Validated leaf spring suspension models

by

Cor-Jacques Kat

Submitted in partial fulfilment of the requirements for the degree

Philosophiae Doctor (Mechanical Engineering)

in the

FACULTY OF ENGINEERING, THE BUILT ENVIRONMENT AND
INFORMATION TECHNOLOGY (EBIT)

UNIVERSITY OF PRETORIA
Pretoria

April 2012

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Denkleiers • Leading Minds • Dikgopolo tša Dihlalefi

Summary of thesis

Title: Validated leaf spring suspension models

Author: Cor-Jacques Kat

Supervisor: Prof. P.S. Els

Department: Mechanical and Aeronautical Engineering, University of Pretoria

Degree: Philosophiae Doctor (Mechanical Engineering)

Mathematical and computer modelling have been playing an increasingly important role in the Computer Aided Engineering (CAE) process. Simulation offers great advantages in the development and analysis phase of products and offers a faster, better and more cost effective way than using physical prototypes alone. The ever increasing demand for new and improved products in the vehicle industry has decreased the time available for the development of new vehicles, but at the same time the demands on quality, reliability and mass that are set for the vehicle are becoming ever more stringent. These requirements have lead to the investigation of procedures and methodologies such as virtual prototyping that will reduce the development time of new vehicles without inhibiting the quality of the vehicle.

In order to perform effective and reliable simulations in the CAE process, accurate simulation models of the vehicle and its associated systems, subsystems and components are required. In the vehicle dynamics context simulation models of the tyres, suspension, springs, damper, etc, are needed. This study will look at creating a validated model of a leaf spring suspension system used on commercial vehicles. The primary goal set for the model is to be able to predict the forces at the points where the suspension system is attached to the vehicle chassis as the model is to be used in full vehicle durability simulations. The component which will receive a considerable amount of attention in this study is the leaf spring. Leaf springs have been used in vehicle suspensions for many years. Even though leaf springs are frequently used in practice they still hold great challenges in creating accurate mathematical models. It is needless to say that an accurate model of a leaf spring is required if accurate full vehicle models are to be created.

As all simulation models in this study are required to be validated against experimental measurements a thorough experimental characterisation of the suspension system of interest, as well as two different leaf springs, are performed. In order to measure the forces between the suspension attachment points and the chassis, two six component load cells were developed, calibrated, verified and validated.

This study will primarily focus on the modelling of a multi-leaf spring as well as a parabolic leaf spring. The study starts with a literature study into the various existing modelling techniques for leaf springs. A novel leaf spring model, which is based on a macro modelling view point similar to that used for modelling material behaviour, is developed. One of the modelling techniques found in the literature, i.e. neural networks, is also used to model the leaf spring. The use of neural networks is applied and some of the challenges associated with the method are indicated. The accuracy and efficiency of the physics-based elasto-plastic leaf spring model and the non physics-based neural network model are compared. The modified percentage relative error metric is compared to two other quantitative validation metrics that were identified from the literature study. It is concluded that the modified percentage relative error has certain limitations but that it is able to give an accurate and representative account of the agreement/disagreement between two periodic signals around zero. The modified percentage relative error is used to obtain the accuracies of the elasto-plastic leaf spring models and the neural network model. Both models give good results with the neural network being almost 3 times more computationally efficient.

The elasto-plastic leaf spring model, for the multi-leaf spring, is further extended to model the behaviour of a parabolic leaf spring. Qualitative validation using experimental data shows that the elasto-plastic leaf spring model is able to accurately predict the vertical behaviour of both the multi-leaf spring as well as the parabolic leaf spring. The elasto-plastic leaf spring model was also combined with a method that is able to capture the effect of changes in the spring stiffness due to changes in the loaded length. Quantitative validation shows that the method proposed for accounting for the change in stiffness due to changes in the loaded length is able to capture this characteristic of the physical leaf spring.

Following a systematic modelling approach the elasto-plastic multi-leaf spring model is incorporated into a model of a simplified version of the physical suspension system. The qualitative validation results from this model show that the model is able to accurately predict the forces that are transmitted from the suspension system to the chassis. The models created in this study can be used in future work and, with the addition of more detail the models, can be extended to create a model of the complete suspension system.

