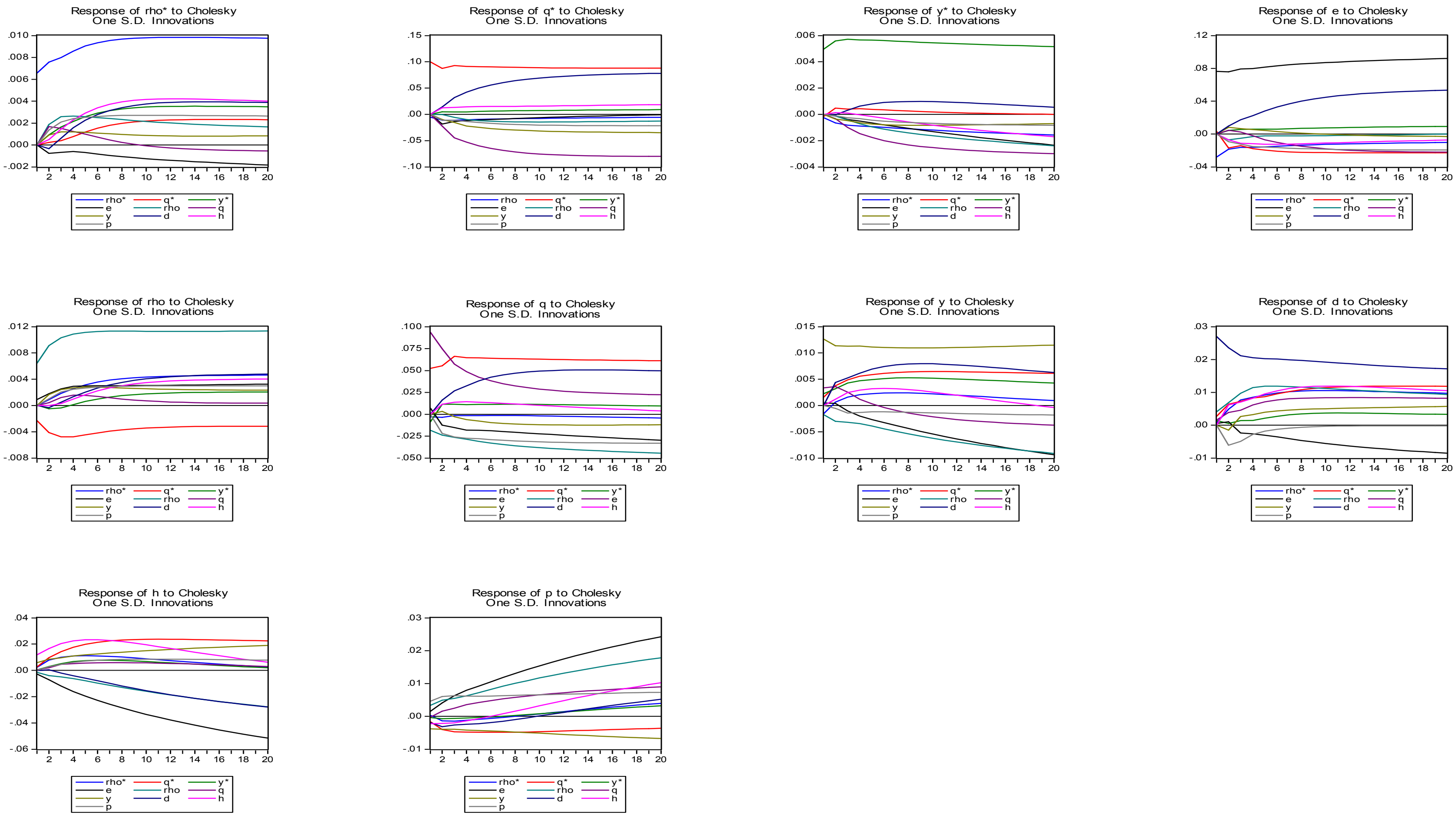
Roots of Characteristic Polynomial:
VECM specification imposes 8 unit root(s)



3.8 DYNAMIC PROPERTIES OF THE VECM

Although the estimation output indicates the elasticity of the model dependent variables to changes in the independent variables, a more satisfactory method of analysing the full dynamics and interdependencies of the model variables is to use general impulse response analysis (GIRF). Due to the unit root properties of the VECM model it is expected that shocks or changes in variables as imposed by general impulse response analysis should have transitory as well as permanent effects on other model variables. Although it is true that it is not "…possible to provide 'structural' or economic interpretation of these shocks... GIRF provides a theoretically consistent account of the interdependencies of idiosyncratic shocks" (PSW (2004)). Due to its small open economy status, South Africa is prone to be influenced significantly by international market developments. As such the impact of a generalised one standard deviation positive shock to all the variables in the VECM is considered and the focus falls on the time it takes for these impacts to manifest in new equilibrium levels, but more importantly, focus falls on whether the adjustment process is smooth.

Figure 3.3 GIRF analysis of global macroeconomic shocks



The results of the GIRF analysis are summarised in figure 3.3. In general, the impact response graphs display a smooth pattern as the endogenous variables adjust to shock from the system. While foreign and domestic interest rates, equity prices, foreign output, the exchange rate and household debt to income levels adjust quite quickly to new levels (8 to 10 quarters) domestic prices, output and house prices needs up to 20 quarters (i.e. 5 years) to result in new equilibrium levels for shocks to some variables in the system, particularly to exchange rate movements. This is consistent with the small open economy status of the South African economy and highlights the sensitivity of the domestic market to global market developments. The results presented here indicate that the VECM model and specification provides reasonable impact dynamics and that the model is appropriate for use in forecast and scenario generation within a credit portfolio model framework.

3.9 STOCHASTIC FORECAST PROPERTIES OF THE VECM

The final test which is performed on the VECM in order to assess its appropriateness within the proposed credit portfolio model context is to test the model's ability to provide reasonable stochastic forecasts. Since a typical bank assesses its conditional loss distribution over a one year period in order to allocate capital, it is essential that the VECM provide consistent forecasts over a one-year period. The estimated VECM is therefore simulated over a one-year period, in- and out-of-sample, i.e. from 2006Q1 to 2006Q4 and from 2007Q1 to 2007Q4 to assess forecast robustness.