Opsomming van proefskrif

Titel:	Gevalideerde bladveersuspensie modelle
Outeur:	Cor-Jacques Kat
Studieleier:	Prof. P.S. Els
Departement:	Meganiese en Lugvaartkundige Ingenieurswese, Universiteit van Pretoria
Graad:	Philosophiae Doktor (Meganiese Ingenieurswese)

Wiskundige- en rekenaargesteunde modellering se rol in die Rekenaargesteunde Ingenieursproses word al meer belangrik. Simulasie hou baie voordele in wanneer dit korrek gebruik word in die ontwikkeling en analise fase van produkte aangesien dit ‘n vinniger, beter en meer koste effektiewe manier is as slegs die gebruik van fisiese prototipes. Die toenemende aanvraag vir nuwe en beter produkte in die voertuigindustrie het die tyd wat beskikbaar is vir die ontwikkeling van nuwe voertuie verminder maar terselfdertyd het die vereistes t.o.v. kwaliteit, betroubaarheid en massa wat gestel word vir die voertuig, deurlopend strenger geword. Hierdie vereistes het gelei tot die ondersoek na prosedures en metodieke, soos virtuele prototipes, wat die ontwikkelingstyd van nuwe voertuie verminder sonder om die kwaliteit van die voertuig in te boet.

Om effektiewe en betroubare simulasies in die rekenaargesteunde ingenieursproses te kan doen, word akkurate simulasiemodelle van die voertuig en sy geassosieerde stelsels, substelsels, en komponente benodig. In die voertuigkonteks word simulasiemodelle van die bande, suspensie, vere, dempers, ens., benodig. Hierdie studie is gemik op die skep van gevalideerde modelle van die bladveer suspensiesisteem soos gebruik op kommersiële voertuie. Die primêre doelwit wat gestel word vir die modelle is dat hul in staat moet wees om die kragte te voorspel wat inwerk op die bakwerk waar die suspensie vasgeheg word aangesien die modelle gebruik gaan word in duursaamheidsimulasies. Die komponent wat ‘n noemenswaardige hoeveelheid aandag sal kry in die studie is die bladveer. Bladvere word al vir baie jare lank in voertuigsuspensies gebruik. Selfs al word bladvere gereeld gebruik in die praktyk, is daar nog steeds verskeie uitdagings om akkurate modelle van bladvere te skep. Dit is vanselfsprekend dat ‘n akkurate model van die bladveer benodig word indien ‘n akkurate volvoertuig model geskep wil word.

Aangesien alle simulasiemodelle wat in hierdie studie ontwikkel word, gevalideer word teen eksperimentele metings, is daar ‘n deeglike eksperimentele karakteriseringoefening uitgevoer. Die suspensiestelsel sowel as twee verskillende bladvere is gekarakteriseer. Om dit moontlik te maak om die kragte tussen die suspensiestelsel en die onderstel te kan meet is twee ses komponent lasselle ontwikkel, gekalibreer, geverifieer en gevalideer.

Hierdie studie fokus hoofsaaklik op die modellering van ‘n multi-blad bladveer sowel as ‘n paraboliese bladveer. Die studie begin met ‘n deeglike literatuurstudie wat ondersoek instel na die verskeie modelleringstegnieke wat tans bestaan vir bladvere. ‘n Unieke bladveer model, wat gebaseer is op ‘n makro modelleringsoogpunt, soortgelyk aan die tegniek wat gebruik word om materiaalgedrag te modelleer, is ontwikkel. Een van die modelleringstegnieke wat uit die literatuur geïdentifiseer is, nl. neurale netwerke, is ook gebruik om die bladveer te modelleer. ‘n Neurale netwerk is gebruik en van die uitdagings geassosieer met die metode word uitgewys. Die akkuraatheid en effektiwiteit van die fisika gebaseerde elasto-plastiese bladveer model en die nie-fisika gebaseerde neurale netwerk model is vergelyk. Die akkuraatheid is bereken deur ‘n nuwe kwantitatiewe validasiemaatstaf te gebruik wat ‘n intuïtiewe en verteenwoordigende aanduiding gee van die fout tussen twee seine. Die kwantitatiewe validasiemaatstaf is gebaseer op die bekende, en algemeen gebruikte, relatiewe fout. Die aangepaste persentasie relatiewe fout maatstaf wat ontwikkel is neem die uitdagings wat geassosieer is met die gebruik van die relatiewe fout, op seine met periodiese gedrag om nul, in ag. Die gemodifiseerde persentasie relatiewe fout word vergelyk met twee ander kwantitatiewe validasiemaatstawwe wat geïdentifiseer is uit die literatuurstudie. Die gevolgtrekking word gemaak dat die persentasie relatiewe fout sekere beperkings het maar dat dit ‘n akkurate en verteenwoordigende aanduiding van die ooreenkoms tussen twee periodiese seine om nul gee. Die gemodifiseerde persentasie relatiewe fout is gebruik om die akkuraatheid van die elasto-plastiese model en die neurale netwerk model te bepaal. Beide modelle gee goeie resultate, maar die neurale netwerk is omtrent drie keer meer berekeningseffektief.