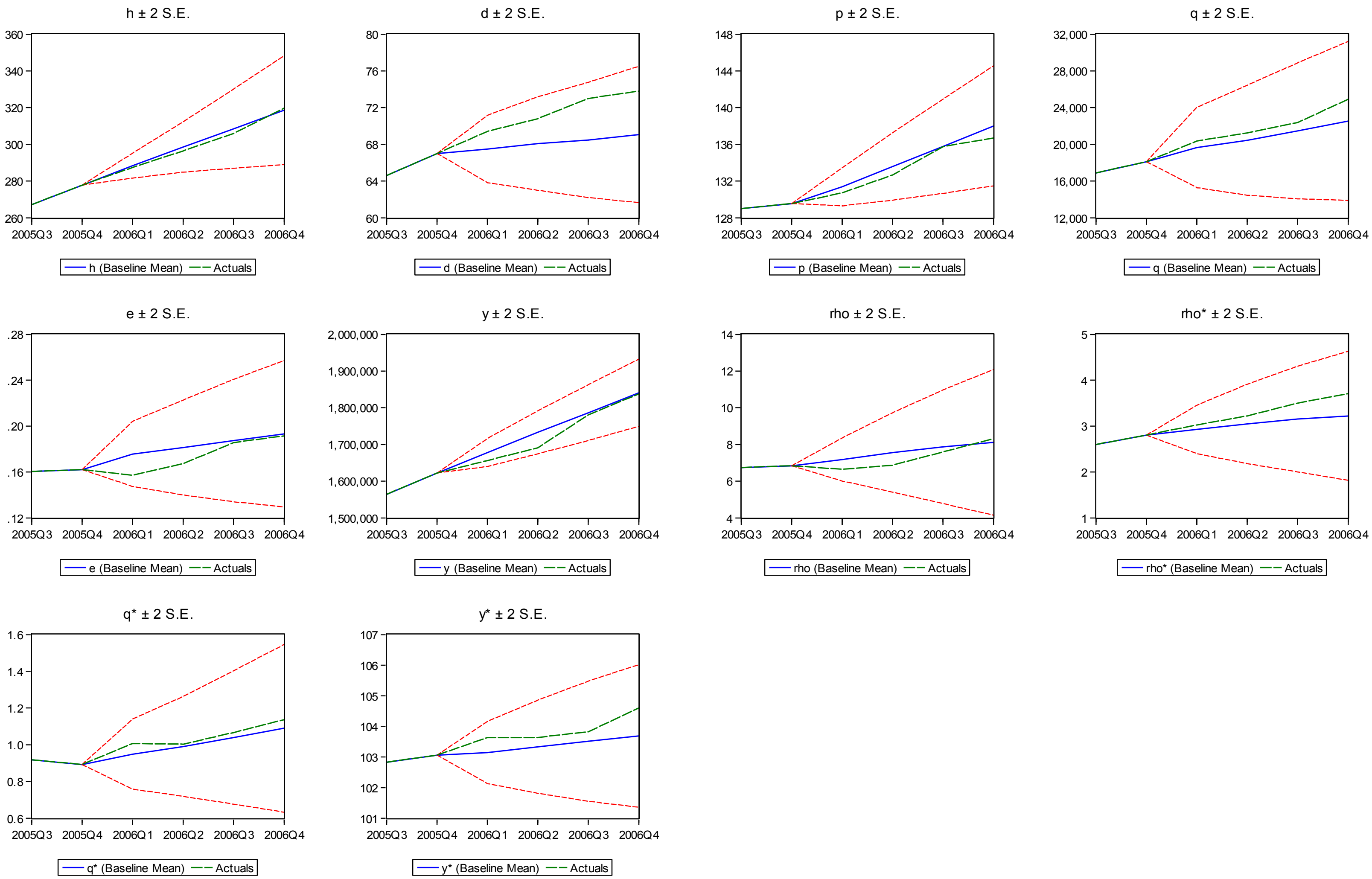
The results of the in- and out-of-sample stochastic simulations are presented in figures 3.4 and 3.5. Analysing the in-sample simulations, except for the household debt to income ratio the VECM predicts the actual realisation of the model variables with a high degree of accuracy and does not display any significant degree of bias in variable predictions. Moreover, although the actual variable realisation may have deviated from the model predicted or expected outcomes no such deviations fell outside the model 2-standard-error confidence intervals. Moreover, by analysing the out-of-sample results we are able to benchmark the results to the some actual realisations of these variables. Specifically, the model forecasts are in line with actual GDP realisations while the end of year results for the house price index and the household debt to income ratio (actual realisations of 360 and 77.6% respectively) are also in line with its actual outcomes. As such, it is concluded that the stochastic model simulation results in appropriate simulation variance to capture and include model forecast error and also provide additional variance to allow for unexpected economic events.

3.10 CONCLUSION

Based on the methodology proposed by PSW (2004) and PSTW (2006) this chapter proposes a South African specific credit market correlation model which could be linked to the current GVAR model proposed by PSW (2004). The model is based on a VECM system which includes credit-market-related domestic and global economic variables. Although PSW (2004) only places statistically exact identifying restrictions on their individual country VECM model, a set of theoretically consistent over-identifying restrictions is proposed for this study's VECM system in order to identify coefficient estimates that conform to theoretical expectations.

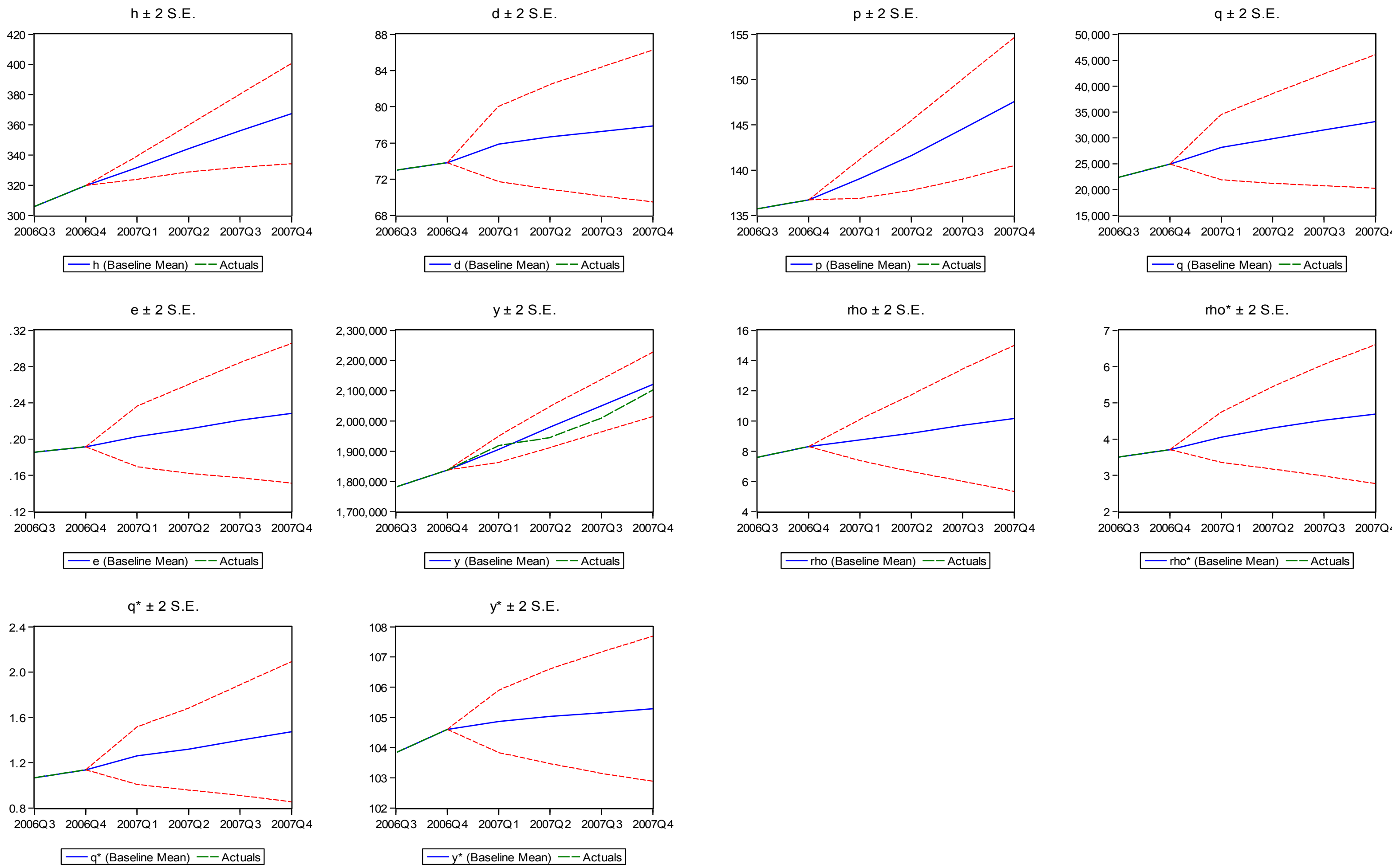
Although it is not the aim of the model to provide forecast results for global factors, but rather to provide South African specific elements to the GVAR model, in- and out-of-sample forecasts have shown that stochastic simulations are in line with actual variable realisation and expectations. As such, it is argued that the correlation model could be employed as a stand-alone model within a South African specific credit portfolio management tool.

Figure 3.4 In-sample stochastic simulation results[3]



[3] Domestic variables have been converted to nominal terms for illustration and benchmarking purposes. Global variables are presented in real terms.

Figure 3.5 Out-of-sample stochastic simulation results[4]



[4] Domestic variables have been converted to nominal terms for illustration and benchmarking purposes. Global variables are presented in real terms.

CHAP7TER 4

DEFAULT RISK AND CONDITIONAL CREDIT PORTFOLIO LOSS SIMULATION

4.1 INTRODUCTION

Active credit portfolio management is becoming a central part of capital management within the banking industry. The estimation of risk-sensitive credit capital is pivotal to success in an environment where there is ever-increasing competition. If any risk mitigation or value-enhancing activity is to be pursued, a credit portfolio manager must be able to identify the interdependencies between exposures in a portfolio but more importantly, also translate macroeconomic credit risk into tangible portfolio effects.

This chapter uses the macroeconometric vector error correcting model (VECM) developed in chapter 3 and applies the proposed methodology of PSTW (2006) to a fictitious portfolio of corporate bank loans within the South African economy. We illustrate that it is not only possible to link macroeconomic factors to a South African specific credit portfolio, but also that scenario and sensitivity analysis can be performed within the credit portfolio model. These results can be used in credit portfolio management or stand-alone credit risk analysis which is ideal for practical credit portfolio management applications.

4.2 DEFAULT THRESHOLDS BY RATING CATEGORY

As explained in chapter 3, a company would default if the log equity threshold falls below a specific level i.e. the default threshold ratio. In general, a particular company will only have one default observation and therefore it is obvious that obtaining empirical estimates of default thresholds on a per company basis is not possible. For the latter-mentioned reason, the identification condition presented in equation 13 of chapter 2 is adopted i.e. at any given time any firms that have received similar ratings will have the same default equity-threshold ratios. As such, the condition allows for different threshold levels, allowing for heterogeneity between firm equity growth paths, but assumes that the same ratio applies. This assumption therefore allows the use of historical data to estimate equity threshold ratios empirically using historical default observations over a sufficiently long time period.

Default equity threshold ratios are taken as presented by PSTW (2006). Using S&P default and rating histories spanning 1981-1999, PSTW (2006) estimates the one to four quarter-ahead threshold equity ratios. Since default experiences and default data are significantly scarce it is unlikely that a replication of the threshold ratio calculation would lead to significantly different threshold estimates. As such this study rather focuses on using the PSTW results in a new and previously unexplored African bank credit portfolio. Nevertheless, for completeness the estimation procedure followed by PSTW (2006) is explained in detail below.

Recall from chapter 2 equation 11 that the log equity threshold level can be estimated as:

$$\hat{\lambda}_R(t,H) = H\hat{\mu}_R + \hat{Q}_R(t,H)\hat{\sigma}_R\sqrt{H}$$

with $\hat{Q}_R(H)$ given by

$$\hat{Q}_R(H) = T^{-1}\sum_{t=1}^{T}\left\{\Phi^{-1}\left[\hat{\pi}_R(t,H)\right]\right\}$$

Clearly, the estimates of the default threshold level per rating category are determined by three variables i.e. default probabilities, $\hat{\pi}_R(t,H)$, average returns, $\hat{\mu}_R$, and the variance of the returns, $\hat{\sigma}_R$.

Applying the methodology presented by Lando and Skodeberg (2002), the default rate estimates $\hat{\pi}_R(t,H)$ are based on a transition-intensity approach. Essentially, default probability is estimated taking into account the migration effects of companies, i.e. a highly rated company will typically migrate from a good rating through a number of rating categories before defaulting (as apposed to "jump-to-default" from its original rating category). As such, this migration effect is taken into account and default probability is attributed to the original rating category. Further to this, PSTW (2006) also recognizes the fact that although the S&P rating agency has rated a significant portion of companies over time, low default experiences in high rating categories makes empirical estimation quite difficult. As such a default probability which is in line with the Basel II requirement of 0.025 basis points per quarter has been assigned to the historical observations. According to PSTW (2006) this floor is particularly relevant if the default thresholds are to be applied to a broader sample of firms not covered by the S&P sample used.

The rating-specific average return, $\hat{\mu}_R$ and volatility estimates $\hat{\sigma}_R$ are computed through a specific estimation process as discussed by a technical note provided by Pesaran and Schuermann (2004) as a supplement to the PSTW (2006) and PSW (2004) papers. For each firm (*i*) in a specific rating category (*R*) the cum dividend total daily returns, $r_{i,R,t}(d)$, for a sample of U.S. companies have been collected over the sample period January 1981 to December 2002. Following market convention daily returns are scaled by the number of trading days in a quarter i.e. 63. As such, quarterly returns and variances are estimated as follows:

$$\hat{\mu}_R = \hat{\mu}_R(d) \times 63$$
$$\hat{\sigma}_R = \hat{\sigma}_R(d) \times \sqrt{63}$$

(93)

As before, the set of firms with rating R at the end of a particular day can be represented by R_t. As can be expected, when a firm's financial conditions change, the set of firms in a particular rating category will vary over time. As such the set of firms and the number of firm days in a particular rating category R over time period $t = 1,2,...T$ can be defined as:

$$\widetilde{R}_T = \bigcup_t^T R_t \quad \text{and} \quad n_R = \sum_t^T n_{Rt}$$

Although Pesaran and Schuermann (2004) presents three methods for estimating $\hat{\mu}_R$ and $\hat{\sigma}_R$, only the method finally used in this study in the estimation process is presented here.

One each day, $t = 1,2,..T$, a specific firm is randomly picked in each rating category, R, and its return $r_{R,t}^{(j)}(d)$ used to represent the rating category in question. By sampling with replacement, a sequence of returns over time can be obtained, i.e. $\left\{r_{R,t}^{(j)}(d)\right\}_{t=1}^{T}$, and as such the sequencing process can be replicated J times. Using the J sequences, the mean and volatility of returns can easily be estimated as:

$$\bar{r}_R^{(j)}(d) = \frac{1}{T}\sum_{t=1}^{T} r_{R,t}^{(j)}(d)$$

$$\hat{\sigma}_R^{(j)}(d) = \frac{1}{T-1}\sqrt{\sum_{t=1}^{T}\left(r_{R,t}^{(j)}(d) - \bar{r}_R^{(j)}(d)\right)^2}$$

Finally, for a sufficiently large J, the average returns and volatilities for typical firms in rating category R can be calculated as:

$$\bar{r}_R^{C}(d) = \frac{1}{T}\sum_{j=1}^{J} \bar{r}_R^{(j)}(d)$$

$$\hat{\sigma}_R^{C}(d) = \frac{1}{T}\sum_{j=1}^{J} \hat{\sigma}_R^{(j)}(d)$$

Using this methodology, PSTW (2006) estimates the 1 to 4 quarter-ahead return, volatility and equity threshold levels. The 1 and 4 quarter-ahead threshold levels are presented in table 4.1 for rating categories AAA to B. Similar results are obtained for quarters 2 and 3.

Table 4.1 One and four quarter ahead return, volatility and default threshold estimates per rating category

Rating Grade	$\hat{\mu}_R$ (J=1000)	$\hat{\sigma}_R$ (J=1000)	$\frac{\hat{\mu}_R}{\hat{\sigma}_R}$ Sharpe	$\hat{\lambda}_R(t,1)$	$\frac{\hat{C}(t,1)}{E(t)}$	$\hat{\lambda}_R(t,4)$	$\frac{\hat{C}(t,4)}{E(t)}$	# Obs
AAA	4.54%	13.87%	32.72%	-0.588	0.555	-0.999	0.368	1,177
AA	4.06%	15.16%	26.76%	-0.648	0.523	-1.096	0.334	6,272
A	4.13%	15.31%	26.99%	-0.645	0.525	-1.042	0.353	12,841
BBB	3.80%	17.38%	21.86%	-0.688	0.503	-0.988	0.372	9,499
BB	3.21%	24.72%	12.99%	-0.870	0.419	-1.218	0.296	7,002
B	2.04%	34.82%	5.86%	-0.908	0.403	-1.211	0.298	6,493

As expected, the results indicate that the average volatility increases monotonically as rating quality decreases over the rating spectrum. This is in line with rating-agency criteria which place a premium on return stability over time. Intuitively it is also expected that higher rated firms' returns would be influenced much less by the average economic cycle while lower rated firms (usually start-up companies) are impacted more directly by economic conditions. From the Sharpe-ratio it is clear that over this longer investment horizon period (1981-2002), on a risk-adjusted basis, investors were not compensated for increased risk. Moreover, the huge difference in this ratio, between investment (BBB and above) and non-investment (BB and below) grade firms, clearly indicates the huge divide between the two types of investment over time. Clearly, except for short-term speculative returns, investors were not adequately rewarded for the higher default risk in these rating categories.

In order to forecast default in the credit portfolio environment, the equity default threshold level is of relevance. Relative to the four quarter ahead threshold levels the one quarter ahead levels are much higher, implying that given bad performance by a particular firm, default is less likely to occur over the short term. However, a sustained period of bad performances e.g. one year, is more conducive to default risk. In line with the rating agency methodology, this implies that a firm is given time to recover from bad short-run performances and that default would occur if a firm's value deteriorated over a "through the cycle" period.

Although PSTW (2006) focus on the one quarter ahead result, a typical bank's credit analysis, return and capital sensitivity planning are usually calculated over a one-year horizon, and as such the four quarter ahead default thresholds are of particular concern. Intuitively one would expect the equity threshold ratios to decrease, implying that firms with a higher rating would need to suffer worse losses than their lower-rated counterparts in order to default. However, the equity default threshold ratios show little variation over rating categories. In fact there is only a 7 per cent difference between the threshold levels of AAA and B-rated firms. In order to understand the role of the threshold level, the variance of returns should be analyzed concurrently. As an example, an AAA-rated firm and a B-rated firm, each with an equity level of 100 today, would be able to sustain a drop in value of 0.37*100=37 and 0.30*100=30 respectively before defaulting. However, the likelihood of such an event is driven by the variance of return. For the AAA-rated firm the likelihood is quite low relative to the B-rated firm since $\hat{\sigma}_{AAA} = 13.87\%$ and $\hat{\sigma}_{B} = 34.82\%$. Clearly the-B rated firm would pierce the equity threshold much more often than the AAA-rated company.