Die elasto-plastiese bladveer model is ook gebruik om ‘n paraboliese bladveer se gedrag te modelleer. Kwalitatiewe validasie, met die gebruik van eksperimentele data, wys dat die elasto-plastiese bladveer model wel in staat is om die vertikale gedrag van beide die multi-blad bladveer sowel as die paraboliese bladveer te voorspel. Die elasto-plastiese bladveermodel is ook gekombineer met ‘n metode wat in staat is om die effek van die verandering in die veerstyfheid, as gevolg van veranderinge in die belaaide lengte, vas te vang. Kwantitatiewe validasie toon dat die metode wel die veranderinge in die veerstyfheid as gevolg van verandering in die belaaide lengte vasvang.

Volgens die sistematiese modelleringsaanslag wat gevolg is, is die elasto-plastiese bladveer model van die multi-blad bladveer geïnkorporeer in ‘n model van ‘n vereenvoudigde weergawe van die suspensiestelsel. Die kwalitatiewe validasie resultate van die model toon dat die model in staat is om die kragte, wat van die suspensiestelsel na die onderstel oorgedra word, akkuraat kan voorspel. Die model wat in die studie geskep is kan in toekomstige werk gebruik word en met die byvoeging van addisionele detail kan die modelle uitgebrei word om modelle te skep van die volledige suspensiestelsel.

Acknowledgements

This research was made possible through the support and funding received from Afrit and the Department of trade and industry's Technology and Human Resources for Industry Programme (THRIP) initiative.

The following people's contribution added to the successful completion of this research and I would like to extend my gratitude to:

- Gerrit van de Wetering, for his supportive role from Afrit and for the hours he generously spent in proof reading reports and articles.
- Schalk Els, for his friendship, mentorship and his guidance throughout my studies.

I will be forever grateful to the following people as they taught me the virtues that have helped formed me into the man I am today. Without them it would have been impossible to achieve success and meaning in life:

- To my mother, for teaching me courage, determination and perseverance.
- To my grandfather, for teaching me integrity and to be just.
- To my brother Tjaart, for teaching me loyalty.
- To my brother Arné, for teaching me patience.
- To my grandmother, for teaching me commitment and consideration
- To my friend Tjaart van der Walt, for teaching me the importance of dependability.
- To my girlfriend Somarié, for strengthening my believes and principles

I can do everything through Him who gives me strength

Philippians 4:13

Table of contents

Summary of thesis III
Opsomming van proefskrif V
Acknowledgements VII
List of symbols XV
List of abbreviations XXI
List of figures XXIII
List of tables XXXI

Chapter 1 – Introduction **1**

1. Problem statement 3
2. Introduction to suspension system of interest 3
3. Literature study 6
    - 3.1. Leaf spring models in previous studies 7
        - 3.1.1.Analytical/Empirical models 7
        - 3.1.2.Equivalent models 8
        - 3.1.3.Discrete methods (or finite segment methods) 9
        - 3.1.4.Finite element methods 11
        - 3.1.5.Neural network models 13
    - 3.2. Summary of leaf spring modelling techniques 14
    - 3.3. Conclusion 18
4. Overview of study 19

Chapter 2 – Experimental characterisation 21

1. Six component load cell (6clc) 21
2. Characterisation of the suspension system using the multi-leaf spring 23

2.1. Force-displacement characteristic 24
    2.1.1.In-service setup 27
        2.1.1.1.Effect of U-bolt preload on the force-displacement characteristic 30
    2.1.2.Spring only setup 31
        2.1.2.1.Effect of longitudinal spacing of hangers 33
        2.1.2.2.Deflection shape of the multi-leaf spring 34
3. Characterisation of the suspension system using the parabolic leaf spring 36
    3.1. Force-displacement characteristic 36
        3.1.1.In-service setup 37
            3.1.1.1.Effect of U-bolt preload on force-displacement characteristics 38
        3.1.2.Spring only setup 39
            3.1.2.1.Effect of longitudinal spacing of hangers 40
            3.1.2.2.Deflection shape of the parabolic leaf spring 41
4. Conclusion 42

Chapter 3 – Leaf spring modelling 43
1. Introduction 43
2. Elasto-plastic leaf spring model 44
    2.1. The behaviour of materials and leaf springs 44
        2.1.1.Deformation behaviour and models of materials 45
        2.1.2.Mechanisms in crystalline materials vs. mechanisms in multi-leaf springs 46
            2.1.2.1.Mechanisms in crystalline materials 46
            2.1.2.2.Mechanisms in multi-leaf springs 47
            2.1.2.3.Solid-solid contact (Tribological process) 49
            2.1.2.4.Conclusion 51
    2.2. Mechanical properties of a multi-leaf spring 51
    2.3. Elasto-plastic leaf spring models 54
        2.3.1.Elastic-linear model 54
        2.3.2.Elastic-nonlinear model 55
    2.4. Validation of the elasto-plastic leaf spring model 57
        2.4.1.Elastic-linear model 57
        2.4.2.Elastic-nonlinear model 59
    2.5. Conclusion 60
3. Elasto-plastic leaf spring model applied to the parabolic leaf spring 60
    3.1. Extracting mechanical properties for the elastic-linear parabolic leaf spring model 61

3.2. Validation of elastic-linear leaf spring model emulating the parabolic leaf spring 62
3.3. Conclusion 64
4. Loaded length changes of a simply supported leaf spring 64
4.1. Method to account for loaded length changes 64
4.2. Validation of loaded length calculation combined with EPLS model 66
4.2.1.Multi-leaf spring 67
4.2.2.Parabolic leaf spring 67
4.3. Conclusion 68
5. Artificial neural networks 69
5.1. Neural network model 70
5.1.1.Reducing noise on neural network predictions 72
5.1.2.Generalization 73
6. Conclusion 80

Chapter 4 – Multi-leaf spring suspension system model 81
1. Introduction 82
2. Modelling of the spring only setup 83
2.1. ADAMS/Car leaf spring model 83
2.1.1.ADAMS/Car leaf spring Model 1 84
2.1.2.ADAMS/Car leaf spring Model 2 84
3. Validation of the spring only model 87
3.1. Validation of the spring only model using Model 1 87
3.2. Validation of the spring only model using Model 2 89
4. Conclusion 92

Chapter 5 – Verification and Validation 93
1. Introduction 93
2. Qualitative validation metrics 97
2.1. Literature survey 98
2.1.1.Russell’s error measure 101
2.1.2.Sprague & Geers’ metric 101
2.2. Validation metric based on relative error 102
2.2.1.Relative error (*RE*) 102
2.2.2.Challenges in using the *%RE* as validation metric 103
2.2.3.Summary of the modified *%RE* validation metric 111

3. Comparison of validation metrics 112
    3.1. Analytical functions 113
        3.1.1. Ability to rank models and identify the best model 114
        3.1.2. Reliability and usefulness of validation metrics 116
        3.1.3. Combination of S&G and the modified *%RE* 117
    3.2. Case studies 118
        3.2.1. Case study 1: Known error between signals 119
        3.2.2. Case study 2: Elasto-plastic leaf spring model 120
        3.2.3. Case study 3: Comparison of accuracy and efficiency of leaf spring modelling methods 122
4. Conclusion 124

Chapter 6 – Conclusions and Recommendations 127
1. Conclusions 127
2. Recommendations 128
    2.1. Chapter 2 128
    2.2. Chapter 3 129
        2.2.1. Elasto-plastic leaf spring model 129
        2.2.2. Neural network model 130
    2.3. Chapter 4 130
    2.4. Chapter 5 131

Bibliography 133

Appendix A – Six component load cell (6clc) 139
A.1. Calibration of uni-axial load cells 139
A.2. Verification and Validation of the physical and virtual 6clc 144
    A.2.1. Verification of 6clc 145
        A.2.1.1. Derivation of analytical equations 145
        A.2.1.2. Verification of 6clc concept 149
        A.2.1.3. Verification of the 6clc ADAMS/Car model 150
    A.2.2. Validation of 6clc models 153
        A.2.2.1. Load case 1 and Load case 2 154
        A.2.2.2. Load case 3 157
        A.2.2.3. Load case 4 158