4.3 THE SAMPLE PORTFOLIO

In order to estimate the impact of the macroeconomy on a bank loan portfolio, a fictitious South African-specific corporate loan portfolio is constructed. Only a small number of South African firms are currently rated by S&P and because firm ratings are one of the major inputs for applying the identification condition in equation 13 to the proposed framework, this presents a severe constraint on the number of counterparts available for inclusion in the sample portfolio. In order to overcome this constraint, propriety ratings from FirstRand Bank as obtained through their credit rating models and credit processes are used. These ratings are calibrated in such a way that they are similar to S&P default ratings so it would not introduce inconsistencies between the log equity threshold values estimated by PSTW (2006) for specific S&P rating categories. Due to the fact that such rating information is highly sensitive, this study will refrain from providing any link between firm and rating, focusing rather on the portfolio aggregates of the fictitious portfolio. Firm ratings have been assigned and assessed as at 1 January 2007 (the beginning of forecast period) and have then been mapped into the major rating categories as used in the portfolio model. In order to facilitate later calculation of the equity threshold ratios, the cum dividend return data per name over time is obtained from publicly available share data sourced from the McGregor equity database.

Although the analysis starts off with a sample of 696 firms, the final portfolio consists of 145 exposures, spanning the rating spectrum in South Africa. All firms included are listed on the Johannesburg stock exchange and were or are currently listed for at least 16 quarters, i.e. 4 years. In order to avoid biased results for any particular rating category and to prevent the introduction of unrealistically large exposure concentration, exposure size has been assigned randomly for each counter party in the portfolio through a random number generation process. These exposure values were added together to give the total portfolio notional amount. In general, this portfolio can therefore be thought of as being representative of the benchmark or market portfolio of the South African corporate loan market. Details of the 145 exposure sample portfolio are illustrated in table 4.2. On a percentage-of-exposure basis, the portfolio is concentrated within the BBB rating category and reflects the composition of the South African corporate rating spectrum which is perceived to be more risky relative to international standards. A share of 38 per cent of the portfolio is concentrated in the sub-investment grade ratings, with only 15 per cent of exposures obtaining a single-A rating status.

Table 4.2 Sample portfolio composition

Major rating category	Exposure % of total portfolio	% of total firms
A	14.75	12.41
BBB	47.44	45.52
BB	25.31	28.28
B	12.47	13.79
	100.00	100.00

When using a fixed LGD assumption of 45 per cent together with the historical default probabilities per rating category as estimated by S&P, the expected portfolio loss over a one-year period is estimated at 0.53 per cent of exposure value. As such, this provides the benchmark for the estimated conditional portfolio loss from our simulated credit portfolio model. If the simulated conditional portfolio loss is estimated as being below this level it implies that there was positive migration over the forecast period while a higher expected loss would imply that the portfolio quality has deteriorated.

4.4 FIRM-SPECIFIC RETURN REGRESSIONS

As discussed in detail in chapter 3, individual firm return equations are used to translate the macroeconomic conditions into firm-specific equity return outcomes. These multifactor return equations can be estimated through least squares analysis assuming that the firm-specific and macroeconomic innovations are uncorrelated.

PSTW (2006) uses pooled mean-group estimators (MGE) to facilitate the estimation of return regressions. In commercial available models the MGE procedure and estimates are usually applied as is. The only heterogeneity within such a framework is generated through the differentiation between countries' firms with respect to macroeconomic factor sensitivities. More differentiation and therefore diversification can be allowed through panel-fixed effects or random-effects modelling. In general, fixed- or random-effect heterogeneity implies that each firm would have a different intercept but that the sensitivity to macroeconomic factors would still be similar across all firms i.e. the factor loadings would be similar. Effectively, average returns per firm would be different but the sensitivity to macro-factors would still be the same. Such a generalisation would be applicable if firms have been classified into homogenous groups e.g. industries or sectors, where a group-specific multifactor panel model could be estimated for each homogenous group. Without such homogenous grouping, a panel estimation model could still overstate risk, as the differential sensitivity of each firm is not taken into account when estimating overall portfolio risk. The difference between a single risk factor versus a multi-factor model can also contribute to risk being overstated. This difference has indeed been criticised by various practitioners as being a major drawback of the single risk factor approach.

As a result, this study proposes to first estimate a pooled panel data model, followed by fixed and random effects panel data models to test overall economy-wide sensitivity to macroeconomic factors. This estimation would be used to inform and guide expectation into the final firm-specific return estimates. A single risk-factor model is then estimated and the results of the single-factor framework are tested against the results from the final model specification. The final estimation models would allow for maximum diversification between the 145 firms as each firm's return model would be estimated on an individual basis. Short names and notation of variables in all estimation results are adopted from chapter 3 but represent the ***log difference transformation*** of the variables here in order to obtain stationary representations of the factors as required in multi-factor models. In summary, the variable abbreviations are – where all variables are the log-difference form and starred variables again represent global variables (also in log differenced form):

y = real output	*q = real equity prices*
p = price index	*ρ = interest rates*
m = real money supply	*h = real house prices*
d = household debt-to-income ratio	*e = real effective exchange rate*

Equity returns are calculated as the cum dividend log differences of equity prices.

4.4.1 POOLED PANEL DATA MODEL FOR PORTFOLIO-WIDE RETURN ESTIMATES

As discussed above, the first step in order to derive a specification for firm-specific return dynamics, is to estimate a pooled multi-factor model for the dummy portfolio. This estimation procedure assumes that there is no cross-section heterogeneity between firms, i.e. it is implicitly assumed that all firms would react similarly to macroeconomic factors.

Estimation results are illustrated in table 4.3 and show that only world interest rates and domestic equity prices are rejected as return factors on a pooled basis and that all other variables are significant determinants of firm returns.

Table 4.3 Pooled estimation results

Dependent Variable: Equity return				
Method: Pooled Least Squares				
Cross-sections included: 696				
Total pool (unbalanced) observations: 26934				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
c	0.421503	0.115481	3.649992	0.0003
ρ^*	0.017514	0.023760	0.737121	0.4611
q^*	-2.683237	0.257293	-10.42874	0.0000
y^*	-0.203933	0.032683	-6.239804	0.0000
e	0.213667	0.092027	2.321782	0.0203
ρ	0.780490	0.029617	26.35260	0.0000
q	0.164608	0.109128	1.508399	0.1315
y	1.631373	0.130061	12.54315	0.0000
d	-0.463861	0.096354	-4.814118	0.0000
h	1.620473	0.183589	8.826623	0.0000
p	3.018423	0.408633	7.386640	0.0000
R-squared	0.054486	Mean dependent var		-0.004473
Adjusted R-squared	0.054169	S.D. dependent var		0.386164
S.E. of regression	0.375559	Akaike info criterion		0.879570
Sum squared resid	3793.678	Schwarz criterion		0.882618
Log likelihood	-11823.29	Hannan-Quinn criter.		0.880553
F-statistic	1.792826	Durbin-Watson stat		1.854387
Prob(F-statistic)	0.000000			

While these results are in line with our expectations it seems unreasonable that interest rates and in particular domestic equity prices should not be included in our specifications. We therefore propose to move forward and estimate a panel regression model which allows more heterogeneity amongst firms in the estimation process.

4.4.2 FIXED AND RANDOM EFFECTS PANEL DATA MODEL FOR PORTFOLIO-WIDE RETURN ESTIMATES

The next step is to estimate a panel multi-factor model for our portfolio of corporate loans allowing for firm-specific heterogeneity through either fixed or random effects panel data modelling.

From table 4.4, the chi-squared test for random effects indicates that the null hypothesis for correlation between the random effects and the regressors can clearly be rejected, pointing to endogeneity issues, and placing doubt on whether the technique is indeed applicable in this context. The F- and Chi-squared tests for fixed effects however rejects the null hypothesis that all fixed effects are jointly insignificant, i.e. firms in this portfolio context are indeed heterogeneous, and would react uniquely to changes in the global and domestic macroeconomic environment.

Table 4.4 Random versus fixed effects panel tests

Correlated Random Effects – Hausman Test			
Test cross-section random effects			
	Chi-Sq. Statistic	Chi-Sq. d.f.	Prob.
Cross-section random	142.86	10	0.0000
Redundant Fixed-Effects Tests			
Test cross-section fixed effects			
Effects Test	Statistic	d.f.	Prob.
Cross-section F	1.308	(694,26)	0.0000
Cross-section Chi-square	916.05	694	0.0000

The result of the fixed effects model is of particular importance and is entirely consistent with the motivation for estimating individual firm-specific return equations as opposed to a pooled model. Not controlling for heterogeneity would disregard significant diversification benefits which are present in the portfolio. Within a portfolio context, risk would be overstated if diversification is not allowed.

The estimation result for the fixed effects panel data model is presented in table 4.5. Despite allowing for more firm heterogeneity, real world equity, domestic interest rates and domestic house prices are still not significant determinants of returns within the dummy portfolio while real house price growth is also insignificant. Although all other variables are significant determinants of equity returns, the adjusted R-squared statistic indicates that only 6 per cent of the variance is explained by the model specification.

Table 4.5 Fixed effects panel estimation results

Dependent Variable: Equity return				
Method: Pooled Least Squares				
Cross-sections included: 696				
Total pool (unbalanced) observations: 26934				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
c	-0.040796	0.008597	-4.745442	0.0000
ρ^*	0.237786	0.120050	1.980722	0.0476
q^*	0.015331	0.024037	0.637816	0.5236
y^*	-2.327328	0.259455	-8.970071	0.0000
e	-0.163703	0.032794	-4.991889	0.0000
ρ	-0.029725	0.100489	-0.295806	0.7674
q	0.780752	0.029745	26.24841	0.0000
y	0.234861	0.110277	2.129739	0.0332
d	1.509952	0.134238	11.24831	0.0000
h	0.086892	0.140674	0.617683	0.5368
p	3.018423	0.408633	7.386640	0.0000
R-squared	0.087182	Mean dependent var		-0.004473
Adjusted R-squared	0.062656	S.D. dependent var		0.386164
S.E. of regression	0.373870	Akaike info criterion		0.896037
Sum squared resid	3662.492	Schwarz criterion		1.110892
Log likelihood	-11349.83	Hannan-Quinn criter.		0.965334
F-statistic	3.554703	Durbin-Watson stat		1.854387
Prob(F-statistic)	0.000000			

While it can be confidently stated that this specification allows more heterogeneity than the country-specific pooled MGE the specification does not capture enough firm-specific return dynamics. It is therefore proposed that the methodology be significantly enhanced by estimating individual multi-factor models on a name-by-name basis, allowing maximum heterogeneity within the study's portfolio. Estimating individual regressions for all firms in the portfolio by applying regressor selection is indeed suggested as a possible approach by PSTW (2006:1239) to account for firm diversification and heterogeneity.

4.4.3 FIRM-SPECIFIC MULTI-FACTOR MODELS

As argued above, the multi-factor MGE model estimation procedure is enhanced by estimating firm-specific multi-factor models using variables from the South African-specific VECM as possible risk factors.

While such a process is modelling-intensive this study proposes overcoming some of the intensities associated with searching for the correct specification in each of the 145 firm multi-factor models by utilizing stepwise regression techniques. Stepwise regression techniques have been criticized by statisticians for a number of shortcomings. Most importantly *p*-values listed in the final regression output and all subsequent testing procedures do not account for the regressions that were run during the selection process which complicates interpretation of results. Other problems include an upwardly biased final R-squared, possibly upwardly biased coefficient estimates, and narrow confidence intervals. It is also often pointed out that the selection methods themselves use statistics that do not account for the selection process.

However, it is argued here that since a set of theoretical multi-factor specifications for each firm is not available, and barring actually searching for and specifying 145 individual specifications manually, step-wise regression provides the closest approximation for a specification procedure. In this study a set of estimation procedures is used which aims to limit the risk of incorrect specification due to the shortcomings of the stepwise technique.

A "stepwise-forwards" technique is used for finding the final estimation model for each firm specific multi-factor model. The stepwise-forwards methodology begins with no regressors in the regression, and then adds the variable with the lowest *p*-value as the first variable in the specification. The variable with the next lowest *p*-value given that the first variable has already been chosen, is then added. Next both of the added variables are checked against the backwards *p*-value criterion. Any variable whose *p*-value is higher than the criterion, is removed. Once the removal step has been performed, the next variable is added i.e. the variable with the lowest p-value. At this, and each successive addition to the model, all the previously added variables are checked against the backwards criterion and possibly removed. The stepwise-forwards routine ends when the lowest *p*-value of the variables not yet included is greater than the specified forwards-stopping criterion.

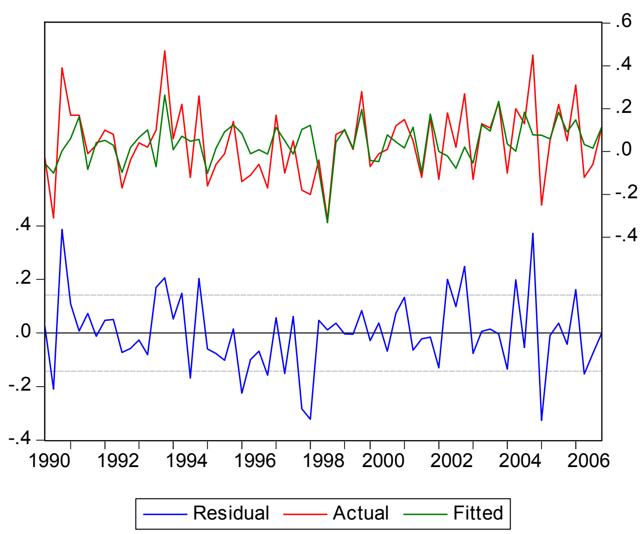
Variables are included based on a *p*-value specification test which allows variables within the multi-factor models with individual *p*-values smaller than 0.20 – although in general the final variables included in the multi-factor models have individual *p*-values below 0.15. An intercept is also included in all specifications in order to obtain the average return estimate for each company. In addition to the individual significance of variables, the overall combination of variables is evaluated using the F-statistic to ensure that the overall set of variables is jointly significant in explaining firm returns. Residual diagnostic tests such as the White-heteroskedasticity and Durbin-Watson serial correlation test are also performed on each model to test specification applicability. Although the set of search variables may contain variables that are collinear, these variables are dropped from the search set upfront. In a case where two or more of the search variables are collinear, the variable listed first in the list of search variables is selected. Based on *a priory* exogeneity expectations, it is proposed that the sequence of evaluation of variables is as follows (all variables are used in log difference form): ρ^*, q^*, y^*, e, ρ, q, y, d, h and p. Therefore although stepwise regressions are used as a tool to expedite the estimation procedure, several other model specifications and diagnostic tests were performed on a name-by-name basis before a final choice for a multi-factor model were made.

The process followed provides a total of 145 firm-specific multi-factor models, allowing the maximum degree of firm heterogeneity possible within the current framework. Model outputs and diagnostic estimates for 145 firm-return models are presented in Appendix A and shows that model specifications range across firms within the dummy portfolio. In general, the estimation procedure followed has increased the percentage of variation explained by the different models considerably, with adjusted R-squared being around 25 per cent for most specifications (increasing to as high as 65 per cent). As an example, the multi-factor model for one of the biggest cement producers in the country is provided below and the procedure followed is discussed in order to evaluate each multi-factor model individually.

The estimation output indicates that 31 per cent of the variation in firm returns is explained by the model specification. Although not fully compatible with the correlation assumptions in an asymptotic single risk-factor model which translates the R-squared estimate directly into an asset correlation, the R-squared estimate in the multi-factor model shows that a substantial proportion of firm returns can be attributed to the correlation with economic activities. This result again illustrates the importance and contribution made by this study in providing a mechanism through which credit risk portfolio managers can link the idiosyncratic component of firm risk to cyclical- or market-driven risk. The average quarterly return estimate (as illustrated by the significant constant term) is 3.8 per cent which can loosely be translated into a 16 per cent annual return assuming no change in the underlying macro economic factors. Assuming an average annual inflation rate of 6 per cent, the estimate is in line with a target real return of 10 per cent to investors. In general this is in line with most large corporations' target equity return of inflation plus 10 per cent.