A.2.3. Model refinement 160
A.2.3.1. Orientation of applied force 161
A.2.3.2. Application point of applied force 162
A.2.4. Validation results for the rear 6clc 167
A.3. Conclusion 171

Appendix B – Theoretical stiffness of the multi-leaf spring

B.1. Calculating the theoretical stiffness 173
B.1.1. Principle of superposition 174
B.1.2. SAE spring design manual 176
B.1.3. Calculating equivalent spring stiffness 177
B.1.3.1. Symmetrical loading 178
B.1.3.2. Asymmetrical loading 179
B.1.3.3. Conclusion 181
B.1.3.4. Neglecting the initial angle of the cantilevers 182
B.1.4. Validation of theoretical stiffness calculation 183
B.2. Effect of the clamping assumption on the theoretical stiffness 184
B.3. Additional validation tests 187
B.3.1. Test 1: 3 blade, clamped length = 0.076m 187
B.3.2. Test 2: 3 blade, clamped length = 0.22m 188
B.3.3. Test 3: 3 blade, clamped length = 0m 191
B.4. Conclusion 192

List of symbols

English symbols:

A	Cross-sectional area	[m^2]
a	Length between applied force and the front support	[m]
b	Length between applied force and the rear support	[m]
C_R	Russell's comprehensive error	
$C_{S\&G}$	Sprague & Geers' comprehensive error	
d_{X1y}	Distance in y-direction from the centre of volume of the 6clc to the line of action of the 1st uni-axial load cell in the longitudinal direction (X_1)	[m]
d_{X2y}	Distance in y-direction from the centre of volume of the 6clc to the line of action of the 2nd uni-axial load cell in the longitudinal direction (X_2)	[m]
d_{X12z}	Distance in z-direction from the centre of volume of the 6clc to the line of action of both uni-axial load cells in the longitudinal direction (X_1 and X_2)	[m]
d_x	Distance in x-direction from centre of volume to the application point of the applied force (F_A)	[m]
d_{Yx}	Distance in x-direction from the centre of volume of the 6clc to the line of action of the uni-axial load cell in the lateral direction (Y)	[m]
d_{Yz}	Distance in z-direction from the centre of volume of the 6clc to the line of action of the uni-axial load cell in the lateral direction (Y)	[m]
d_y	Distance in y-direction from centre of volume to the application point of the applied force (F_A)	[m]
d_{Z1x}	Distance in x-direction from the centre of volume of the 6clc to the line of action of the 1st uni-axial load cell in the vertical direction (Z_1)	[m]

d_{Z23x}	Distance in x-direction from the centre of volume of the 6clc to the line of action of the 2nd and 3rd uni-axial load cell in the vertical direction (Z_2 and Z_3)	[m]
d_{Z2y}	Distance in y-direction from the centre of volume of the 6clc to the line of action of the 2nd uni-axial load cell in the vertical direction (Z_2)	[m]
d_{Z3y}	Distance in y-direction from the centre of volume of the 6clc to the line of action of the 3rd uni-axial load cell in the vertical direction (Z_3)	[m]
d_z	Distance in z-direction from centre of volume to the application point of the applied force (F_A)	[m]
E	Young's modulus	[Pa]
F	Force	[N]
F_A	Force applied to 6clc	[N]
F_{Ax}	Component of force applied to 6clc in x-direction	[N]
F_{Ay}	Component of force applied to 6clc in y-direction	[N]
F_{Az}	Component of force applied to 6clc in z-direction	[N]
$F_{applied}$	Applied force	[N]
F_x	Equivalent force in x-direction that acts on the centre of volume of the 6clc	[N]
F_{xF}	Force in longitudinal direction at front hanger	[N]
F_{xR}	Force in longitudinal direction at rear hanger	[N]
F_y	Equivalent force in y-direction that acts on the centre of volume of the 6clc	[N]
F_z	Equivalent force in z-direction that acts on the centre of volume of the 6clc	[N]
$F_{y,elaslin}$	Elastic-linear frictional yield force	[N]
$F_{y,elaslin,L}$	Elastic-linear frictional yield force when leaf spring is loaded	[N]
$F_{y,elaslin,UL}$	Elastic-linear frictional yield force when leaf spring is unloaded	[N]
$F_{y,L}$	Frictional yield force when leaf spring is loaded	[N]
$F_{y,UL}$	Frictional yield force when leaf spring is unloaded	[N]