Table 4.6 Example: individual multi-factor model

Dependent variable: Equity return Method: stepwise regression				
Independent variables	Coefficient	Std. Error	t-Statistic	Prob.
c *q* *e*	0.038278 0.876086 -0.465580	0.017744 0.165017 0.222017	2.157206 5.309061 -2.097052	0.0348 0.0000 0.0400
R-squared	0.318509	Mean dependent var		0.040455
Adjusted R-squared	0.296875	S.D. dependent var		0.169019
S.E. of regression	0.141727	Akaike info criterion		-1.025440
Sum squared resid	1.265450	Schwarz criterion		-0.925911
Log likelihood	36.83953	Hannan-Quinn criter.		-0.986111
F-statistic	14.72221	Durbin-Watson stat		2.320085
Prob(F-statistic)	0.000006			



White heteroskedasticity test:			
F-statistic	0.948290	Prob. F(5,60)	0.4567
Obs*R-squared	4.833623	Prob. Chi-Square(5)	0.4365
Scaled explained SS	6.077828	Prob. Chi-Square(5)	0.2987
Breusch-Godfrey serial correlation LM test:			
F-statistic	1.622092	Prob. F(2,60)	0.2095
Obs*R-squared	3.332849	Prob. Chi-Square(2)	0.1889
Normality test: Jarque-Bera	2.161070	Prob.	0.3395

The multi-factor model indicates that two additional factors are significant determinants of firm returns. As expected, the sign and size of the coefficient of real domestic equity returns, indicate that firm-specific returns are very closely related to overall movements in equity markets. The second significant factor included in the multi-factor model is the effective exchange rate which illustrates that depreciation in the exchange rate would lead to a loss in equity return, naturally this implies that the import component of the cost base of the production process is sensitive to the effective price of imported goods.

The F-statistic for overall significance of the regressors confirms the individual regressors' high t-statistics (low p-values) and indicates that the variables are jointly significant in explaining returns. Analysing the actual versus expected graph, it is clear that the model captures returns dynamics over the 17 year period and that the residual does not possess significant patterns pointing to some information not adequately accounted for. For this reason, the Durbin-Watson statistic for serial correlation and the White-test for residual heteroscedasticity are performed in order to formally test the residuals for any misspecification. The Durbin-Watson statistic of 2.32 falls just inside the acceptable range of the critical values implying that we cannot reject the null hypothesis of no serial correlation. No existence of serial correlation is also confirmed by the Breuch-Godfrey test for serial correlation, while the p-value of the White-test indicates that the null-hypothesis of no residual heteroskedasticity cannot be rejected.

Taking all factors into consideration, it can safely be stated that the multi-factor model estimated is appropriate and that it reflects the correlation of firm returns with the economy.

Following a similar process for all other multi-factor return models it can be concluded that they are all appropriate for inclusion in a stochastic simulation and forecasting model to determine firm-specific default dynamics. In conclusion it is argued that constructing individual return equations provides a significant enhancement over the pooled or even fixed or random effects panel estimation techniques normally performed in the literature and commercially available models. As such, the models would provide significant benefit in identifying true risk and return dynamics in a South African specific credit portfolio model context. Specifications and diagnostics of these individual multi-factor return models are presented in Appendix A.

4.5 CONDITIONAL LOSS ESTIMATION AND SCENARIO-ANALYSIS RESULTS

With the conditional return dynamics defined by means of the multi-factor models and the log-equity threshold levels determined from historical observations as provided by PSTW (2006), the conditional default and expected loss for the portfolio based on the macroeconomic VECM model as constructed in chapter 3 are simulated.

Expected loss and capital sensitivity analysis is generally conducted over a one year period and this study therefore only illustrates the conditional loss distribution estimates for the four quarters ahead forecast horizon (although one-to-three-quarter estimates are also available). From a practitioner’s perspective the simulated loss distribution is of particular interest since it allows inference to be drawn with respect to the likelihood of various loss events taking place over the forecast horizon. Contrary to the analytical approach which provides estimated default and expected loss estimates of exposures and portfolios, a simulated loss procedure provides significant information to the credit portfolio manager with respect to the magnitude of risks faced and the likelihood of such risks within the credit portfolio, since it provides a complete loss distribution from the simulation procedure. As such, while the analytical estimates have been used to test the reasonability of the simulated loss and individual default probability estimates, the results are not provided in this study.

The focus is rather placed on the simulated loss distribution and the practical application of such a conditional loss estimation process.

Using the methodology proposed by PSW (2004) and PSTW (2006), the conditional loss distribution of the dummy portfolio based on 95 000 simulations are generated. Since the main benefit of conditional loss simulation is to provide a direct way of estimating the impact of macroeconomic factors on portfolio loss, scenario analysis results are also constructed. In practise, portfolio expected loss scenario analysis is done based on either a single factor stress tests or sensitivity analysis or on a combination of stresses applied to a set of risk drivers to assess the combined effects of such a scenario on the portfolio. These scenario analyses are usually done through a level movement in the underlying risk factors which would move the whole distribution of factor simulations up or down. We therefore present both these sets of analysis: the sensitivity analysis is done by applying a 15 per cent up and down adjustment on the level of world equities in an attempt to assess the sensitivity of the portfolio to global effects. The scenario analysis focus on South African specific factors and tests the combined effects of shocking two variables simultaneously to create an "upturn" and "downturn" scenario. The upturn scenario is characterised by a 4 per cent decrease in interest rates combined with a 15 per cent increase in equity prices while the downturn is based on the opposite but equal magnitude increase and decrease in interest rates and equity prices.

However, credit risk does not only originate from the level of variables but also from the volatility of variables over time. In fact, as indicated already by the log equity threshold levels above the absolute levels across rating categories is not necessarily that different but the volatility and therefore the credit risk of the underlying assets increase as rating deteriorate. We therefore present a third scenario analysis to simulate a highly volatile economic environment over the forecast period by increasing and decreasing the standard deviation of the stochastic error terms of the exchange rate, South African and world equities equations in the VECM model by 1.5 and 0.5 times respectively.

Summary statistics of the conditional expected and unexpected (SD) loss estimate are provided in table 4.7 below. As illustrated by the baseline distribution mean of 0.31

per cent (31 bps), the simulated one-year conditional expected loss of the portfolio indicates that the portfolio has experienced positive migration over the one year horizon from the 0.53 bps expected loss estimate provided by prior expectations and benchmark estimates.

The scenario expected loss estimates show the asymmetric behaviour of the portfolio; for the level adjustment analyses the higher risk environment sensitivity and scenario analysis expected loss and standard deviation increases over baseline is much higher than the less risky environment results. However, from the volatility scenario analysis one can see that the expected loss and standard deviation changes relative to the baseline is quite similar.

Table 4.7 Conditional expected loss simulation summary results*

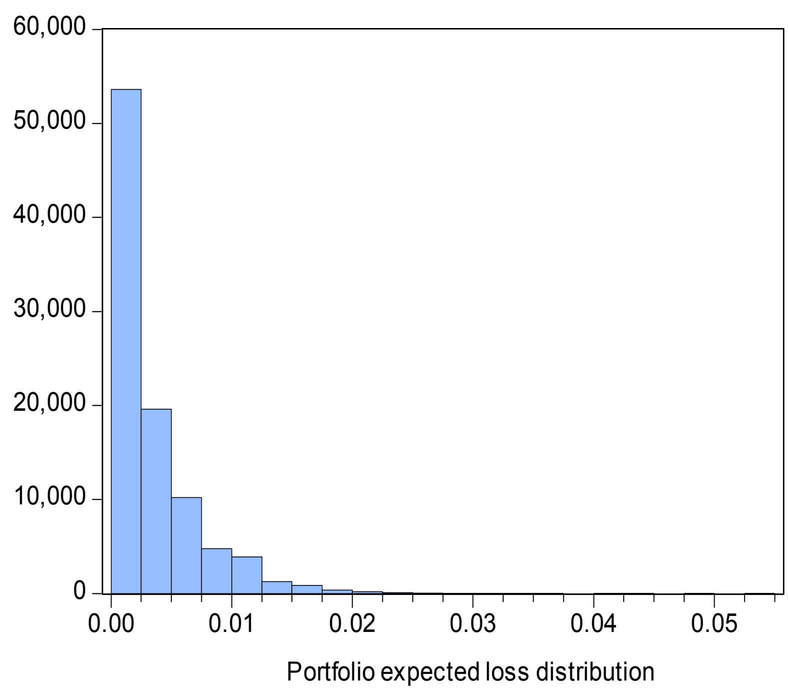
	Mean	SD	90th	99th	99.9th
Baseline	0.31	0.41	0.93	1.67	2.52
<u>Sensitivity analysis</u>					
World equity decrease	0.49 (0.18)	0.51 (0.10)	1.17 (0.24)	2.08 (0.41)	2.92 (0.40)
World equity increase	0.29 (-0.02)	0.35 (-0.06)	0.73 (-0.20)	1.41 (-0.26)	2.15 (-0.37)
<u>Scenario analysis</u>					
Downturn scenario	0.52 (0.21)	0.47 (0.08)	0.99 (0.07)	1.73 (0.06)	2.70 (0.18)
Upturn scenario	0.29 (-0.01)	0.40 (-0.01)	0.90 (-0.03)	1.62 (-0.05)	2.44 (-0.08)
<u>Volatility analysis</u>					
High volatility	0.43 (0.12)	0.50 (0.09)	1.08 (0.15)	2.05 (0.38)	3.16 (0.64)
Low volatility	0.20 (-0.11)	0.30 (-0.11)	0.57 (-0.36)	1.19 (-0.48)	1.76 (-0.76)

*Results presented in percentage terms with changes over baseline in brackets

The benefit of the simulation approach is apparent in its ability to provide insight into the severity associated with tail events or the “body” of the distribution. As an example; the losses associated with a 1 in 10, 1 in 100 and 1 in 1000 year event (i.e. the 90^{th}, 99^{th} and 99.9^{th} percentile values of the loss distribution) for each scenario are also presented in table 4.7.

These results clearly illustrate the asymmetric reaction of the loss distribution for similar positive and negative shocks i.e. an equal increase or decrease in the level of risk results in substantial higher increases in loss events in riskier environments than the decrease in loss in less riskier environments. Moreover, the difference between changes in scenario expected losses, over the baseline, increases marginally more the further in the tail one moves. However, an interesting result is than a decrease in volatility has a significantly more pronounced impact on expected losses than a similar increase in volatility. This might imply that the environment under which the portfolio is operating under the baseline scenario is significantly volatile already (i.e. the baseline simulations are already vary volatile) and that a reduction in this risk have a significant impact in credit riskiness of the portfolio.

Figure 4.1 Conditional loss distributions (baseline)



The conditional loss distribution is presented in figure 4.. The shape of the loss distribution clearly shows the asymmetric behaviour described above. The expected loss of the portfolio is comparatively small but the long tail of the distribution clearly indicates that there is a small probability of incurring a severe credit-related loss.

4.6 CONCLUSION

Credit portfolios are ultimately exposed to macroeconomic cycles even though idiosyncratic risk within a portfolio can to some extent be limited through diversification in a large corporate loan portfolio. In order to perform credit portfolio scenario analysis, the portfolio manager must be able to link firm-specific dynamics to macroeconomic factors through statistical models.

The main elements in such a framework include a structural macroeconomic risk driver engine, a default model which governs the default states within the macroeconomic environment and finally a translation function which transforms macroeconomic conditions into firm credit risk. In this study the South African specific VECM suggested in chapter 3 is used as the macroeconomic engine together with the Merton-type default model proposed by PSTW (2006) as evidence that conditional loss credit portfolio modelling is possible in South Africa. This study's credit portfolio model provides stochastic simulation results and allows for correlation between macroeconomic factors, the correlation of firms with these macro-factors as well as the correlation of firms amongst themselves. The commercial available methodology is extended in that an individual multi-factor model for each exposure in the portfolio is provided and it is then argued that the enhancement allows for more diversification to be recognised in the portfolio than what is assumed in normal asymptotic single risk factor type models.

The methodology provides a theoretically consistent and direct means of estimating credit risk as well as performing scenario analysis.

CHAPTER 5

SUMMARY AND CONCLUDING REMARKS

5.1 INTRODUCTION

During the late 1980s, 1990s and towards the end of the credit cycle in 2007 banks across the globe suffered large default experiences within their credit and traded credit portfolios. Driven by intense competition for market share, banks allowed credit portfolios to become less diversified (across all dimensions – country, industry, sector and size) and were willing to accept lesser quality assets on their books. As a result, even well-capitalised banks came under severe solvency pressure when global economic conditions turned. Banks soon realised the need for more sophisticated loan origination, and credit and capital management practices.

The core principle for addressing practical questions in credit portfolio management lies in the ability to link the cyclical or systematic components of firm credit risk to the firm's own idiosyncratic credit risk as well as the systematic credit risk component of every other exposure in the portfolio. Simple structural credit portfolio management approaches have opted to represent the general economy or systematic risk by a single risk factor. The systematic component of all exposures, the process generating asset values and therefore the default thresholds are homogeneous across all firms. Indeed, this Asymptotic Single Risk Factor (ASRF) model has been the foundation for Basel II. While the ASRF framework is appealing due to its analytical closed-form properties for regulatory and generally universal application in large portfolios, the single risk factor characteristic is also its major drawback. Essentially it does not allow for enough flexibility in answering real-life questions. Commercially available credit portfolio models make an effort to address this by introducing more systematic factors in the asset value-generating process but from a practitioner's point of view, these models are often a "black-box" allowing for little economic meaning or inference to be attributed to systematic factors.

This study aimed to address these shortcomings and provide a useable and practical application for credit portfolio management by using theoretical and empirical techniques from the economic, econometric, finance and credit risk disciplines. By combining the different disciplines, more insight has been obtained in terms of the impact of macro-economic or systematic credit risk on a South African credit portfolio.

5.2 SUMMARY RESULTS OVERVIEW

5.2.1 BASIC MODEL OUTLINE AND METHODOLOGY

Using macroeconometric models and linking them to the return process of individual firms, the credit portfolio model framework has been applied in the standard Merton-type credit portfolio default model context. Based on the methodology proposed by Pesaran, Schuermann, Treutler and Weiner (2006) a model for conditional credit losses has been provided which combines systematic risk with the idiosyncratic component of each exposure and also included an explicit channel for default correlation.

The methodology can be summarised as follows. The macroeconometric risk driver model specifies and represents the macroeconomic environment in which the credit portfolio operates. Using Monte Carlo simulations, various possible simulation paths of the economy are forecast over a forecast period. These macro-factors are fed into the firm-specific return models in order to obtain the value-generating process of each firm. Using the return dynamics and the estimated equity default thresholds from Pesaran, Schuermann, Treutler and Weiner (2006), the probability of default can be obtained using the structural-based credit default model described above. Finally a conditional loss distribution for the credit portfolio can be obtained and used to estimate various credit-related parameters such as economic capital, further allowing for various scenario analysis to be performed.

The practical appeal of the methodology is particularly appealing in that it is not only flexible in answering practical portfolio questions (through scenario analysis) but it also steers away from the data confidentiality problem that most practitioners face when using commercially-available credit portfolio models. The return of individual firms is driven by the domestic economy as well as the impact of the international business cycle. Using the firm's credit rating with this dynamic return process the portfolio loss distribution can be obtained through a much richer and more practically applicable means, allowing one to obtain more information through scenario analysis.

From an academic point of view the framework is appealing in that it bridges the gap between finance, credit and econometric literature and combines the three disciplines to provide a theoretically sound credit portfolio model.

5.2.2 EMPIRICAL RESULTS

The macroeconomic simulation engine or GVAR model used in PSTW (2006) comprises a total of 25 countries which are grouped into 11 regions and account for 80 per cent of world production. In the case of South Africa the GVAR model lacks applicability since it does not include an African component. Here a country-specific macroeconometric risk driver engine has been constructed which is compatible with and could feed into the GVAR model and framework of PSW (2004) using VECM techniques. On a stand-alone basis this allows for conditional loss estimation of a South African-specific credit portfolio. As part of a bigger credit portfolio model it opens the door for credit portfolio modelling on a global scale because the proposed VECM model can easily be linked into the GVAR model. Furthermore the set of domestic factors has been extended beyond those used in PSW (2004) in such a way that the risk driver model is applicable to both retail and corporate credit risk. As such, the model can be applied to a total bank balance sheet, incorporating the correlation and diversification between both retail and corporate credit exposures.

Assuming statistical identification restrictions as applied in PSTW (2006) the empirical results indicate a South African component for the GVAR model is possible and that such a component could easily be integrated into a global content. When economic over-identification restrictions are imposed the results becomes less encouraging. As such, the study reduces the dimensions of the VECM and proposes a smaller but theoretically more correct system. Although not as rich as the dimensions used in the GVAR model it is argued that this system is still a fair reflection of the macroeconomic variables which impact on the credit market and is also compatible with the GVAR model.