F_u	Ultimate frictional yield force	[N]
$F_{u,L}$	Ultimate frictional yield force when leaf spring is loaded	[N]
$F_{u,UL}$	Ultimate frictional yield force when leaf spring is unloaded	[N]
$F_{preload}$	Force due to preload in U-bolts	[N]
F_s	Spring force	[N]
F_{zF}	Force in vertical direction at front hanger	[N]
F_{zR}	Force in vertical direction at rear hanger	[N]
f_y	Yield fraction	[Dimensionless]
$f_{y,L}$	Yield fraction when leaf spring is loaded	[Dimensionless]
$f_{y,UL}$	Yield fraction when leaf spring is unloaded	[Dimensionless]
I	Area moment of inertia	[m^4]
k_a	Stiffness of front cantilever beam	[N/m]
k_b	Stiffness of rear cantilever beam	[N/m]
k_{eq}	Equivalent stiffness of cantilever beams in parallel	[N/m]
k_L	Stiffness of the layered beam during loading	[N/m]
k_{UL}	Stiffness of the layered beam during unloading	[N/m]
δk	Incremental change in stiffness	[N/m]
δk_1	1st incrementally changes stiffness	[N/m]
δk_2	2nd incrementally changes stiffness	[N/m]
L	Loaded length	[m]
l	length	[m]
l_f	Length between axle seat and front hanger	[m]
l_r	Length between axle seat and rear hanger	[m]
M_x	Equivalent moment about the x-axis that acts on the centre of volume of the 6clc	[N.m]
M_y	Equivalent moment about the y-axis that acts on the centre of volume of the 6clc	[N.m]

M_z	Equivalent moment about the z-axis that acts on the centre of volume of the 6clc	[N.m]
M_R	Russell's magnitude error	
$M_{S\&G}$	Sprague & Geers' magnitude error	
m	Measured signal	
N	Number of data point is signal	
n	Value in neuron that is sent to transfer function	
P	Probability	
P	Applied force	[N]
P_R	Russell's phase error	
$P_{S\&G}$	Sprague & Geers' phase error	
p	Predicted signal	
TP_L	Turning point which indicates the change from unloading to loading	[N]
TP_{UL}	Turning point which indicates the change from loading to unloading	[N]
V	Relative error bounded by the tanh function	
X_1	Force measured in 1st uni-axial load cell orientated in longitudinal direction	[N]
X_2	Force measured in 2nd uni-axial load cell orientated in longitudinal direction	[N]
x	Displacement (or deflection)	[m]
Y	Force measured in uni-axial load cell orientated in lateral direction	[N]
Z_1	Force measure by the 1st uni-axial load cell orientated in vertical direction	[N]
Z_2	Force measure by the 2nd uni-axial load cell orientated in vertical direction	[N]
Z_3	Force measure by the 3rd uni-axial load cell orientated in vertical direction	[N]

Greek symbols:

α	Angle of slope at contact points between leaf spring and hanger	[°]
α_f	Angle of slope at contact points between leaf spring and front hanger	[°]
α_r	Angle of slope at contact points between leaf spring and rear hanger	[°]
ε	Strain	[Dimensionless]
ε_e	Elastic strain	[Dimensionless]
ε_p	Plastic strain	[Dimensionless]
σ_y	Yield stress	[Pa]
σ	Stress	[Pa]
θ	Angle between resultant force and horizontal line that goes through the contact point	[°]
υ	Deflection of beam at applied force	[m]
$\upsilon_F^{'}$	Slope of beam at applied force	

List of abbreviations

acar	ADAMS/Car
ADAMS	Automatic Dynamic Analysis of Mechanical Systems
ASTM	American Society for Testing and Materials
CAD	Computer Aided Design
CAE	Computer Aided Engineering
cv	centre of volume
DTW	Dynamic Time Warping
EPLS	Elasto-Plastic Leaf Spring
HRC	Rockwell hardness, C-scale
HV	Vickers hardness
HB	Brinell hardness
IEEE	Institute of Electrical and Electronics Engineers
Inf	Infinite
MBS	Multi-Body Simulation
m%RE	modified percentage relative error
m%RE^{m}	modified percentage relative error defined by the mean of the percentage relative error
m%RE^{s}	modified percentage relative error defined by a specific percentage relative error
NaN	Not-a-Number
NISE	Normalized Integral Square Error