From an economic theory perspective it is important that the VECM results are consistent with theoretical expectations. The estimation results of the cointegration equations are of particular interest as they would govern the long-run relationship of the model simulations in the portfolio model and should therefore provide theoretically consistent estimates of the interaction between the variables. According to the Likelihood-ratio test, the Chi-square statistic of 6.95 (probability equal to 0.14) indicates that the theoretical constraints placed on the system are valid and binding and identify all cointegrating vectors. At the same time the R-squared values from the dynamic estimation output indicate that a significant portion of the variation in the dynamics of the variables is explained by the system. The long-run cointegration coefficient estimate results conform to economic theory and individual t-statistic results show a high degree of dependence between the dependent and independent variables.

As presented in chapter 3 the results of the general impulse response function (GIRF) analysis of the VECM are satisfactory. As an example, shocks to real world equity and output levels take approximately 20 quarters (i.e. 5 years) to result in new equilibrium levels to be achieved in the domestic market. This is consistent with the small open economy status of the South African economy and highlights the sensitivity of the domestic market to global market developments. The GIRF analysis is used in portfolio simulation analysis and forms an integral part of the credit portfolio simulation process. The favourable results from the GIRF analysis indicate that the VECM provided in this study has reasonable impact dynamics and that the model is appropriate for use in forecast and scenario generation within a credit portfolio model framework.

The results of the in- and out-of-sample stochastic simulations are also analysed. The simulation results show that the VECM predicts the actual realisation of the model variables with a high degree of accuracy and does not display any significant amount of bias in variable predictions. Moreover, although the actual variable realisation may have deviated from the model's predicted or expected outcomes, no such deviations fell outside the model's 2 standard-error confidence intervals. As such it is concluded that the stochastic model simulation results include appropriate simulation variance to capture and include model-forecast error and also provide additional variance to allow for unexpected economic events.

In order to estimate the impact of the macroeconomy on a bank loan portfolio, a fictitious South African-specific corporate loan portfolio was constructed. As discussed in chapter 4 the final portfolio consists of 145 exposures, spanning the rating spectrum in South Africa, all of which are listed on the Johannesburg stock exchange and were or are currently listed for at least 16 quarters i.e. 4 years. In order not to bias results towards any particular rating category or introduce any unduly large exposure concentration, exposure size have been assigned randomly for each counter party in the portfolio. In general, this portfolio can therefore be thought of as being representative of the benchmark or market portfolio of the South African corporate loan market.

As discussed in detail in chapter 2, individual firm return equations are used to translate the macroeconomic conditions into firm-specific return outcomes. These multifactor return equations can be estimated through least-squares analysis assuming that the firm-specific and macroeconomic innovations are uncorrelated. PSTW (2006) uses pooled mean group estimators (MGE) to estimate country-specific return regressors. The only heterogeneity within such a framework is generated through the differentiation between countries' firms with respect to macroeconomic factor sensitivities. More differentiation and therefore diversification can be facilitated through panel fixed effects or random effects or individual firm-return modelling. The multi-factor MGE model estimation procedure proposed by PSTW (2006) was enhanced by estimating name-by-name firm-specific multi-factor models using the variables from the South African-specific VECM as possible risk factors.

In order to inform and guide expectation into the final firm-specific return estimates, a pooled and then a fixed and random effects panel data model were estimated to test overall economy-wide return sensitivity to macroeconomic factors. Allowances were then made for maximum diversification between the 145 firms i.e. an individual return model for each firm was estimated.

The estimation process followed provided a total of 145 firm-specific multi-factor models, allowing the maximum degree of firm heterogeneity possible within the current framework. Individual return models are presented in Appendix A. In general, the estimation procedure followed has considerably increased the percentage of variation explained by the models relative to the pooled and effects panel data models, with adjusted R-squared statistics being around 25 per cent for most specifications (increasing to as high as 65 per cent). In conclusion it is argued that constructing individual return equations provides a significant enhancement over the pooled or even effects panel estimation techniques normally performed in the literature and found in commercially-available models. As such it is believed that the models would provide significant benefit in identifying true risk and return dynamics in a South African-specific credit portfolio model context.

Using the methodology proposed by PSW (2004) and PSTW (2006) the conditional loss distribution of the study's dummy portfolio based on 95 000 simulations was generated. Since the main benefit of conditional loss simulation is to provide a direct way of estimating the impact of macroeconomic factors on the portfolio loss, scenario analysis results of a positive and negative 2 standard deviation shock to the random errors of South African interest rates in the first quarter of 2007 were also provided. This represented an equal magnitude up and down movement in 2007Q1 interest rates.

Summary statistics of the conditional expected and unexpected (standard deviation) loss estimate illustrated that the baseline distribution mean portfolio loss was 0.54 per cent. The scenario analysis' expected loss estimates showed the asymmetric behaviour of the portfolio; the increasing interest rate shock scenario led to a 7 basis points (bps) increase in expected loss but an equal-sized decrease in interest rates only led to a 1 bps decrease in expected losses. Similar observations were observed for the standard deviation of the loss distribution which indicated that increasing interest rates increased the risk substantially more than the subsequent decrease in risk in a similar but decreasing interest rate environment.

The benefit of the simulation approach is apparent in its ability to provide some insight into the whole loss distribution such as the severity associated with tail events.

As an example; the loss associated with a 1 in 10, 1 in 100 and 1 in 1000 year event (i.e. the 90^{th}, 99^{th} and 99.9^{th} percentile values of the loss distribution) for each scenario were also inferred from the simulated loss distribution. The results more clearly illustrated the asymmetric reaction of the loss distribution for similar positive and negative interest rate shocks. Moreover, results indicated that the difference between the two scenarios' changes over the baseline increases marginally, the further movement into the tail. These results indicate that a negative interest rate shock leads to a proportionally bigger loss outcome than the gains obtained in loss outcomes in positive interest rate shock environments.

5.3 AREA OF FUTURE RESEARCH

Although it was not the aim of the VECM, developed in chapter 3, to provide forecast results for the global factors, but rather to provide South African specific elements to the GVAR model, in- and out-of-sample forecasts indicated that stochastic simulations were in line with actual variable realisation and expectations. As such, we could argue that the correlation model could be employed as a stand-alone model within a South African specific credit portfolio management tool. However, one limitation of the forecasting exercise performed is that it does not compare the forecasting performance of the benchmark VECM with that obtained from alternative standard, possibly atheoretical, econometric models. Ideally, as part of our future research, we would want to compare the predictive capabilities of the VECM with that of the Bayesian VAR (BVAR) and Bayesian VECM (BVECM). The motivation behind using Bayesian models is mainly due to the fact that, unlike standard classical VARs and VECMs, the Bayesian models can help us to retain the small open economy structure of the South African economy by setting up priors for the parameters of the model in a way that allows us to model the influence of foreign variables on domestic variables, but not the other way round. In other words, we can devise an interaction matrix in the Bayesian methodology that allows us to treat the domestic and foreign variables differently, and more along the lines of economic theory governing a small open economy. Besides, with the Bayesian models being estimated using the Theil's (1971) mixed estimation technique that involves supplementing data with prior information on the distribution of the coefficients, such that, for each restriction imposed on the parameter estimates the number of observations and degrees of freedom are increased by one, the loss of degrees of freedom due to over parameterization (associated with a VAR and VECM) is not a concern in the Bayesian models. This is likely to reduce the problem of overparameterization that could result in multicollinearity, inefficient estimates and, hence, large out-of-sample forecasting errors with the VARs and VECMs.

5.4 CONCLUSION

Active credit portfolio management is becoming a central part of capital management within the banking industry. Stimulated by the Basel II capital accord the estimation of risk sensitive credit capital is central to success in an increasingly competitive environment. If any risk-mitigation or value-enhancing activity is to be pursued, a credit portfolio manager must be able to identify the interdependencies between exposures in a portfolio but more importantly be able to translate macroeconomic credit risk into tangible portfolio effects.

In this study it is illustrated that it is not only possible to link macroeconomic factors to a South African-specific credit portfolio but that scenario and sensitivity analysis can also be performed within the credit portfolio model. These results can be used in credit portfolio management or stand-alone credit risk analysis which is ideal for practical credit portfolio management applications.

It is shown that for a fictitious South African corporate loan portfolio this credit portfolio model provides results that are significantly consistent with prior expectations based on S&P rating and default estimates. The scenario analysis results provided confirm the asymmetric behaviour of credit risk, i.e. negative economic shocks translate into proportionally much higher increases in portfolio risk than a decrease in risk from a similar positive economic shock.

The methodology provides a theoretically consistent and direct means of estimating credit risk as well as performing scenario analysis. Combining finance, economic, econometric and credit management disciplines the macroeconometric-based credit portfolio model makes a significant contribution to the credit risk management literature in South Africa and also adds a South African extension previously lacking from the PSTW (2006) application.