NN	Neural Network
RE	Relative Error
%RE	Percentage relative error
rme	relative magnitude error
SF	Stiffening Factor
SME	Subject Matter Expert
SRQ	System Response Quantity
SRQ^{m}	Measured system response quantity (obtained from physical system)
SRQ^{p}	Predicted system response quantity (obtained from simulation model)
V&V	Verification and Validation
6clc	Six component load cell

List of tables

Chapter 1 – Introduction

Table1.1. Summary of leaf spring modelling techniques 15

Chapter 2 – Experimental characterisation

Table 2.1. Spacing of hangers 34

Chapter 3 – Leaf Spring Modelling

Table 3.1. Elastic-linear equations 55
Table 3.2. Turning point force, frictional yield force and yield fraction 56
Table 3.3. Elastic-nonlinear equations 57
Table 3.4. Dimensions of layouts (see Figure 3.24) 67

Chapter 4 – Multi-leaf spring suspension system model

Table 4.1. Angle of slope at front contact point 86
Table 4.2. Angle of slope at rear contact point 86

Chapter 5 – Verification and Validation

Table 5.1. Summary of Error Measures and Metrics 99
Table 5.2. Effect of $\%RE$ not being bounded on the results of the $m\%RE^m$ (Not bounded) 108
Table 5.3. Effect of $\%RE$ not being bounded on the results of the $m\%RE$ (Bounded) 109
Table 5.4. Known $\%RE$ between true and approximate data 110
Table 5.5. Results for the $m\%RE^m$ using different *%RE threshold* values 110
Table 5.6. Summary of the two formulations of the modified $\%RE$ validation metric 112
Table 5.7. Equation for the various analytical functions 113
Table 5.8. Ranking of comparisons by different validation metrics (Functions 1 to 8) 114
Table 5.9. Ranking of comparisons by SME's (Functions 1 to 8) 114
Table 5.10. Ranking of comparisons by different validation metrics (Functions 9 to 15) 115
Table 5.11. Ranking of comparisons by SMEs (Functions 9 to 15) 115
Table 5.12. Comparison between the error measures' ability to quantify the accuracy (Function 21(a) and 21(b)) 116

Table 5.13. Comparison between the error measures' ability to quantify the accuracy (Function 22(a) and 22(b)) 117
Table 5.14. Comparison between error measures for models with same phase shift but different magnitudes 118
Table 5.15. Relative error between Model 1, Model 2 and the measured data 119
Table 5.16. Comparison between error measures for known *%RE* 120
Table 5.17. Results with noise on measured data around zero 121
Table 5.18. Results with noise on measured data around zero removed 122
Table 5.19. Results with noise on measured data around zero removed and Model 1 refined 122
Table 5.20. Accuracy of elastic-nonlinear and neural network model 123
Table 5.21. Accuracy of elastic-nonlinear and neural network model for the inner loop only 124

Chapter 6 – Conclusions and Recommendations

No tables in this chapter

Appendix A – Six component load cell (6clc)

Table A.1. Cross-sectional area of the uni-axial load cells 140
Table A.2. Results of harness tests 141
Table A.3. Calibrated Young's modules 144
Table A.4. Load cases used in verification process 145
Table A.5. Results from analysis with no load applied to 6clc 151
Table A.6. Maximum difference between analytical results and ADAMS/Car results (Load case 1) 152
Table A.7. Maximum difference between analytical results and ADAMS/Car results (Load case 2 to 4) 153
Table A.8. Application point 164
Table A.9. Application point (Rear 6clc) 168

Appendix B – Theoretical stiffness of the multi-leaf spring

Table B.1. Equation for calculating stiffness of prismatic and non-prismatic cantilever beams 176
Table B.2. Results from two cantilevers compared with results from simply supported beam 181
Table B.3. Equations for calculating stiffness of cantilevers (Neglecting initial angle of rotation) 182
Table B.4. Results from two cantilevers (initial angle of rotation neglected) compared with results from simply supported beam 183
Table B.5. Blade thickness measurements 185